Uncertainty Analysis for Four Terminal-Pair Capacitance and Dissipation Factor Characterization at 1 MHz and 10 MHz[‡]

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Introduction

The Electricity Division at the National Institute of Standards and Technology (NIST, formerly NBS) has implemented a system to characterize capacitance and dissipation factor for four terminal-pair (4TP) air dielectric capacitors at frequencies from 1 kHz to 10 MHz [1]. The method is based on work by Cutkosky [2,3] and Jones [4,5] of NBS and recent developments by Yokoi, Suzuki, and Aoki [6,7] as well as Yonekura and Wakasugi [8] of Hewlett-Packard Japan. This paper describes an extensive uncertainty analysis of the measurement system. The analysis has been divided into three areas: 1 kHz capacitance measurements; network analyzer impedance measurements (covering frequencies from 40 MHz to 200 MHz); and a mathematical extrapolation algorithm that regresses the high-frequency characterization down to frequencies of 10 MHz and below [6,7]. This algorithm is referred to as the capacitor frequency characteristic prediction (CFCP) method. The capacitance and dissipation factor characteristics at 1 MHz and 10 MHz are produced by applying the CFCP algorithm to the 1 kHz capacitance as well as the high-frequency (40 MHz to 200 MHz) impedance measurements.

Capacitors characterized using this technique will be used as impedance reference standards for a general-purpose digital impedance bridge recently developed at NIST to calibrate inductors and ac resistors [9]. The technique is also to be employed in a future NIST Special Test for 4TP capacitance and dissipation factor.

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1 kHz Capacitance Measurement Uncertainty

Figure 1 shows the simple 4TP capacitor circuit model, where C_{lh} is the low-to-high capacitance, and C_{lg} and C_{hg} are low-to-ground and high-to-ground leakage capacitances, respectively. These capacitance components are repeatedly measured over time to establish repeatability using a 1 kHz capacitance meter. Table 1 presents Type A relative standard uncertainties for 1 kHz measurements of the 1 pF, 10 pF, 100 pF, and 1000 pF standard capacitors. Uncertainties are given as parts in 10^6 and labeled ppm. The Type B relative standard uncertainty for the 1 kHz capacitance meter is about 10 ppm [10].

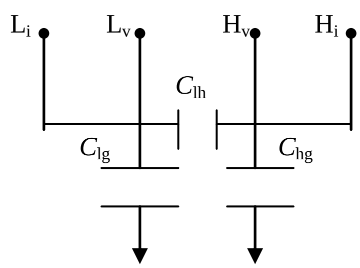

Figure I. Four Terminal-Pair Capacitor: Simple Model

Since the results of the 1 kHz capacitance measurements are used in the CFCP method, there will be an uncertainty contribution from the 1 kHz measurements to the final capacitance and dissipation factor results. Software simulations were performed to determine the uncertainty components of the 4TP capacitance and dissipation factor due to the 1 kHz capacitance measurements.

Table 1. Type A Standard Uncertainties for 1 kHz Measurements

Nominal Capacitance (pF)	Measured Parameter	Measured Capacitance (pF)	Type A Uncertainty (ppm)
1	Clh	1.0000142	5

	Clg	48.82658	37
	Chg	17.67269	86
10	Clh	10.000745	4
	Clg	29.16876	14
	Chg	25.02418	30
100	Clh	99.99892	7
	Clg	32.27685	30
	Chg	31.03866	37
1000	Clh	1000.0739	5
	Clg	34.98904	164
	Chg	33.36330	263

In 1990, Yonekura and Wakasugi described a circuit model of 4TP capacitors (of the type described in this paper) [8]. They disassembled several of these capacitors and measured and published the values of typical model components, e.g., connector and lead inductances. The following simulations are based on this model and component values (see Fig. II).

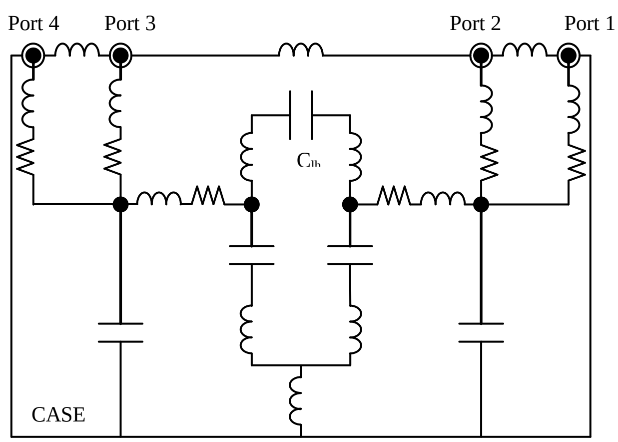

Figure II. Yonekura Component Model

Equations were developed to describe the Yonekura model in terms of the model components. Using published values from reference [8], we computed the 4TP capacitance and dissipation, as well as the single-port data needed for the CFCP algorithm for this 'reference' capacitor.

The sensitivity of the CFCP method was tested by applying a fixed set of single-port (simulated network analyzer) data, while randomly varying the 1 kHz capacitance data based on the values given in Table 1. Predictions of the 4TP capacitance and dissipation factor from the CFCP method were obtained for the multiple simulations at 1 MHz and 10 MHz. The predictions were compared with the exact solution of the reference capacitor to estimate the Type A uncertainty contribution in the final results due to the 1 kHz measurements. The Type B uncertainty simulations were run just as the Type A simulations except that the 1 kHz capacitance values were offset by 10 ppm instead of randomly scattered prior to running the CFCP algorithm. The uncertainties produced from this simulation are reported in the Uncertainty Results section as 1 kHz Types A and B standard uncertainties.

