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NIST Special Test Service for Four-Terminal-Pair Capacitance Standards from 0.01 µF to 100 µF

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Table of Contents

Page

List of Figures

List of Tables

List of Tables, cont'd

1. Introduction

Precision LCR (impedance) meters typically operate with a four-terminal-pair (4TP) configuration and are calibrated using 4TP capacitance standards. The measurement system described herein has been developed in response to a need for improved LCR meter calibrations.

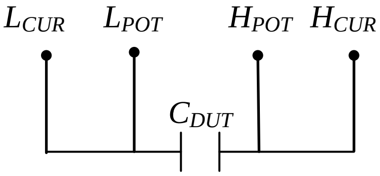

Figure 1. Four-terminal-pair capacitor model.

Figure 2. Commercial set of four-terminal-pair capacitors.

Figure 1 shows a simple circuit model for a 4TP capacitance standard. A capacitance scaling method is used to calibrate 4TP standard capacitors of values from 10 nF to 100 µF. Aoki and Yokoi introduced this technique in 1997 [1]. References [1] and [2] describe the general method and provide a detailed uncertainty analysis. Aoki and Yokoi developed a calibration procedure based on reference [1]. Figure 2 shows a set of commercial standards[**] to which this calibration system may be applied. Note that convention allows for the standards to be identified with units of nF or μF. Therefore, the 0.01 μF standard is also referred to as the 10 nF standard and the 0.1 μF standard is referred to as the 100 nF standard. Note, also, that the typical commercial set of 4TP ceramic capacitance standards contains values from 10 nF to 10 μF. The 100 μF capacitor must be obtained separately.

The capacitance scaling system uses a commercial automatic capacitance bridge to measure reference points, and it uses an LCR meter along with a single-decade inductive voltage divider (IVD) and interface circuitry (together called Scaling Fixture) as an impedance comparator. The capacitance scaling system measures 4TP capacitors in decade (10:1) steps from 10 nF to 100 μF. The measurements are performed at frequencies of 100 Hz, 1 kHz, 10 kHz, and 100 kHz.

The initial procedure used at NIST with the capacitance scaling system was described in reference [3]. This Technical Note more fully describes the method, modified procedure, and error analysis.

2. Description of Capacitance Scaling Method

Modern instrumentation is designed for automated control and can be used to create custom calibration procedures. In the case when very precise and specialized tests are necessary for a metrology application, it is particularly challenging to establish computer control of an entire procedure. Accurate calibration of capacitors that range in value from 0.01 μF to 100 μF over the frequency range from 100 Hz to 100 kHz is desired.

There are several instruments available commercially to measure the impedance of a capacitor. LCR meters are general impedance-measuring instruments that have limited uncertainty, while automatic capacitance bridges are available commercially with very low uncertainties but with more limited measurement ranges.

An automatic capacitance bridge is very convenient for measuring three-terminal (3T) standard capacitors with precision, reliability, and uncertainty at metrological levels. Measurement errors are on the order of parts in 10^6. The measurement uncertainty depends directly on the standard capacitor that resides in one branch of the bridge as well as on the inductive voltage divider used to scale the measured capacitance to that standard. The limitations of the automatic bridge include a limited frequency band and limited capacitance range. A single-frequency commercial capacitance bridge has been available for several years. Recent versions allow measurement from near dc to 20 kHz. The capacitance measurement upper limit is typically around 1 μF but measurement uncertainties grow significantly near the upper limit.

Since the capacitance bridge is a 3T instrument and the measured capacitors and the LCR meter are 4TP, the 3T-to-4TP conversion must be addressed. The details of the measurement setup and

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[**] Identification of commercial products does not imply endorsement by the U.S. Government, nor does it imply that such products are necessarily the best available.

their analysis are given in section 6 (see Figure 6). The study showed negligible influence of the lead impedance on the capacitance measurements.

LCR meters that operate in a four-terminal-pair configuration have been widely available for many years. They operate on the principle of sourcing a known current through unknown impedance and measuring the voltage drop. The impedance is determined based on the magnitude and phase of the known current and the measured voltage. Instruments typically operate over a broad frequency range (up to MHz) and can measure inductors, capacitors, and resistors with a wide range of values and losses. Uncertainties are typically on the order of fractions of a percent for the most accurate meters.

Both the automatic capacitance bridge and the LCR meter play major roles in evaluating capacitors. The goal is to calibrate capacitors up to 100 μF over the frequency range from near dc to 100 kHz. The reference measurements are made using an automatic capacitance bridge. All of the remaining measurements are performed using the LCR meter and are traceable to the reference measurements.

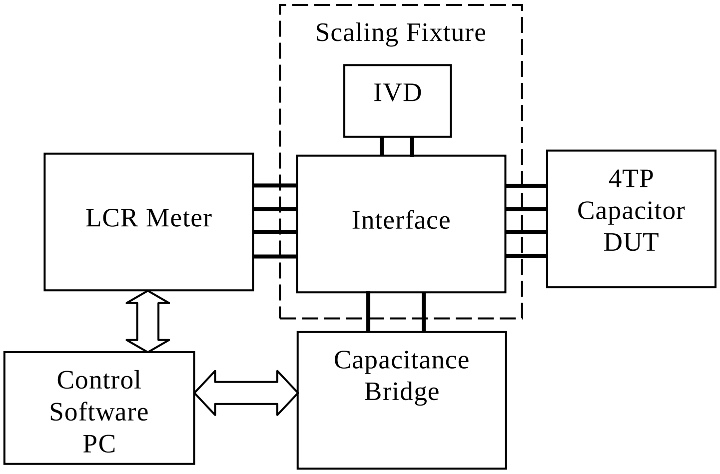


Figure 3. Capacitance scaling measurement system.

The goal of the newly developed measurement procedures is to use commercially available instrumentation as much as possible and design specialized equipment so it is easily integrated into an automatic measuring system.

The capacitance scaling system consists of an LCR meter, an automatic capacitance bridge, and a custom-made interface (Scaling Fixture), as shown schematically in Fig 3. Figure 4 shows a photograph of the measurement system.

Capacitors calibrated using this procedure have values of 10 nF, 100 nF, 1 μF, 10 μF and 100 μF. The scaling fixture uses a fixed 10:1 ratio IVD. Reference 100 pF and 1 nF standard capacitors are used to calibrate the scaling fixture and the 1 nF reference is also used to begin the

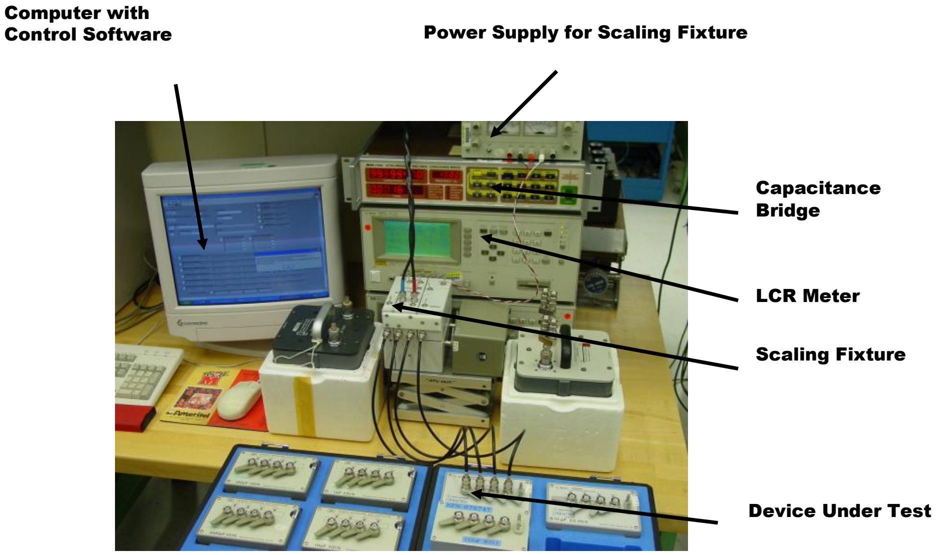


Figure 4. Photograph of the measurement system.

scaling process. The fixed ratio IVD limits the scaling system to the measurement of standards that are nominally 1, 10, or 100 in value.

The scaling fixture controls the circuitry used in switching measurement instruments, measurement ratios (1:1 and 10:1), high-frequency and low-frequency IVDs, offset correction configurations, and signal levels. The LCR meter and scaling fixture combination is self-calibrated by comparing calibrated capacitors. This essentially establishes the ratio of the scaling fixture as a 1:1 and 10:1 interface.

2.1. Calibration of the Scaling Ratio

Four measurements are required to calibrate the 10:1 scaling ratio of the scaling fixture circuitry. The measurement process is controlled by software that prompts the user to set the appropriate switches on the measurement interface to achieve the proper settings during measurement.

First, a 100 pF standard capacitor, $Z_{100\ \mathrm{pF}}$, is measured using a 3T capacitance bridge, $Z_{100\ \mathrm{pF}\_CB}$, shown in Eq. (1). The capacitor measured is a four-terminal-pair device and the capacitance bridge is 3T, so the scaling fixture provides an appropriate 4TP-to-3T connection switch (see the discussion associated with Figure 6).

The scaling fixture switches are then configured so that the capacitance standard is connected to the LCR meter. The impedance of this standard is then measured using the LCR meter with the 10:1 IVD switched into the circuit, yielding $Z_{100\ \mathrm{pF\_LCR}}$, as in Eq. (2). The IVD makes the 100 pF capacitor appear to the LCR meter to have the impedance of a 1 nF capacitor.

