# Dissipation Factors of Fused-Silica Capacitors in the Audio Frequency Range

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***Abstract*—**We describe dissipation factor measurements of 10 pF fused-silica capacitance standards from 50 Hz to 20 kHz, using a toroidal cross capacitor and a 10 pF nitrogen-filled capacitor as the references. The relative combined standard uncertainties are $0.56\times10^{-6}$, $0.16\times10^{-6}$, and $0.26\times10^{-6}$ at 100 Hz, 1 kHz, and 10 kHz, respectively.

***Index Terms*—Capacitance, cross-capacitor, dielectric loss, dissipation factor, farad, frequency dependence, fused-silica capacitor.**

# I. INTRODUCTION

We have recently reported progress on determining the frequency dependence of capacitance standards in the audio frequency range [1]. This effort is in part a response to industrial needs. Recently, ultra-precision multi-frequency (from 50 Hz to 20 kHz) capacitance bridges have become commercially available and secondary calibration laboratories have started using this new type of bridge for their impedance calibrations. Another closely related calibration need is the determination of dissipation factors of capacitance standards, needed not only for calibrations of capacitance bridges, but also for LCR meters and network analyzers. New measurement capabilities of dissipation factors will also enable improved traceability of energy and power measurements and better characterization of dielectric materials.

Dissipation factors of capacitors have previously been studied at NIST for various applications. Astin [2] studied loss mechanisms of air capacitors at 60 Hz, 200 Hz, and 1000 Hz, achieving uncertainties as low as $0.5\times10^{-6}$. Shields [3] established a dissipation factor standard using a 0.5 pF toroidal cross capacitor, $C_{0.5}$, with an estimated uncertainty of $0.02\times10^{-6}$ at 1592 Hz. So and Shields [4] used a variable parallel-plate guard-ring capacitor as the reference for dissipation factor measurements, achieving uncertainties as low as $0.01\times10^{-6}$ at 1592 Hz. However, these previous studies have yet to be extended to other frequencies in the audio frequency range. Absolute determinations of the dissipation factor of capacitors have also been carried out in other National Metrology Institutes. Inglis [5] performed a thorough study of electrode surface film effects on the frequency dependence and dissipation factor of parallel-plate capacitors in the frequency range from 11 Hz to

52 kHz. He found that the dissipation factor of a well cleaned parallel-plate capacitor with a 1 mm vacuum gap is less than $1.5 \times 10^{-7}$ over the frequency range. His work was motivated by the need of a new electrostatic ac-dc transfer standard and unfortunately, his dissipation factor measurements have apparently not been transferred to other capacitance standards. With a variable parallel-plate guard-ring capacitor as the reference, Eklund [6] has determined the dissipation factors of 100 pF capacitors at 1 kHz, 4 kHz, and 10 kHz. Using a combination of equivalent circuit modeling for capacitor and ac resistor, a programmable two-channel ac voltage source and a sampling voltmeter, Ramm and Moser [7] have developed a multi-frequency method for determining the dissipation factor of capacitors and the time constant of resistors simultaneously below a few kHz with uncertainties of $6 \times 10^{-7}$.

## II. KRONIG-KRAMERS RELATIONS

For a simple parallel-plate capacitor, the dominant loss mechanisms include the dielectric loss between the electrodes and the resistance of the electrodes and their leads. Conductivity of a dielectric can be written as two components: $\sigma_o + \omega\varepsilon''(\omega)$, where the first term results from the dc conductivity and the second term is due to dielectric relaxation with $\varepsilon''$ being the imaginary part of the dielectric constant. When the dc conductivity is negligible as is the case for fused-silica capacitors, the dissipation factor

$$\tan\delta = \frac{G}{\omega C} = \frac{\varepsilon''}{\varepsilon'}, \qquad (1)$$

where $C$ is the capacitance and $G$ is the conductance of the dielectric and $\varepsilon'$ is the real part of the dielectric constant. $\varepsilon''$ and $\varepsilon'$ are related via the Kronig-Kramers relations:

$$\varepsilon'(\omega) - \varepsilon_\infty = \frac{2}{\pi}\int_0^\infty \frac{\varepsilon''(u)u^2}{\omega^2 - u^2} d\ln u\,. \qquad (2)$$