Note that the Type B uncertainty of the capacitance meter has been approximated conservatively at 10 ppm.

Network Analyzer Measurement Uncertainty

The high-frequency single-port measurements used by the CFCP method are provided by a precision network analyzer. Measurements are made from 40 MHz to 200 MHz, depending on the capacitor. The actual measured quantity is the scattering parameter, S11, which is converted into impedance. The network analyzer contributes both Type A and Type B uncertainties. The Type A component of uncertainty is the standard deviation of capacitance and dissipation factors based on repeated network analyzer measurements applied to the CFCP algorithm. Note that this value also includes random variations of the capacitor.

The Type B component of uncertainty due to the network analyzer was estimated using software simulations. Again, the solution of the Yonekura circuit model [8] was used to compute the true 4TP capacitance and dissipation factor of the reference capacitor at frequencies of 1 MHz and 10 MHz. Ideal S11 parameters (that would be generated by an errorless network analyzer) were also computed from the circuit equations and applied to the CFCP algorithm to predict 1 MHz and 10 MHz capacitance and dissipation factor behavior. The calculated S11 values were then modified to simulate a network analyzer with offset, gain, and frequency response errors within the manufacturer's specifications [11]. S11 measurements of a NIST-calibrated precision 20 cm air line indicated that the network analyzer was within these specifications.

The Types A and B standard uncertainties in the 4TP characterization due to the network analyzer are reported in the Uncertainty Results section.

Regression Algorithm Uncertainty Due to Variations in Capacitor Manufacturing

Still more simulations were performed to determine the uncertainty components of the 4TP capacitance and dissipation factor introduced by the CFCP method. The regression algorithm extrapolates the network analyzer impedance measurements (made over a range of frequencies chosen somewhere between 40 MHz and 200 MHz) down to 10 MHz and below using the 1 kHz capacitance measurement values, described briefly above, as references. The regression parameters were selected to optimally predict the capacitance frequency characteristic for nominal capacitor values. This test was set up to determine the sensitivity of the method to variability in manufacturing of the capacitor standards.

The circuit solution of the Yonekura model provides reference values of capacitance and dissipation factor at the frequencies of interest. It is also used to obtain high-frequency single-port values. Simulated network analyzer measurement data were used to iteratively extrapolate to the frequencies of 10 MHz and 1 MHz with normally distributed random errors injected into the Yonekura model components according to each component's uncertainty [8]. For each reference capacitor, the exact solution and the value predicted by the CFCP method was compared. The Type B standard uncertainties attributed to the method are reported in the Uncertainty Results section.

Uncertainty Results

Table 2 labels the standard uncertainty components and Table 3 shows the values of the components as well as the expanded standard uncertainties for capacitance and dissipation factor characterization of the standard 4TP capacitors (1 pF, 10 pF, 100 pF, and 1000 pF) at frequencies of 1 MHz and 10 MHz. The uncertainty components are root-sum-squared and then multiplied by two to produce the expanded standard uncertainty values. All capacitance uncertainty components are given in parts in 10^6, labeled as ppm, and all dissipation factor uncertainty components are given in microradians, labeled as :rad. These values will be refined and reevaluated as the authors gain experience with the measurement system.

Table 2. Uncertainty Component Descriptions

a1	Type A 1 kHz capacitance standard uncertainty
a2	Type A network analyzer standard uncertainty
b1	Type B 1 kHz capacitance standard uncertainty
b2	Type B network analyzer standard uncertainty
b3	Type B CFCP standard uncertainty

Table 3. Relative Uncertainty Components and Expanded Uncertainties (k = 2)

	a1	a2	b1	b2	b3	U (k=2)
1 pF Capacitance						(ppm)
1 MHz	5	0	10	250	250	710
10 MHz	5	8	10	5000	5000	14000
1 pF Dissipations Factor						(μrad)
1 MHz	5	5	10	100	100	300
10 MHz	5	153	10	2000	2000	5700
10 pF Capacitance						(ppm)
1 MHz	4	1	10	5	1	24
10 MHz	4	76	31	520	75	1100
10 pF Dissipation Factor						(μrad)
1 MHz	4	0	10	9	2	28
10 MHz	4	2	10	295	70	610
100 pF Capacitance						(ppm)
1 MHz	7	0	10	5	0	26
10 MHz	7	9	10	545	7	1100
100 pF Dissipation Factor						(μrad)
1 MHz	7	0	10	1	0	24
10 MHz	7	0	10	51	2	105
1000 pF Capacitance						(ppm)
1 MHz	5	0	10	5	2	25
10 MHz	5	8	36	474	255	1100
1000 pF Dissipation Factor						(μrad)
1 MHz	5	0	22	6	8	50
10 MHz	5	0	610	203	266	1400

Future Work

The uncertainty analysis reported in this paper consists of simulations that determine the statistical variation of the components of the 4TP capacitor characterization technique. Some assumptions are made regarding the errors and the circuit model derived from measurements of a set of standard capacitors. A theoretical analysis of the network analyzer should be performed in order to compare with the simulations performed and reported upon here.

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