As an example, a 100 pF capacitor measured at 1 kHz has an impedance of $\mathrm{Z}_{100\,\mathrm{pF}} = \dfrac{1}{2\pi(10^{3})(10^{-10})} = 1591.55\,\mathrm{k}\Omega$. A 1 nF capacitor has an impedance of 159.155 kΩ at 1 kHz.

The 100 pF capacitor is measured with the scaling fixture set to the ratio of 0.1 and the 1 nF capacitor is measured with the scaling fixture set to the ratio of 1.0. Connecting the 100 pF capacitor to the LCR meter through the 10:1 IVD reduces the voltage across the voltage ports of the LCR meter and thus causes the LCR meter to see the same nominal impedance value as is produced by the 1 nF capacitor connected to the LCR meter without the IVD. In the derivation of the scaling ratio, below, we use the notation $K_{1\,\mathrm{nF}\_LCR}$ to describe the scale factor of the LCR meter when measuring 1 nF.

The impedance of the 1 nF capacitor, $Z_{1\mathrm{nF}}$, is measured using the LCR meter with the scaling ratio set to 1.0, yielding $Z_{1\,\mathrm{nF}\_LCR}$, shown in Eq. (3). And finally, the impedance of the 1 nF standard is measured using the capacitance bridge, yielding $Z_{1\,\mathrm{nF}\_CB}$, shown in Eq. (4).

The scaling ratio, K, is calculated from the described measurements. The ratio of impedances measured using the LCR meter is equal to the ratio of impedances derived from the capacitance bridge measurements multiplied by the scaling ratio, represented in Eq. (5). This provides the basis to compute the scaling ratio, as shown in Eq. (6).

$$Z_{100\,\mathrm{pF}\_CB} = Z_{100\,\mathrm{pF}} \tag{1}$$

$$Z_{100\,\mathrm{pF}\_LCR} = K\ Z_{100\,\mathrm{pF}}\ K_{1\,\mathrm{nF}\_LCR} \tag{2}$$

$$Z_{1\,\mathrm{nF}\_LCR} = 1.0\ Z_{1\,\mathrm{nF}}\ K_{1\,\mathrm{nF}\_LCR} \tag{3}$$

$$Z_{1\,\mathrm{nF}\_CB} = Z_{1\,\mathrm{nF}} \tag{4}$$

$$\frac{Z_{100\,\mathrm{pF}\_LCR}}{Z_{1\,\mathrm{nF}\_LCR}} = K\ \frac{Z_{100\,\mathrm{pF}\_CB}}{Z_{1\,\mathrm{nF}\_CB}} \tag{5}$$

$$K = \frac{Z_{1\,\mathrm{nF}\_CB}}{Z_{100\,\mathrm{pF}\_CB}}\frac{Z_{100\,\mathrm{pF}\_LCR}}{Z_{1\,\mathrm{nF}\_LCR}} \tag{6}$$

Equations (7) and (8) represent formulas used to estimate the in-phase and quadrature components of the scaling ratio. Empirically, the quadrature component is on the order of a few µrad. Since the angle is small, the imaginary component represents the phase of the scaling ratio. The in-phase component is simply called the scaling ratio.

$$K_{ratio} = \mathrm{Re}\left(K\right) \tag{7}$$

$$K_{\varphi} = \operatorname{Im}(K) \tag{8}$$

A detailed explanation of scaling ratio measurements will be presented in the Section 6.

2.2. Calibration of 10 nF Capacitor

Once the scaling ratio, *K*, is determined, the system can be used to calibrate capacitors. The calibration of the 10 nF capacitor will be presented in sufficient detail here. More information will be presented in the section 6.

A 1 nF reference capacitor, $Z_{1\ \mathrm{nF}}$, is measured using the capacitance bridge, yielding $Z_{1\ \mathrm{nF}\_CB}$, shown in Eq. (9), and then is measured using the LCR meter with the scaling ratio, *K*, set to 0.1, yielding $Z_{1\ \mathrm{nF}\_LCR}$, shown in Eq. (10). Then, a 10 nF device under test, $Z_{DUT\_10\ \mathrm{nF}}$, is measured using the LCR with the scaling ratio, *K*, set to 1.0, yielding $Z_{10\ \mathrm{nF}\_LCR}$, represented in Eq. (11). These measurements are used to find the impedance of the device under the test, as in Eqs. (12) and (13).

$$Z_{1\ \mathrm{nF}\_CB} = Z_{1\ \mathrm{nF}} \tag{9}$$

$$Z_{1\ \mathrm{nF}\_LCR} = K\ Z_{1\ \mathrm{nF}}\ K_{10\ \mathrm{nF}\_LCR} \tag{10}$$

$$Z_{10\ \mathrm{nF}\_LCR} = 1.0\ K_{10\ \mathrm{nF}\_LCR}\ Z_{DUT\_10\ \mathrm{nF}} \tag{11}$$

$$\frac{Z_{10\ \mathrm{nF}\_LCR}}{Z_{1\ \mathrm{nF}\_LCR}} = \frac{Z_{DUT\_10\ \mathrm{nF}}}{K\ Z_{1\ \mathrm{nF}\_CB}} \tag{12}$$

$$Z_{DUT\_10\ \mathrm{nF}} = K\ \frac{Z_{10\ \mathrm{nF}\_LCR}}{Z_{1\ \mathrm{nF}\_LCR}}\ Z_{1\ \mathrm{nF}\_CB} \tag{13}$$

The goal of this calibration process is to calibrate the 10 nF capacitor by using a known 1 nF capacitor. Comparison of the measurement of the 1 nF standard with the scaling ratio and the measurement of the 10 nF standard without the scaling ratio, factoring the scaling calibration back into the equation, allows the accurate determination of the 10 nF standard capacitor.

2.3. Calibration of 100 nF Capacitor

The next capacitor, 100 nF, is measured by using the result of the 10 nF, and making the same two sets of measurements mentioned above (Eqs. (11) and (12)). The 10 nF capacitor, $Z_{DUT\_10\ \mathrm{nF}}$, is measured using the LCR meter, yielding $Z_{10\ \mathrm{nF}\_LCR}$, and then the 100 nF capacitor, $Z_{DUT\_100\ \mathrm{nF}}$, is measured using the LCR meter, yielding $Z_{100\ \mathrm{nF}\_LCR}$. The result is obtained using Eq. (14).

$$Z_{DUT\_100\ \mathrm{nF}} = K\frac{Z_{100\ \mathrm{nF}\_LCR}}{Z_{10\ \mathrm{nF}\_LCR}}\ Z_{DUT\_10\ \mathrm{nF}} \tag{14}$$

This same process is repeated to characterize all of the other capacitors in the set. It is important to mention that all of the errors are accumulated during this bootstrap process.

There are several measuring methods that are available for calibrating 1 nF capacitors. They include calibration using a vector network analyzer in combination with a capacitance bridge. This method covers a very wide frequency range (up to 10 MHz) and is complex to perform. Its expanded uncertainty with $k = 2$ is on the order of 25 parts in 10^6 at 100 kHz. A multi-frequency automatic capacitance bridge may be used to measure a 1 nF capacitor up to 20 kHz with an expanded uncertainty on the order of 10 parts in 10^6 at 10 kHz. In the field of metrology, it is important to independently verify results. Since the proposed capacitance scaling method derives the values from a known standard that is traceable to the U.S. representation of the farad, it is considered a valid and reliable measurement procedure. Another important property of the described method is that it uses the ratio of measurements to establish the result. The LCR meter is an instrument that produces repeatable results with good linearity but has significant systematic errors. Since the impedances measured using the LCR meter are compared in a ratio format, the effects of systematic errors are reduced, providing a lower measurement uncertainty than taking absolute measurements.

This concludes the theoretical presentation of the capacitance scaling method. In practice, there are several additional test method considerations that are necessary to improve the estimation of the scaling factor, K, and the estimation of the capacitance of the device under test. These will be discussed in section 3.