Approximating the integral factor $u^2/(\omega^2\text{-}u^2)$ by the unit-step function [8], we obtain

$$\varepsilon'(\omega) - \varepsilon_\infty = \frac{2}{\pi}\int_\omega^\infty \varepsilon''(u) d\ln u\,. \qquad (3)$$

This derivation effectively assumes that the distribution of the dielectric relaxation times is very broad, leading to relatively flat curves of $\varepsilon''(\omega)$ and $\varepsilon'(\omega)$. Differentiating Eq. (3), we have

$$\tan\delta = -\frac{\pi}{2}\frac{d\varepsilon'(\omega)}{\varepsilon' d\ln\omega}\,. \qquad (4)$$

Eq. (4) is useful for estimating the dissipation factor of a capacitance standard from its frequency dependence of capacitance. For example, the relative capacitance change due to electrode surface films of a parallel-plate capacitor tends to follow a ln ($\omega$) law over the audio frequency range [5]. Using this frequency dependence for $\varepsilon'(\omega)$ in Eq. (4), we conclude that the dissipation factor tan δ of the capacitor is a constant in the frequency range, in agreement with the experimental observations.

## III. EXPERIMENT

The ultimate reference for dissipation factor measurements at NIST is the toroidal cross capacitor $C_{0.5}$ which is made of stainless steel and sealed in a vacuum housing [3]. It has been shown that the net contributions of thin dielectric films on the electrodes of such a cross capacitor to the dissipation factors of the two cross capacitances are negligible to the first order. The toroidal arrangement also contains another cylindrical 10 pF capacitance, $C_{10}$, between two of the four active electrodes. The electrode separation of $C_{10}$ is about 3 mm. Since all electrodes were made from the same material and were finished and cleaned in the same manner, Shields was able to determine that $C_{10}$ has a dissipation factor less than $0.02\times10^{-6}$ at 1592 Hz with comparable uncertainty. The dissipation factor due to the dielectric films is inversely proportional to the electrode separation and is less than

$0.15\times10^{-6}$ in the audio frequency range when the electrodes are well cleaned and their separation is 1 mm or more [3, 5]. Comparing $C_{10}$ with an identically made capacitor shows that the difference of their dissipation factors is within the detection limit in the audio frequency range. Comparing with another 10 pF nitrogen-filled cylindrical capacitor whose frequency dependence of capacitance had been determined earlier with respect to a 1 pF cross capacitor shows that the frequency dependence of $C_{10}$ is no more than $0.2\times10^{-6}$ per decade change in frequency. Using this estimate of frequency dependence in Eq. (4), we conclude that the dissipation factor of $C_{10}$ is less than $0.14\times10^{-6}$ in the audio frequency range. Simple substitution techniques are employed to measure dissipation factors of 10 pF fused-silica capacitors with respect to $C_{10}$, using ac bridges which have been described previously [1].

In principle, the calculable capacitor could also serve as the reference for dissipation factor measurements at various frequencies. However, this approach is very tedious to cover the entire audio frequency range. The NIST calculable capacitor involves difference measurements between 0.2 pF and 0.7 pF; calibration of the bridge transformer used for comparison with other standards involves multiple steps at each frequency. The ac bridge system for the NIST calculable capacitor has been calibrated only for in-phase capacitance measurements at 1000 Hz and 1592 Hz. Nevertheless, the bridge transformer errors at 1592 Hz appear stable within a few parts in $10^9$ over a period of more than 20 years, the twice annual calculable capacitor measurements at NIST have recorded relative variations of dissipation factors of several fused-silica capacitors. Typical results will also be discussed in the next section.

## IV. RESULTS AND UNCERTAINTY ANALYSIS

Shown in Fig. 1 is the measured dissipation factor of a 10 pF fused-silica transfer standard, $C_{112}$, as a function of frequency from 50 Hz to 20 kHz. The main sources of uncertainties for the measurements are listed on Table 1 for four representative frequencies. The Type A uncertainty, which is directly linked to the signal-to-noise ratio of the ac bridge systems and the stabilities of the standards, dominates at low frequencies. The reference standard $C_{10}$ is a four-terminal-pair capacitor, and its loss due to the leads and contacts is negligible in the frequency range. However, $C_{112}$ is a three terminal capacitor and its lead resistance is the dominant loss mechanism at high frequencies. In the frequency range from 300 Hz to 6 kHz, the uncertainties of the reference standard dominates. The relative combined standard uncertainties are shown in Fig. 1 together with the dissipation factor data.