3. Description of Graphical Control Software

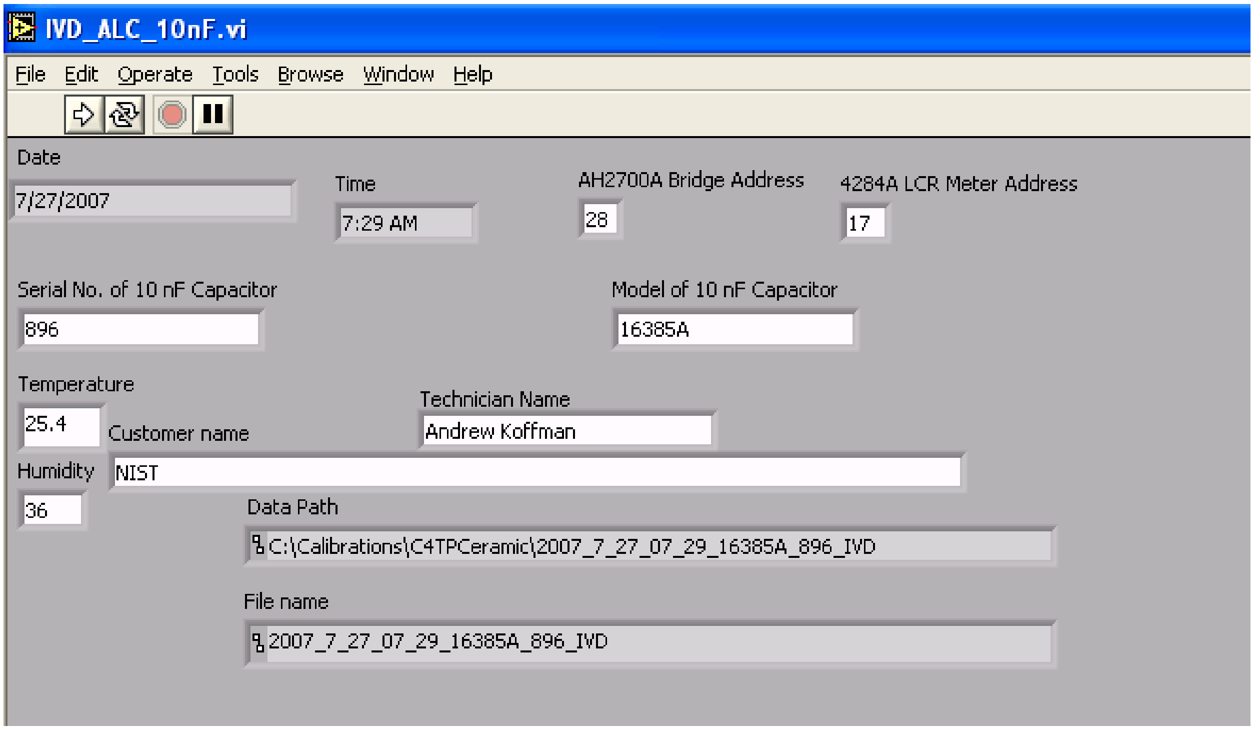
A control software program was written with an extensive set of on-line instructions in order to make the calibration process user-friendly (see Figure 5). The program will be presented step by step.

The first step is the scaling ratio calibration. It is recommended that this calibration be performed before each calibration of a capacitor set, in order to minimize the measurements errors.

The program creates one file to store the measurements and another file where the report summarizing the calibration results will be stored. All of the file names are unique and include the capacitor value and the date and time of calibration. Extra care is introduced in order to keep the serial numbers of the calibrated devices in order. This is done in order to organize the measurements in unique storage and report files.

It is not possible to accurately provide a scaling ratio over the frequency range from 100 Hz to 100 kHz with a single transformer. Therefore, the system is implemented using a low-frequency transformer operating from 100 Hz to 10 kHz, and a high-frequency transformer operating from 10 kHz to 100 kHz. The software program prompts the user to select the appropriate transformer by setting switches on the interface.

The user is guided through the calibration process with prompts on the computer screen. The procedure pauses until the user responds to the prompt.



Date: 7/27/2007
Time: 7:29 AM
AH2700A Bridge Address: 28
4284A LCR Meter Address: 17
Serial No. of 10 nF Capacitor: 896
Model of 10 nF Capacitor: 16385A
Temperature: 25.4
Technician Name: Andrew Koffman
Customer name: NIST
Humidity: 36
Data Path: C:\Calibrations\C4TPCeramic\2007_7_27_07_29_16385A_896_IVD
File name: 2007_7_27_07_29_16385A_896_IVD

Figure 5. Example of a user-friendly interactive window.

All instrumentation control is performed via software. The sequencing of the measurements taken, the specific values for the voltages applied, and the appropriate frequencies are all prescribed and controlled without user intervention. Also, data averaging and formatting is performed, with averages and standard deviations recorded for further analysis.

Another program has been developed to perform the actual capacitor calibration. It requires selection of a file containing the scaling ratio information.

3.1. Offset Measurements

There are several additional details associated with the capacitance scaling procedure. For higher values of capacitance at higher frequencies, the offset voltage of the LCR meter high potential port measurement circuitry is measured by shorting the high and low potential ports.

Without offset corrections, capacitance scaling calculation consists of combining four quantities: the known standard impedance, Z_{STD}, the scaling ratio, K, and the LCR meter measurements of the device under the test and the standard impedance, Z_{DUT_LCR} and Z_{STD_LCR}. In equation form, the impedance of the device under test, Z_{DUT}, is

$$Z_{DUT} = K \frac{Z_{DUT\_LCR}}{Z_{STD\_LCR}} Z_{STD} . \tag{15}$$

Equation (15) is the general representation of the capacitance scaling method and straightforward modification is applied to incorporate offset corrections. The modified equation is

$$Z_{DUT} = K \frac{Z_{DUT\_LCR} - Z_{DUT\_OFFSET}}{Z_{STD\_LCR} - (0.1) Z_{STD\_OFFSET}} Z_{STD}. \qquad (16)$$

The scaling fixture allows a direct measurement of the offset with the LCR meter in the same measurement state as for the measurement of the standard and device under test. The (0.1) multiplier is an approximation of the scaling ratio, *K*, and is used to scale the offset measurement.

The software prompts the user to throw the necessary switches on the scaling fixture at appropriate times to measure the offset (see Table 1). The software also incorporates the offset corrections into the calculation of the impedance.

Table 1. Offset switch operation for the standard capacitor (STD) and the device under test (DUT) for different capacitance values over the frequency range of interest.

	Frequency			
Capacitance	100 Hz	1 kHz	10 kHz	100 kHz
STD 1 nF	off	off	off	off
DUT 10 nF	off	off	off	on
STD 10 µF	off	off	off	on
DUT 100 nF	off	off	on	on
STD 100 nF	off	off	on	on
DUT 1 µF	off	on	on	on
STD 1 µF	off	on	on	on
DUT 10 µF	on	on	on	on
STD 10 µF	on	on	on	N/A
DUT 100 µF	on	on	on	N/A

3.2. Selecting Voltage Ranges for the Capacitance Scaling Calibration

The LCR meter has limited current and voltage capability when measuring the impedance of the device under test (DUT). In practice, the LCR meter limits the test voltage applied to the DUT to 1 V_{rms} and the current to 20 mA_{rms}. The ALC function (Automatic Level Control) allows the control of the amplitude of the applied voltage. The reason for using this command is that without it, even when the user selects a 1 V_{rms} test voltage, the instrument lowers that voltage to optimize the current. This change is performed without specifically informing the user. This results in uncontrolled application of the applied voltage to the capacitor under test. Proper determination of the capacitance requires knowledge of the test voltage. When ALC is ON, the requested voltage is guaranteed. If the instrument can not achieve the requested voltage, the information is sent to the user.

For higher values of capacitance, the LCR meter allows the ALC to operate only as long as AUTO is selected for the impedance range. Table 2 shows the test voltages used in the scaling procedure. The goal for selection was to apply as high a test voltage as possible while maintaining a small standard deviation of measurement and allowing ACL to be ON.

Table 2. Selected test voltages for the capacitance scaling system.

Capacitance	Frequency			
	100 Hz	1 kHz	10 kHz	100 kHz
10 nF	600 mV	600 mV	600 mV	600 mV
100 nF	600 mV	600 mV 𝒜(10k)	600 mV 𝒜(1k)	200 mV 𝒜(100)
1 µF	600 mV 𝒜 (1k)	600 mV 𝒜 (100)	200 mV 𝒜 (100)	20 mV 𝒜(10)
10 µF	600 mV 𝒜 (100)	200 mV 𝒜 (100)	20 mV 𝒜 (10)	ALC OFF 5mV 𝒜 (10)
100 µF	200 mV 𝒜 (100)	20 mV 𝒜 (10)	ALC OFF 5 mV 𝒜 (10)	N/A

Letter 𝒜 in the table shows when AUTO impedance range must be selected, allowing the meter to select the impedance measurement range. The impedance range actually used is shown in parentheses. This information is necessary because the offset measurements must be performed at the same operating range and applied voltage to minimize linearity errors of the meter. The ALC feature on the LCR meter is not operational for the 5 mV voltage range. Such a small voltage is applied due to the fact that the measured impedance at high frequencies and high values of capacitance is less than 1 Ω.

Tables 3-7 present the combinations of frequencies, capacitance values and voltage settings incorporated into the control software routines that calibrate each capacitor. For example, the 100 nF capacitor calibration is performed in three steps (see Table 4). The first step is the calibration of the scaling ratio at 600 mV for frequencies of 100 Hz, 1 kHz, and 10 kHz, and at 200 mV for 100 kHz. The second step is the calibration of the 10 nF capacitor at the same voltage settings. The final step is the calibration of the 100 nF capacitor at the same settings. Data is collected at each intermediate step in the scaling process as well as at the final step.

Table 3. Selected test voltages for the 10 nF capacitor calibration.

Step	Frequency			
	100 Hz	1 kHz	10 kHz	100 kHz
1. Scaling ratio for 10 nF	600 mV	600 mV	600 mV	600 mV
2. 10 nF calibration				

Table 4. Selected test voltages for the 100 nF capacitor calibration.

Step	Frequency			
	100 Hz	1 kHz	10 kHz	100 kHz
1. Scaling ratio for 100 nF	600 mV	600 mV	600 mV	200 mV
2. 10 nF calibration for 100 nF				
3. 100 nF calibration				

Table 5. Selected test voltages for the 1 μF capacitor calibration.

Step	Frequency			
	100 Hz	1 kHz	10 kHz	100 kHz
1. Scaling ratio for 1 μF	600 mV	600 mV	200 mV	20 mV
2. 10 nF calibration for 1 μF				
3. 100 nF calibration for 1 μF				
4. 1 μF calibration				

Table 6. Selected test voltages for the 10 μF capacitor calibration.

Step	Frequency			
	100 Hz	1 kHz	10 kHz	100 kHz
1. Scaling ratio for 10 μF	600 mV	200 mV	20 mV	5 mV
2. 10 nF calibration for 10 μF				
3. 100 nF calibration for 10 μF				
4. 1 μF calibration for 10 μF				
5. 10 μF calibration				

Table 7. Selected test voltages for the 100 μF capacitor calibration.

Step	Frequency		
	100 Hz	1 kHz	10 kHz
1. Scaling ratio for 100 μF	200 mV	20 mV	5 mV
2. 10 nF calibration for 100 μF			
3. 100 nF calibration for 100 μF			
4. 1 μF calibration for 100 μF			
5. 10 μF calibration for 100 μF			
6. 100 μF calibration			

This set of tables shows all the intermediate steps for which data are collected and the same organization will be followed in section 6. Separate determinations of the scaling ratio, *K*, are necessary for the different calibrated capacitance values because the applied voltages are different for the different capacitance values.

4. Uncertainty Analysis for the Capacitance Scaling Measurement Procedure

4.1. Impedance Parameter Definitions

Impendence, *Z*, is defined as the ratio of voltage, *V*, to current, *I*,

$$Z = \frac{V}{I} = R + jX = \sqrt{R^2 + X^2} e^{j \arctan(\frac{X}{R})} = Me^{j\varphi}, \qquad (17)$$

where *R* and *X* are the real part (resistance) and imaginary part (reactance) of the impedance, respectively. *M* is the magnitude and φ is the phase of the impedance vector.

Impedance requires two parameters for full characterization. In this presentation, the capacitors are modeled as the parallel combination of resistance, R_P and capacitance, C_P. These definitions are related in the following manner.

$$
\begin{aligned}
Z &= \frac{R_P \dfrac{1}{j\omega C_P}}{R_P + \dfrac{1}{j\omega C_P}} = \frac{R_P}{1 + j\omega C_P R_P} = \frac{R_P\left(1 - j\omega C_P R_P\right)}{1 + \left(\omega C_P R_P\right)^2} \\
&= \frac{R_P}{1 + \left(\omega C_P R_P\right)^2} - j\frac{\omega C_P \left(R_P\right)^2}{1 + \left(\omega C_P R_P\right)^2} = R + jX,
\end{aligned}
\tag{18}
$$

where ω is the angular frequency.

In the capacitance scaling measurement procedure, the LCR meter is used to measure R and X, and the capacitance bridge is used to measure capacitance, C_P, and dissipation factor, DF. The following formulas are used to find capacitance and dissipation factor based on the LCR meter readings.

$$
\frac{1}{Z} = \frac{1}{R_P} + j\omega C_P \Rightarrow C_P = \frac{\mathrm{Im}\left\{\dfrac{1}{Z}\right\}}{\omega} = -\frac{1}{\omega M}\sin\left(\varphi\right)
\tag{19}
$$

$$
DF = \frac{R}{X} = \frac{1}{\omega R_P C_P}
\tag{20}
$$

To convert the capacitance bridge readings to impedance, Eq. (21) is used

$$
Z = \frac{\dfrac{1}{\omega C_P DF}}{1 + j\dfrac{1}{DF}} \; .
\tag{21}
$$

4.2. Capacitance Error

Equations (22) through (25) are used to derive the capacitance uncertainties from the measured impedance represented in Eq. (19). Partial derivatives of the parallel capacitance are used to determine the uncertainty components. Symbol Δ denotes the variable with respect to which the derivative is taken.

$$
\partial C_P = \frac{\partial}{\partial \omega}\left(-\frac{1}{\omega M}\sin\left(\varphi\right)\right)\Delta\omega + \frac{\partial}{\partial M}\left(-\frac{1}{\omega M}\sin\left(\varphi\right)\right)\Delta M + \frac{\partial}{\partial \varphi}\left(-\frac{1}{\omega M}\sin\left(\varphi\right)\right)\Delta\varphi
\tag{22}
$$

$$
\partial C_P = \left(\frac{1}{\omega^2 M}\sin\left(\varphi\right)\right)\Delta\omega + \left(\frac{1}{\omega M^2}\sin\left(\varphi\right)\right)\Delta M + \left(-\frac{1}{\omega M}\cos\left(\varphi\right)\right)\Delta\varphi
\tag{23}
$$

Partial derivatives are used throughout this document to derive uncertainty. Equation (24) shows how uncertainties are determined from Eq. (23).

$$u_c(C_P) = \sqrt{\left(\frac{1}{\omega^2 M}\sin(\varphi)\right)^2 u^2(\omega) + \left(\frac{1}{\omega M^2}\sin(\varphi)\right)^2 u^2(M) + \left(-\frac{1}{\omega M}\cos(\varphi)\right)^2 u^2(\varphi)} \tag{24}$$

Relative uncertainty for the parallel capacitance measurement circuit is expressed as

$$\frac{u_c(C_P)}{C_P} = \sqrt{\left(-\frac{u(\omega)}{\omega}\right)^2 + \left(-\frac{u(M)}{M}\right)^2 + \left(\frac{\cos(\varphi)}{\sin(\varphi)}u(\varphi)\right)^2} \tag{25}$$

Since we are measuring capacitors, phase φ is very close to 90°. The largest deviations from 90° measured for this investigation are on the order of 0.05°. The multiplier $\frac{\cos(\varphi)}{\sin(\varphi)}$ is then 0.001, leading to a negligible phase contribution to the capacitance uncertainty.

The relative frequency error $\frac{u(\omega)}{\omega}$ is insignificant since the instrumentation used incorporates a crystal oscillator with a drift rate of less than 0.1 part in 10^6 over the observation time. Since the capacitance scaling procedure uses a ratio of measurements, the systematic error in frequency will not have significant impact.

Therefore the relative error in capacitance is approximated with

$$\frac{u_c(C_P)}{C_P} \approx -\frac{u(M)}{M}. \tag{26}$$

In the capacitance scaling procedure, the impedance of a capacitor is measured as

$$Z_{DUT} = K\frac{Z_{DUT\_LCR}}{Z_{STD\_LCR}}Z_{STD}. \tag{27}$$

When partial derivatives are applied to this formula, the following result is obtained:

$$\frac{\Delta Z_{DUT}}{Z_{DUT}} = \frac{\Delta K}{K} + \frac{\Delta\left(\frac{Z_{DUT\_LCR}}{Z_{STD\_LCR}}\right)}{\frac{Z_{DUT\_LCR}}{Z_{STD\_LCR}}} + \frac{\Delta Z_{STD}}{Z_{STD}} \tag{28}$$

The factor $\frac{Z_{DUT\_LCR}}{Z_{STD\_LCR}}$ is treated as a single contribution since the ratio of measured impedances is very close to one and they are both measured using the same instrument. Equation (29) shows that the relative errors subtract.

$$\frac{\Delta\left(\frac{Z_{DUT\_LCR}}{Z_{STD\_LCR}}\right)}{\frac{Z_{DUT\_LCR}}{Z_{STD\_LCR}}}=\frac{\Delta Z_{DUT\_LCR}}{Z_{DUT\_LCR}}-\frac{\Delta Z_{STD\_LCR}}{Z_{STD\_LCR}} \tag{29}$$

This component remains for consideration because the standard and DUT measurements are not exactly the same and the uncertainty contribution due to this factor will be proportional to the measured difference.

Partial derivatives are investigated to find the uncertainty of the scaling ratio, *K* (see Eq. (6)).

$$\begin{aligned}\partial K=&\frac{\partial}{\partial\frac{Z_{100\,\mathrm{pF}\_LCR}}{Z_{1\,\mathrm{nF}\_LCR}}}\left(\frac{Z_{100\,\mathrm{pF}\_LCR}}{Z_{1\,\mathrm{nF}\_LCR}}\frac{Z_{1\,\mathrm{nF}\_CB}}{Z_{100\,\mathrm{pF}\_CB}}\right)\Delta\left(\frac{Z_{100\,\mathrm{pF}\_LCR}}{Z_{1\,\mathrm{nF}\_LCR}}\right)+\cdots\\ &\cdots\frac{\partial}{\partial\frac{Z_{1\,\mathrm{nF}\_CB}}{Z_{100\,\mathrm{pF}\_CB}}}\left(\frac{Z_{100\,\mathrm{pF}\_LCR}}{Z_{1\,\mathrm{nF}\_LCR}}\frac{Z_{1\,\mathrm{nF}\_CB}}{Z_{100\,\mathrm{pF}\_CB}}\right)\Delta\left(\frac{Z_{1\,\mathrm{nF}\_CB}}{Z_{100\,\mathrm{pF}\_CB}}\right)\end{aligned} \tag{30}$$

$$\frac{\Delta K}{K}=\frac{\Delta\left(\frac{Z_{100\mathrm{pF}\_LCR}}{Z_{1\mathrm{nF}\_LCR}}\right)}{\frac{Z_{100\mathrm{pF}\_LCR}}{Z_{1\mathrm{nF}\_LCR}}}+\frac{\Delta\left(\frac{Z_{1\mathrm{nF}\_CB}}{Z_{100\mathrm{pF}\_CB}}\right)}{\frac{Z_{1\mathrm{nF}\_CB}}{Z_{100\mathrm{pF}\_CB}}} \tag{31}$$

Equation (31) will be used to estimate the uncertainty of the scaling ration, *K*.