Also shown in Fig. 1 is the estimated dissipation factor below 1592 Hz using Eq. (4) and the frequency dependence of capacitance of $C_{112}$ measured earlier [1]. The comparison is restricted to the low frequency region where the dominant source of frequency dependence results from dielectric relaxation. The leads effect becomes significant above 1592 Hz, and we have not attempted to separate the contributions from the two sources.

Shown in Fig. 2 are the measured dissipation factors of four commercial 10 pF fused-silica capacitance standards as a function of frequency from 50 Hz to 20 kHz. The dissipation factors of these standards are uniformly within a few parts in $10^7$ below 1592 Hz; they increase with frequency to a few parts in $10^6$ at 20 kHz with comparable variations from one standard to another. This increase is partly due to the series lead and electrode resistance. However, if we assume the combined series resistance of 0.1 Ω and the combined stray capacitance of 100 pF, the loss is no more than $1 \times 10^{-6}$ at 16 kHz. The observed increase and variations of dissipation factors could be partly due to the dielectric loss in the fused-silica. The dielectric properties of fused-silica have been known to be strongly influenced by the residual hydroxyl content. Dielectric loss peaks as large as several parts in $10^4$ have been observed in the audio frequency range in fused-silica samples at low temperatures (below 4.2 K) and were attributed to a distribution of dipolar two-level systems of hydroxyl anions having energy splittings comparable to the thermal energies of the experiments. Such dipolar systems may also make observable contributions to the dielectric loss of the room temperature fused-silica in the present case.

Variations of the dissipation factor of $C_{112}$ relative to the calculable capacitor at 1592 Hz are shown in Fig. 3 as a function of time from 1988 to present. As can be seen in the figure, the dissipation factor remains stable within $5\text{x}10^{-8}$ over the period. Comparisons of the presently measured dissipation factors of four 10 pF fused-silica standards with those measured by So [10] more than 30 years ago for these same standards are shown in Table 2. The largest change is seen with $C_{127}$, about $1.3\text{x}10^{-7}$, while the dissipation factor of $C_{124}$ shows no observable change.

## V. Conclusion

References have been established for measuring dissipation factor of 10 pF capacitance standards from 50 Hz to 20 kHz. Measurements of both NIST-fabricated fused-silica capacitors as well as similar commercial standards have shown that the dissipation factors of these standards are typically within a few parts in $10^7$ below 1592 Hz; their dissipation factors may increase with frequency to a few parts in $10^6$ at 20 kHz due to their lead and electrode resistances and also possibly due to the dielectric loss resulting from the residual hydroxyl content in the bulk fused-silica. The dissipation factors of several typical fused-silica capacitors have remained stable within $2\text{x}10^{-7}$ over 30 years.

**References:**


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## Figure Captions

**Fig. 1.** Measured dissipation factor of $C_{112}$ as a function of frequency (open circles), with 1σ uncertainty bars, and calculated dissipation factor (solid triangles).

**Fig. 2.** Measured dissipation factors of four commercial 10 pF capacitance standards (s/n 1421: open triangles, s/n 1422: open circles, s/n 1423: solid triangles, s/n 1424: solid circles) as a function of frequency, with 1σ uncertainty bars.

**Fig. 3.** Dissipation factor of $C_{112}$ at 1592 Hz as a function of time from 1988 to present.

## Table Captions

**Table 1.** Contribution of component uncertainties to the total uncertainty at four representative frequencies for $C_{112}$.