4.3. Dissipation Factor Error

The derivation of the dissipation factor error shown below follows from Eq. (20). The dissipation factor is given in radians and is commonly converted to µrad (10^{-6}×radians).

$$\partial DF=\frac{\partial}{\partial R}\left(\frac{R}{X}\right)\Delta R+\frac{\partial}{\partial X}\left(\frac{R}{X}\right)\Delta X=\left(\frac{1}{X}\right)\Delta R-\left(\frac{R}{X^2}\right)\Delta X \tag{32}$$

$$\partial DF=DF\,\frac{\Delta R}{R}-DF\,\frac{\Delta X}{X} \tag{33}$$

The measured dissipation factor is very small for the capacitors addressed in this investigation. The values range from about 1 μrad at low frequency for small capacitors, to 1000 μrad at higher frequencies for larger capacitors. Since for this capacitance calibration procedure the dissipation factor is very small, the following approximation holds: $\arctan\left(\frac{R}{X}\right) \approx \frac{R}{X}$ and $DF \approx \varphi$ (see Eq. (17)). This notion provides for simple phase calculations. From Eq. (27), the phase and dissipation factor estimates for the device under test are given by Eqs. (34) and (35).

$$\varphi_{Z\_DUT} = \varphi_K + \varphi_{\frac{Z_{DUT\_LCR}}{Z_{STD\_LCR}}} + \varphi_{Z\_STD} \tag{34}$$

$$\partial DF_{Z\_DUT} \approx \partial DF_K + \partial DF_{\frac{Z_{DUT\_LCR}}{Z_{STD\_LCR}}} + \partial DF_{Z\_STD} \tag{35}$$

5. Instrumentation Errors

5.1. LCR Meter Uncertainty

An LCR meter is used to measure the impedance of the capacitors. Table 8 presents the nominal measured impedance, |*Z*|. This information is important to establish the range of values and relate the uncertainty of the instrument to the measurements.

Table 8. Nominal measured impedances given in ohms.

	Frequency			
Capacitance	100 Hz	1 kHz	10 kHz	100 kHz
1 nF	1591549.431	159154.943	15915.494	1591.549
10 nF	159154.943	15915.494	1591.549	159.155
100 nF	15915.494	1591.549	159.155	15.915
1 μF	1591.549	159.155	15.915	1.592
10 μF	159.155	15.915	1.592	0.159
100 μF	15.915	1.592	0.159	N/A

Information for the LCR meter error analysis is taken from the LCR meter manual [7]. The manual provides information only on measurement accuracy. The accuracy refers to the manufacturer specifications and will be used as an uncertainty component for the LCR meter throughout this document. As given in the LCR meter manual, absolute measurement accuracy, A_T, when measuring the magnitude of the impendence of a capacitor (in %), is

$$A_T = A_E + A_{CAL}, \tag{36}$$

where A_E is the relative accuracy and A_{CAL} is the calibration accuracy. The absolute measurement accuracy, A_T, will be included as a factor in the computation of the capacitance and phase measurement uncertainties when using the LCR meter.

Relative accuracy, A_E , given in %, is defined as

$$A_E = \pm[A + (K_a + K_{aa} + K_b K_{bb} + K_c)100 + K_d]K_e, \qquad (37)$$

where the terms of the equation are

A	Basic accuracy
K_a	High impedance proportional factor
K_{aa}	Cable length factor for high impedance proportional factor
K_b	Low impedance proportional factor
K_{bb}	Cable length factor for low impedance proportional factor
K_c	Calibration interpolation factor
K_d	Cable length factor
K_e	Temperature factor

Table 9 shows the basic accuracy of the LCR meter [7]. The capacitance scaling measurement procedure uses the MEDIUM option for the measurement time parameter (this information is necessary to determine basic accuracy contribution for each capacitance value and frequency).

Table 9. Basic accuracy, *A*, obtained from manufacturer specifications, given in %.

DUT Capacitor	Capacitance Scaling Measurements	Frequency			
		100 Hz	1 kHz	10 kHz	100 kHz
10 nF	1 nF	0.2	0.05	0.05	0.05
	10 nF	0.1	0.05	0.05	0.05
100 nF	1 nF	0.2	0.05	0.05	0.07
	10 nF	0.1	0.05	0.05	0.07
	100 nF	0.1	0.05	0.05	0.07
1 μF	1 nF	0.2	0.05	0.07	0.1
	10 nF	0.1	0.05	0.07	0.1
	100 nF	0.1	0.05	0.07	0.1
	1 μF	0.1	0.05	0.07	0.1
10 μF	1 nF	0.2	0.07	0.1	0.6
	10 nF	0.1	0.07	0.1	0.6
	100 nF	0.1	0.07	0.1	0.6
	1 μF	0.1	0.07	0.1	0.6
	10 μF	0.1	0.07	0.1	0.6
100 μF	1 nF	0.14	0.1	0.6	N/A
	10 nF	0.14	0.1	0.6	
	100 nF	0.14	0.1	0.6	
	1 μF	0.14	0.1	0.6	
	10 μF	0.14	0.1	0.6	
	100 μF	0.14	0.1	0.6	

The software program that controls the LCR meter is set up to perform MEDIUM integration, so the following formulas apply for the impedance proportional factors.

$$K_a = \frac{0.001}{|Z_m|}\left(1+\frac{200}{V_s}\right) \text{ and } K_b = |Z_m|(1x10^{-9})\left(1+\frac{70}{V_s}\right), \tag{38}$$

where V_s is the test signal voltage given in mV. $|Z_m|$ is the impedance of the measured capacitor in ohms and is equal to $|Z_m| = \frac{1}{2\pi fC}$.

K_a is negligible for impedances above 500 Ω and K_b is negligible for impedances below 500 Ω. The results are summarized in the following tables. The K_a values are dominant in the case of high frequencies and high values of capacitance.

Table 10. Impedance proportional factor, K_a, values.

	Frequency			
Capacitance	100 Hz	1 kHz	10 kHz	100 kHz
1 nF	0	0	0	0
10 nF	0	0	0	0
100 nF	0	0	0	0.0001
1 μF	0	0	0.0001	0.0057
10 μF	0	0.0001	0.0057	0.2576
100 μF	0.0001	0.0057	0.2576	N/A

Table 11. Impedance proportional factor, K_b, values.

	Frequency			
Capacitance	100 Hz	1 kHz	10 kHz	100 kHz
1 nF	0.0018	0.0002	0	0
10 nF	0.0002	0	0	0
100 nF	0	0	0	0
1 μF	0	0	0	0
10 μF	0	0	0	0
100 μF	0	0	0	N/A

The cable length factor, $K_{aa} = 1$, and the cable length factor, $K_{bb} = 1 + 5f$. Parameter f is the frequency in kHz. In our case, the capacitor is connected to the LCR meter via a 0.61 m cable.

The calibration interpolation factor, $K_c = 0$, because all of the measurements are taken at direct calibration frequencies.

The cable length factor, $K_d = 0.00025(1 + 50f_m)$, for the calibration interpolation factor is given for a cable length of 1 m. Frequency, f_m, is entered in units of MHz, so the cable length factor is approximately 0.00025.

The temperature factor, $K_e = 1$. In our case, the temperature in the laboratory is 23 °C and $K_e = 1$ for the temperatures from 18 °C to 28 °C. When all of the manufacturer accuracy factors are combined, the LCR meter relative accuracy is obtained (see Table 12). Table 13 shows the LCR meter calibration accuracy.

Table 12. LCR meter relative accuracy, A_E, obtained from manufacturer specifications, given in %.

DUT Capacitor	Capacitance Scaling Measurement	Frequency			
		100 Hz	1 kHz	10 kHz	100 kHz
10 nF	1 nF	0.3781	0.0681	0.0522	0.0519
	10 nF	0.1180	0.0521	0.0506	0.0524
100 nF	1 nF	0.3781	0.0681	0.0522	0.0719
	10 nF	0.118	0.0521	0.0506	0.0728
	100 nF	0.102	0.0505	0.0512	0.0841
1 µF	1 nF	0.3781	0.0681	0.0726	0.1030
	10 nF	0.1180	0.0521	0.0707	0.1072
	100 nF	0.1020	0.0505	0.0717	0.1581
	1 µF	0.1005	0.0511	0.0829	0.6670
10 µF	1 nF	0.3781	0.0919	0.1068	0.6077
	10 nF	0.118	0.0724	0.1016	0.6276
	100 nF	0.102	0.0706	0.1061	0.8591
	1 µF	0.1005	0.0715	0.1569	3.1776
	10 µF	0.1011	0.0828	0.6659	26.3626
100 µF	1 nF	0.3552	0.161	0.6257	N/A
	10 nF	0.1617	0.1064	0.6055	
	100 nF	0.1424	0.1014	0.6264	
	1 µF	0.1405	0.106	0.858	
	10 µF	0.1411	0.1568	3.1765	
	100 µF	0.1528	0.6657	26.3614	

The factor, f, in Table 13 is frequency given in kHz. The magnitude of the calibration accuracy is given in percent. Note that only amplitude calibration accuracy is taken into consideration since it has the predominant influence on the uncertainty of the capacitors calibrated, as seen in Eq. (26).

Table 13. Calibration accuracy, A_{CAL}, obtained from manufacturer specifications, given in %.

Capacitance	Frequency			
	100 Hz	1 kHz	10 kHz	100 kHz
1 nF	$0.03+1\times10^{-3}f$	$0.03+1\times10^{-3}f$	$0.03+1\times10^{-3}f$	0.05
10 nF	$0.03+1\times10^{-3}f$	$0.03+1\times10^{-4}f$	0.03	0.05
100 nF	$0.03+1\times10^{-4}f$	0.03	0.05	0.05
1 µF	0.03	0.03	0.05	0.05
10 µF	0.03	0.03	0.05	0.05
100 µF	0.03	0.03	0.05	N/A

LCR meter measurements are used in ratios in the capacitance scaling calculations. Because the scaling fixture allows the LCR meter to measure two very similar impedances, Z_{DUT} and Z_{STD}, the manufacturer specified accuracy was modified in the following way to include differential nonlinearity, which comes into play when the two measurements are not the same magnitude, as given in Eq. (39). Throughout the rest of this document, the differential nonlinearity given in Eq. (39) will be included when estimating the measurement uncertainty. It will be called the LCR meter nonlinearity, *u(LCR_NL)*. The LCR meter nonlinearity is the dominant factor in the measurement uncertainty of low impedance capacitors.

$$u\left(LCR\_NL\right) = \frac{(A_E + A_{CAL})}{\sqrt{3}} \frac{Z_{DUT} - Z_{STD}}{Z_{STD}} \tag{39}$$

Treating the LCR meter maximum uncertainty as a Type B uncertainty, the standard uncertainty estimate must be divided by $\sqrt{3}$ to comply with the guidelines for the expression of measurement uncertainty [9].

5.2. Automatic Capacitance Bridge Uncertainty

The capacitance scaling system uses a commercial automatic capacitance bridge [8]. This instrument was calibrated NIST and that information was used in the uncertainty analysis in this report. The manufacturer specifications were used in the cases where NIST calibration was not available.

Table 14 summarizes the uncertainty of the capacitance bridge determined at NIST when measuring 100 pF and 1 nF capacitors.

Table 14. Combined standard uncertainty of the capacitance bridge as calibrated at NIST. The values are given in parts in 10^6.

Frequency	Capacitance	
	100 pF	1 nF
100 Hz	0.7	1.0
1 kHz	0.3	0.6
10 kHz	6.0	6.3

6. Capacitance Uncertainty

6.1. 10 nF Capacitance

The impedance of the 10 nF capacitor is measured as

$$Z_{10\ \mathrm{nF}\_CAL} = K \frac{Z_{10\ \mathrm{nF}\_LCR}}{Z_{1\ \mathrm{nF}\_LCR}} Z_{1\ \mathrm{nF}\_CB}\,. \tag{40}$$

In order to find the uncertainty of the 10 nF capacitor, the uncertainty of the scaling ratio, *K*, must be determined. The scaling ratio is measured using the equation

$$K = \frac{Z_{1\,\mathrm{nF}\_CB}}{Z_{100\,\mathrm{pF}\_CB}} \frac{Z_{100\,\mathrm{pF}\_LCR}}{Z_{1\,\mathrm{nF}\_LCR}}. \tag{41}$$

From Eq. (41), the uncertainty of the scaling ratio is calculated as

$$u_c(K) = \sqrt{\left(s\left(\frac{Z_{100\,\mathrm{pF}\_CB}}{Z_{1\,\mathrm{nF}\_CB}}\right)\right)^2 + \left(s\left(\frac{Z_{100\,\mathrm{pF}\_LCR}}{Z_{1\,\mathrm{nF}\_LCR}}\right)\right)^2 + u^2(LCR\_NL) + u^2(CB\_NL)}, \tag{42}$$

where *s(x)* represents the standard deviation of measurement *x*. *u(LCR_NL)* is the uncertainty contribution from the nonlinearity of the LCR meter and scaling fixture, and *u(CB_NL)* is the uncertainty contribution from the nonlinearity of the capacitance bridge.

While the 100 pF and 1 nF air capacitors have insignificant dissipation factors over the frequency range of interest, their capacitance is not particularly stable (typically 30 parts in 10^6 per degree C). Over the course of several months of measurements using the capacitance scaling system, we observed fluctuations in the 1 kHz capacitance of standard air capacitors on the order of 10 parts in 10^6. In order to discriminate between the uncertainty of the scaling system and the instability of the 4TP air standards, more stable 3T nitrogen capacitors were examined. At 1 kHz, the nitrogen capacitors varied less than 0.1 part in 10^6. The nitrogen capacitors are connected to the 4TP scaling fixture using a coaxial adapter.

A 3T capacitance bridge is used to measure 3T capacitors of values 100 pF and 1 nF at frequencies of 100 Hz, 1 kHz and 10 kHz. The only instance where the 3T capacitance bridge is used to measure 4TP capacitors is at 1 kHz. These measurements are used in calculations to extrapolate the 4TP capacitance value at 100 kHz, [4, 5].

In order to characterize the effects of the leads on capacitance measurements when a 3T capacitance bridge is used in the measurement procedure, the circuit shown in Figure 6 is analyzed. The ratio $\frac{V_2}{V_1}$ is used to estimate the lead influence, as analyzed in Eq. (43).

$$\frac{V_2}{V_1} = \frac{\frac{1}{j\omega C}\,\frac{1}{j\omega C_{SF}}\,\frac{1}{j\omega(C_C + C_{DUT})}}{\left(R + j\omega L + \frac{1}{j\omega C}\right)\left(\frac{1}{j\omega C} + R_{SF} + j\omega L_{SF} + \frac{1}{j\omega C_{SF}}\right)\left(\frac{1}{j\omega(C_C + C_{DUT})} + R_C + j\omega L_C + \frac{1}{j\omega C_{SF}}\right)\ldots cont}$$

$$\cdots \overline{-\left(R + j\omega L + \frac{1}{j\omega C}\right)\left(\frac{1}{j\omega C_{SF}}\right)^2 - \left(\frac{1}{j\omega(C_C + C_{DUT})} + R_C + j\omega L_C + \frac{1}{j\omega C_{SF}}\right)\left(\frac{1}{j\omega C}\right)^2} \tag{43}$$

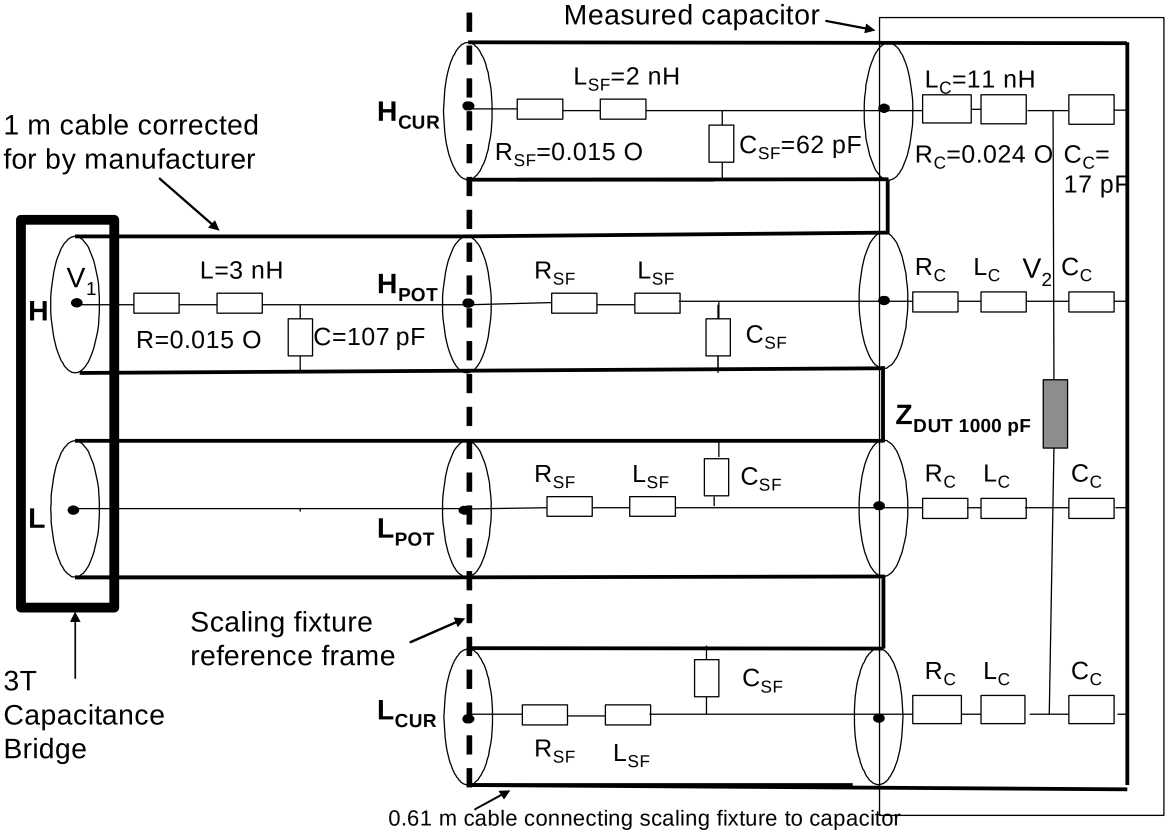


Figure 6. Measurement setup for 4TP 1 nF capacitor using 3T capacitance bridge.

This ratio is nominally one. When the 4TP 1 nF air capacitor is measured at 1 kHz, the deviation of the ratio from one is computed to be less than 1 part in 10^9. Assuming similar circuit parameters for the 1 nF nitrogen capacitor, the computed ratio deviation at 10 kHz is less than 1 part in 10^7. The lead influence is minimized further due to the fact that the 3T capacitance bridge manufacturer automatically compensates for the lead impedances of the 1-meter measurement cables.