**Table 2.** Dissipation factors of four NIST 10 pF capacitance standards measured in 1974, 1990, and 2006.

| Source of uncertainty | Relative standard uncertainty ($\times 10^{-6}$) | | | |
|---|---|---|---|---|
| | 100 Hz | 400 Hz | 1 kHz | 10 kHz |
| Type A | 0.53 | 0.05 | 0.03 | 0.03 |
| Reference capacitor $C_{10}$ | 0.14 | 0.14 | 0.14 | 0.14 |
| Contact resistance of $C_{112}$ | 0.01 | 0.01 | 0.02 | 0.2 |
| Bridge linearity errors | 0.05 | 0.05 | 0.05 | 0.05 |
| **Relative combined standard uncertainty** | 0.56 | 0.16 | 0.16 | 0.26 |

| Standard | Dissipation factor ($\times 10^{-6}$) | | |
|---|---|---|---|
| | 1974 | 1990 | 2006 |
| $C_{109}$ | 0.74 | 0.75 | 0.77 |
| $C_{114}$ | 0.33 | 0.37 | 0.39 |
| $C_{124}$ | 0.35 | 0.35 | 0.35 |
| $C_{127}$ | 0.78 | 0.65 | 0.65 |

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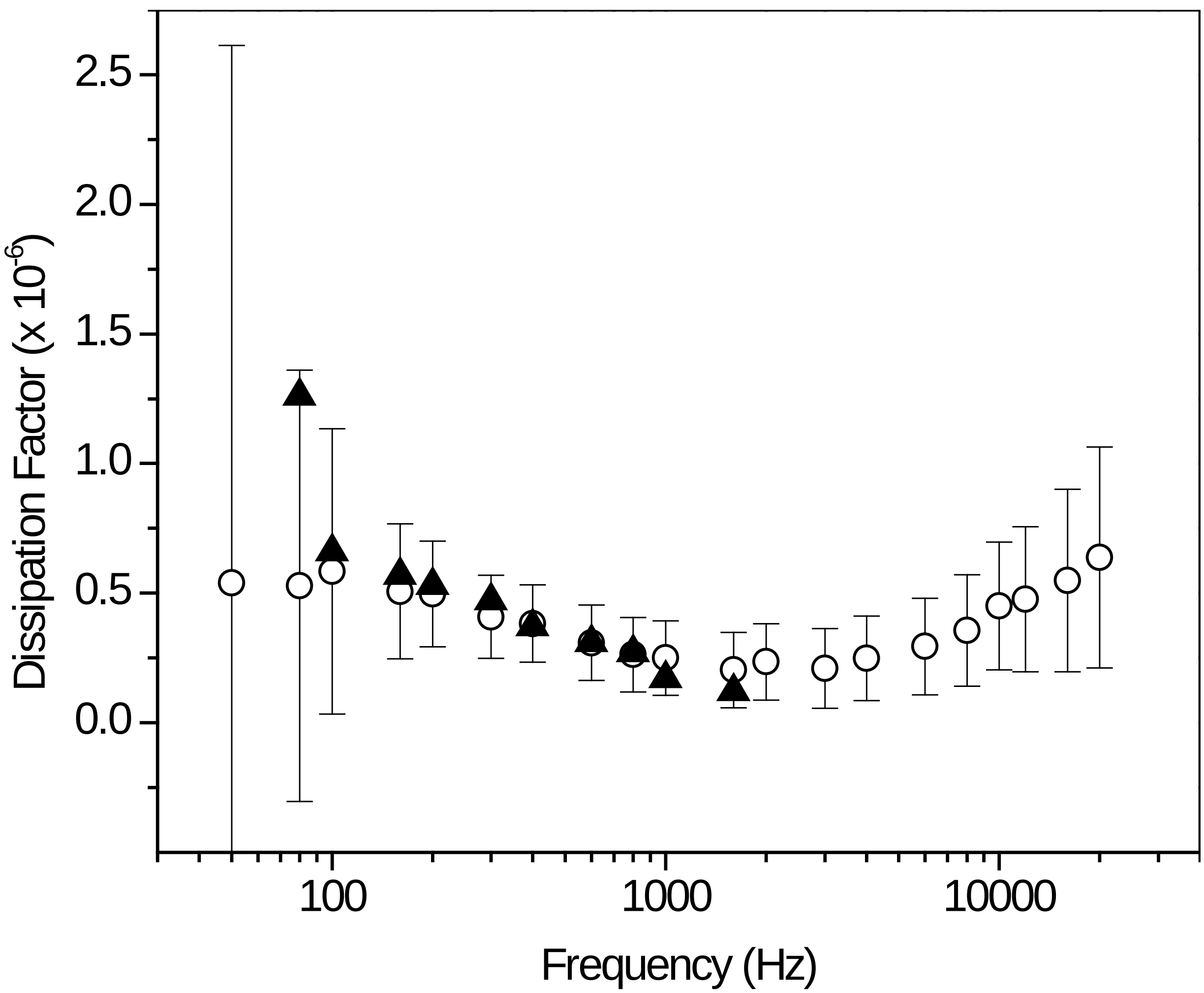