During previous work characterizing 4TP air standard capacitors using a network analyzer, we observed minimal change between the capacitance values at 1 kHz and 100 kHz, when calibrating the 100 pF and 1 nF capacitors [5, 6]. The differences between the capacitances measured at 1 kHz and those measured at 100 kHz are less than 1 part in 10^6, with an expanded uncertainty ($k = 2$) of 25 parts in 10^6. Measured dissipation factors for the same 100 pF and 1 nF air capacitors are less than 1 µrad from 1 kHz to 100 kHz, with an expanded uncertainty ($k = 2$) of 24 µrads, for both capacitors. Based on the uncertainty analysis in section 6, we observe that the measurement noise from the scaling fixture and LCR meter have significantly more effect on the overall measurement uncertainty than the 1 kHz-to-100 kHz extrapolation factor.

Additionally, a 1 nF 4TP air capacitor was measured at 1 kHz and 10 kHz using the 3T capacitance bridge. The difference between the recorded values was on the order of 1 part in 10^6. The combined standard uncertainty of the capacitance bridge, when measuring 1 nF at 10 kHz, is on the order of 6 parts in 10^6. The same capacitor was characterized at the same frequencies using the network analyzer approach, and the differences were less than 1 part in 10^6. The combined standard uncertainty of the network analyzer approach to characterizing capacitance and dissipation factor for 4TP air standard capacitors is 13 parts in 10^6, and 12 μrads, respectively, at 10 kHz.

Therefore, the previous work allows for 1 kHz 3T measurements of the 100 pF and 1 nF air capacitors to be used at 100 kHz. The capacitance measurements at 1 kHz are used to estimate the impedance of the capacitors at 100 kHz, according to $Z_{at\ 100\ \mathrm{kHz}} = \frac{1}{j\omega C_{measured\ \ at\ 1\ \mathrm{kHz}}}$.

Measurements were performed to determine the nonlinearity of the LCR meter. Using capacitors that varied from nominal as much as 100 parts in 10^6, the LCR meter readings were in error by less than 1 part in 10^6, supporting previous studies [2]. The LCR meter nonlinearity is not the dominant error when measuring a 10 nF capacitor, but it becomes more significant for higher valued capacitors at higher frequencies. The nonlinearity of the capacitance bridge was found to be less than 1 part in 10^6.

To investigate the stability of the scaling fixture, we replaced the capacitance scaling IVD with a commercial IVD of known stability and observed the same level of variations in the scaling ratio, *K*. To determine whether this instability is time dependent, we temporarily modified the software routine, using only one measurement point, to shorten the measurement process from 30 minutes to 5 minutes. The standard deviation of the scaling ratio was reduced by nearly an order of magnitude to approximately 0.3 part in 10^6. This indicates that the stabilities of the interface circuitry, the LCR meter, and the capacitors are the main contributors to the uncertainty of the scaling ratio calibration.

Two sets of capacitors are used in the measurement procedure. Measurements at 100 Hz, 1 kHz and 10 kHz are performed using 3T nitrogen capacitors and the capacitance and dissipation factor are measured using a multi-frequency capacitance bridge. The nitrogen capacitors used in this part of the measurement procedure are stable to better than 0.5 part in 10^6. The nitrogen capacitors have significant frequency dependence at 100 kHz, so the 100 kHz measurement is done with 4TP air capacitors exhibiting smaller dissipation factor. An extrapolation method is used to determine the performance of the air capacitors at 100 kHz [3, 4]. The air capacitors are kept in an insulated case so that user handling of the capacitors is eliminated and temperature changes are minimized during measurements.

The LCR meter measures the same nominal impedance while measuring the 100 pF capacitor and the 1 nF capacitor with the scaling ratio of 0.1. However, the two measurements are not exactly the same. The contribution from this difference is labeled LCR nonlinearity and is given in Eq. (39).

Table 15 shows the LCR meter uncertainties and the measured differences. The measurement uncertainty contribution from the differences is the product of the two columns, as shown in column 4 of Table 16, which shows all uncertainty contributions from Eq. (42).

Table 15. Actual LCR meter deviations when measuring 100 pF and 1 nF capacitors for 10 nF scaling ratio calibration.

Frequency	LCR Meter Uncertainty $\frac{(A_E + A_{CAL})}{\sqrt{3}}$ [parts in 10^6]	LCR Meter Deviations $\frac{Z_{100\ \mathrm{pF}\_measured} - Z_{1\ \mathrm{nF}\_measured}}{Z_{1\ \mathrm{nF}\_measured}}$
100 Hz	2362	0.000020
1 kHz	624	0.000030
10 kHz	532	0.000040
100 kHz	588	0.000090

The maximum LCR meter linearity uncertainty component over all measurements performed during this investigation was not more 0.1 part in 10^6. However, for another set of calibrated capacitors with values further from nominal, the measured differences will be larger. The LCR meter nonlinearity contributions should reflect the new measured differences.

Table 16. Combined standard uncertainty of scaling ratio for 10 nF capacitor, in parts in 10^6.

Frequency	$s\left(\frac{Z_{100\,\mathrm{pF}\_LCR}}{Z_{1\,\mathrm{nF}\_LCR}}\right)$ data obtained from a set of measurements	$s\left(\frac{Z_{100\mathrm{pF}\_CB}}{Z_{1\,\mathrm{nF}\_CB}}\right)$ data obtained from NIST capacitance bridge calibration	LCR Meter Nonlinearity for *K* calculated from Equation (39) and Table 14	$u_c(K_{10\ \mathrm{nF}})$ calculated from Equation (42)
100 Hz	8	1	0.0	9
1 kHz	6	0.6	0.1	6
10 kHz	5	6.3	0.0	8
100 kHz	5	13*	0.1	13

(*) The asterisk indicates a measurement using 4TP air capacitors and its capacitance is obtained from measurement at 1 kHz and extrapolation to 100 kHz. The uncertainty of the measurement procedure is 13 parts in 10^6.

At 100 kHz, the uncertainty of the scaling ratio, *K*, is computed using

$$u_c(K) = \sqrt{u^2\left(\frac{Z_{100\ \mathrm{pF}\_CB}}{Z_{1\ \mathrm{nF}\_CB}}\right) + \left(s\left(\frac{Z_{100\ \mathrm{pF}\_LCR}}{Z_{1\ \mathrm{nF}\_LCR}}\right)\right)^2 + u^2(LCR\_NL) + u^2(CB\_NL)}, \tag{44}$$

where the first term is contribution of the capacitance bridge using an extrapolation to 100 kHz and the second term is the contribution of the LCR meter measurements at 100 kHz. The capacitance bridge nonlinearity for measuring 100 pF and 1 nF is less than 1 part in 10^6 for all measurements addressed in this document.

The next step is to calculate the uncertainty in measuring the 10 nF capacitor. Based on the formula used to perform the measurements, we have

$$Z_{10\ \mathrm{nF\_CAL}} = K \frac{Z_{10\ \mathrm{nF}\_LCR}}{Z_{1\ \mathrm{nF}\_LCR}} Z_{1\ \mathrm{nF}\_CB} \,. \tag{45}$$

The uncertainty of the 10 nF capacitance is computed using

$$u_c(C_{10\ \mathrm{nF}}) = \sqrt{u^2\left(Z_{1\ \mathrm{nF}\_CB}\right) + \left(s\left(\frac{Z_{10\ \mathrm{nF}\_LCR}}{Z_{1\ \mathrm{nF}\_LCR}}\right)\right)^2 + u^2\left(LCR\_NL\right) + u^2\left(CB\_NL\right)}, \tag{46}$$

where $u(Z_{1\ \mathrm{nF}\_CB})$ is the Type B uncertainty of the capacitance bridge derived from measuring the 1 nF capacitor as calibrated by NIST. Table 17 presents the uncertainty components for the 10 nF capacitor. Note that the nonlinearity of the capacitance bridge is insignificantly small and is not included.

Table 17. Uncertainty contributions for 10 nF capacitor, in parts in 10^6.

Frequency	$s\left(\frac{Z_{100\ \mathrm{pF}\_LCR}}{Z_{1\ \mathrm{nF}\_LCR}}\right)$ data obtained from a set of measurements	$u_c\left(K_{10\ \mathrm{nF}}\right)$	$u_c\left(Z_{1\ \mathrm{nF}\_CB}\right)$ data obtained from NIST capacitance bridge calibration	LCR Meter Nonlinearity for 10 nF data obtained from a set of measurements and LCR meter uncertainty specifications
100 Hz	8	9	1	0.0
1 kHz	10	6	0.6	0.0
10 kHz	11	8	6.3	0.0
100 kHz	5	13	13*	0.1

*At 100 kHz, the uncertainty of the 10 nF capacitor is modified using

$$u_c(C_{10\ \mathrm{nF}}) = \sqrt{u^2\left(Z_{1\ \mathrm{nF}\_CB}\right) + u^2\left(K\right) + \left(s\left(\frac{Z_{100\ \mathrm{pF}\_LCR}}{Z_{1\ \mathrm{nF}\_LCR}}\right)\right)^2 + u^2\left(LCR\_NL\right) + u^2\left(CB\_NL\right)}. \tag{47}$$

The calculated uncertainty of a 10 nF capacitor is given in Table 18. For comparison, the standard deviation of the measured capacitance is also presented. The agreement is acceptable.

Table 18. Combined standard uncertainty of 10 nF capacitance.

Frequency	Applied voltage [mV]	$u_c(C_{10\text{ nF}})$ [parts in 10^6]	$\frac{s\left(C_{10\text{ nF}}\right)}{C_{10\text{ nF}}}$ [parts in 10^6] data obtained from a set of measurements
100 Hz	600	12	12
1 kHz	600	12	13
10 kHz	600	15	10
100 kHz	600	18	9

6.1.1. Additional Tests of the 10 nF Capacitor

The improved stability of the nitrogen dielectric standard capacitors comes at the expense of increased frequency dependence. The frequency dependence of the 1 nF standard was measured using the NIST 4TP Bridge designed by Cutkosky [10]. The results, taken from [6], are given in Table 19.