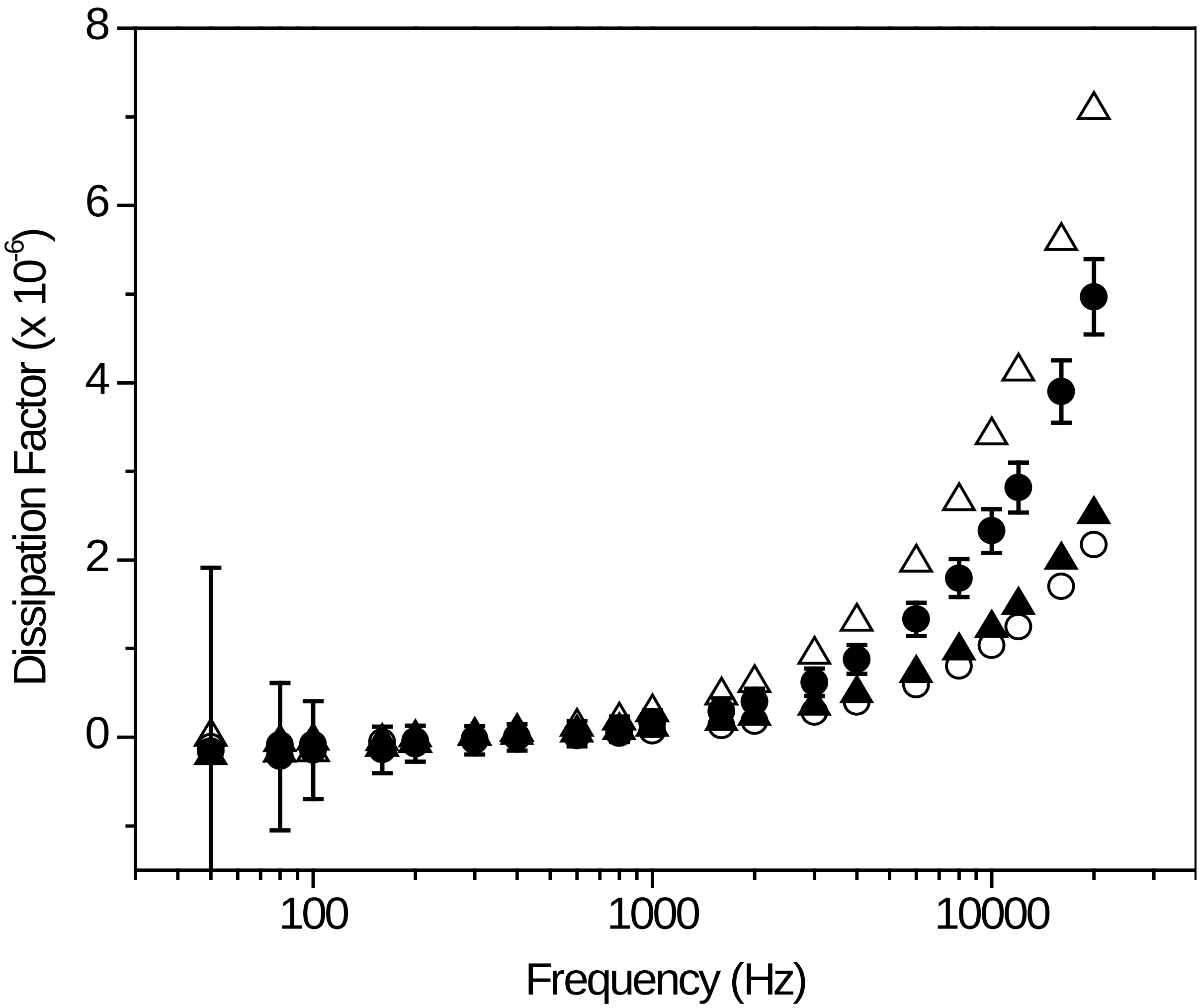
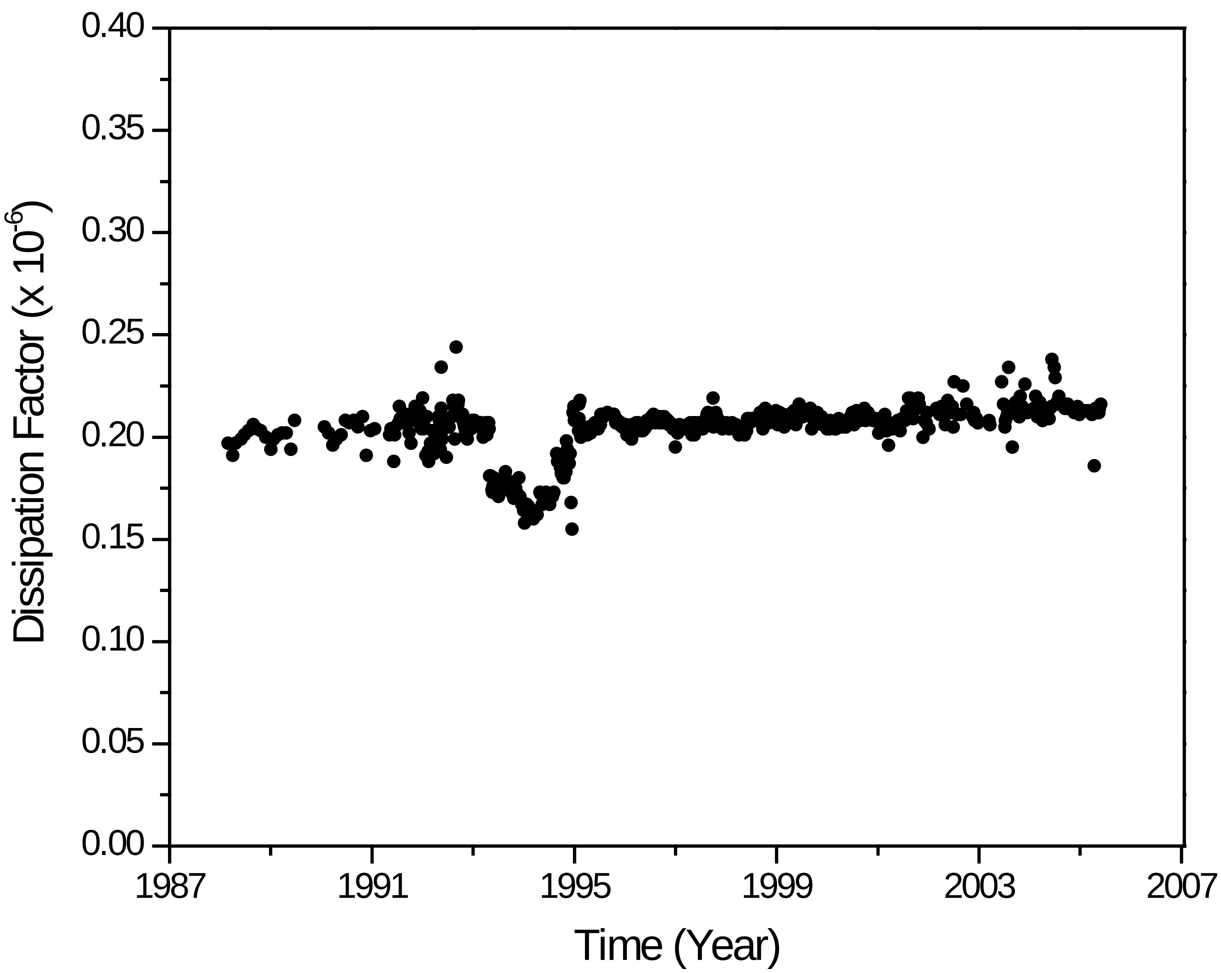


Dissipation Factor (x $10^{-6}$)
2.5
2.0
1.5
1.0
0.5
0.0
100
1000
10000
Frequency (Hz)



8
6
4
2
0
Dissipation Factor (x 10-6)
100
1000
10000
Frequency (Hz)



Dissipation Factor (x 10$^{-6}$)
0.40
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
1987
1991
1995
1999
2003
2007
Time (Year)