Table 19. Frequency dependence of the 1 nF standard capacitor (nitrogen dielectric) measured using the NIST 4TP Bridge, in parts in 10^6.

Frequency [kHz]	0.1	1	10
1 nF Deviation from Nominal Value	7.28	8.74	9.83

A 10 nF nitrogen dielectric capacitor with short-term stability of better than 1 part in 10^7 was measured using the NIST 4TP Bridge and the capacitance scaling system. A summary of the results is shown in Table 20.

Table 20. 10 nF capacitor measurements at several frequencies, in parts in 10^6. The * denotes the Type A uncertainty as calculated based on the observed measurement standard deviations.

	Deviation from NIST 4TP Bridge Measurement			Measurement Uncertainty Coverage Factor (k = 1)		
Frequency [kHz]	0.1	1	10	0.1	1	10
NIST 4TP Bridge	**0**	**0**	**0**	**0.3**	**<0.1**	**0.2**
Capacitance Scaling System	**19**	**1.9**	**1.2**	**23***	**4***	**6***

The results in Table 20 show excellent agreement between the NIST manual bridge and the capacitance scaling system at 1 kHz and 10 kHz using stable 3T nitrogen capacitors. The capacitance scaling measurements were done before the voltage levels were controlled using the ALC implementation and the agreement at 100 Hz is not as good due to the LCR meter performance. The latest measurements have much improved stability.

6.2. 100 nF Capacitance

Derivations similar to those of the 10 nF capacitor are used for the 100 nF capacitor. The formula to find the 100 nF impedance is given in Eq. (48) and its uncertainty is calculated in Eq. (49).

$$Z_{100\text{ nF}\,CAL} = K\,\frac{Z_{100\text{ nF}\_LCR}}{Z_{10\text{ nF}\_LCR}}\,Z_{10\text{ nF}\,CAL} \tag{48}$$

$$u_c(C_{100\text{ nF}}) = \sqrt{u_c^{\,2}(K) + \left(s\left(\frac{Z_{100\text{ nF}\_LCR}}{Z_{10\text{ nF}\_LCR}}\right)\right)^2 + u_c^{\,2}(C_{10\text{ nF}}) + u^2(LCR\_NL)} \tag{49}$$

The LCR meter nonlinearity contributions for *K* and 10 nF are negligible and the data for 100 nF is given in Table 21. Table 22 provides data necessary for the calculation of the uncertainty for the 100 nF capacitor. The notation $C_{10\text{ nF}/100\text{ nF}}$ refers to the intermediate calibration of the 10 nF capacitor used in the calibration of the 100 nF capacitor. Similar notation will be used to reference other intermediate steps in the calibration process. The capacitance uncertainty for the 100 nF capacitor is given in Table 23.

Table 21. LCR meter deviations when measuring the 10 nF and 100 nF capacitors for the 100 nF calibration.

Frequency	LCR Meter Uncertainty $\frac{(A_E + A_{CAL})}{\sqrt{3}}$ [parts in 10^6]	LCR Meter Deviations $\frac{Z_{10\text{ nF}\_measured} - Z_{100\text{ nF}\_measured}}{Z_{100\text{ nF}\_measured}}$	LCR Meter Nonlinearity for 100 nF [parts in 10^6]
100 Hz	763	0.000100	0.1
1 kHz	465	0.000120	0.1
10 kHz	584	0.000080	0.0
100 kHz	774	0.002230	1.7

Table 22. Uncertainty contributions for the 100 nF capacitor, in parts in 10^6. Note that the 10 nF capacitor is calibrated as part of the 100 nF calibration.

Frequency	$s\left(\frac{Z_{100\,\mathrm{pF}\_LCR}}{Z_{1\,\mathrm{nF}\_LCR}}\right)$ data obtained from a set of measurements	$u_c(K_{100\,\mathrm{nF}})$	$s\left(\frac{Z_{10\,\mathrm{nF}\_LCR}}{Z_{1\,\mathrm{nF}\_LCR}}\right)$ data obtained from a set of measurements	$u_c(C_{10\,\mathrm{nF}/100\,\mathrm{nF}})$	$s\left(\frac{Z_{100\,\mathrm{nF}\_LCR}}{Z_{10\,\mathrm{nF}\_LCR}}\right)$ data obtained from a set of measurements
100 Hz	6	6	2	7	2
1 kHz	7	7	11	13	14
10 kHz	9	11	5	13	3
100 kHz	6	10	13	18	4

Table 23. Combined standard uncertainty of 100 nF capacitance.

Frequency	Applied voltage [mV]	$u_c(\mathrm{C}_{100\,\mathrm{nF}})$ [parts in 10^6]	$\frac{s(C_{100\,\mathrm{nF}})}{C_{100\,\mathrm{nF}}}$ [parts in 10^6] data obtained from a set of measurements
100 Hz	600	10	12
1 kHz	600	21	19
10 kHz	600	18	23
100 kHz	200	23	18

6.3. 1 µF Capacitance

The 1 µF capacitor is characterized using the formula

$$Z_{1\,\mu\mathrm{F}\_CAL} = K\,\frac{Z_{1\,\mu\mathrm{F}\_LCR}}{Z_{100\,\mathrm{nF}\_LCR}}\,Z_{100\,\mathrm{nF}\_CAL}\,. \tag{50}$$

The uncertainty for the 1 µF capacitance is given as

$$u_c(C_{1\,\mu\mathrm{F}}) = \sqrt{u_c^{\,2}(K) + \left(s\left(\frac{Z_{1\mu\mathrm{F}\_LCR}}{Z_{100\,\mathrm{nF}\_LCR}}\right)\right)^2 + u_c^{\,2}(C_{100\,\mathrm{nF}}) + u^2(LCR\_NL)}\,. \tag{51}$$

The LCR meter nonlinearity contributions for *K*, 10 nF, and 100 nF are small and the data for 100 nF is given in Table 24. Note the high value of the uncertainty at 100 kHz. Table 25 provides data necessary for the calculation of the uncertainty for the 1 µF capacitor. Table 26 gives the capacitance uncertainty for the 100 nF capacitor calibration for the 1 μF capacitor. Table 27 gives the capacitance uncertainty for the 1 μF capacitor.

Table 24. LCR meter deviations when measuring the 100 nF and 1 μF capacitors for the 1 μF calibration.

Frequency	LCR Meter Uncertainty $\frac{(A_E + A_{CAL})}{\sqrt{3}}$ [parts in 10^6]	LCR Meter Deviations $\frac{Z_{100\ \mathrm{nF}\_measured} - Z_{1\ \mu\mathrm{F}\_measured}}{Z_{1\ \mu\mathrm{F}\_measured}}$	LCR Meter Nonlinearity for 1 μF [parts in 10^6]
100 Hz	753	0.000008	0.0
1 kHz	468	0.000008	0.0
10 kHz	767	0.000518	0.4
100 kHz	4139	0.022370	92.6

Table 25. Uncertainty contributions for the 1 μF capacitor, in parts in 10^6. Note that there is a 10 nF calibration step as part of the 1 μF calibration.

Frequency	$s\left(\frac{Z_{100\ \mathrm{pF}\_LCR}}{Z_{1\ \mathrm{nF}\_LCR}}\right)$ data obtained from a set of measurements	$u_c\left(K_{1\ \mu\mathrm{F}}\right)$	$s\left(\frac{Z_{10\ \mathrm{nF}\_LCR}}{Z_{1\ \mathrm{nF}\_LCR}}\right)$ data obtained from a set of measurements	$u_c\left(C_{10\ \mathrm{nF}/1\ \mu\mathrm{F}}\right)$
100 Hz	8	8	1	8
1 kHz	7	7	6	10
10 kHz	19	20	10	23
100 kHz	3	13	7	19

Table 26. Capacitance uncertainty for the 100 nF capacitor, in parts in 10^6.

Frequency	$s\left(\frac{Z_{100\ \mathrm{nF}\_LCR}}{Z_{10\ \mathrm{nF}\_LCR}}\right)$ data obtained from a set of measurements	$u_c\left(C_{100\ \mathrm{nF}/1\ \mu\mathrm{F}}\right)$	$s\left(\frac{Z_{1\ \mu\mathrm{F}\_LCR}}{Z_{100\ \mathrm{nF}\_LCR}}\right)$ data obtained from a set of measurements
100 Hz	3	12	2
1 kHz	7	14	5
10 kHz	7	32	2
100 kHz	8	24	7

Table 27. Combined standard uncertainty of 1 μF capacitance.

Frequency	Applied voltage [mV]	$u_c(C_{1\,\mu\mathrm{F}})$ [parts in 10^6]	$\frac{s(C_{1\,\mu\mathrm{F}})}{C_{1\,\mu\mathrm{F}}}$ [parts in 10^6] data obtained from a set of measurements
100 Hz	600	15	25
1 kHz	600	16	31
10 kHz	200	38	63
100 kHz	25	97	9

Note the significant discrepancy between the low standard deviation of measurements at 100 kHz and the high uncertainty. The main contribution comes from the LCR nonlinearity component.

6.4. 10 μF Capacitance

The 10 μF capacitor is characterized using Eqs. (52) and (53).

$$Z_{10\,\mu\mathrm{F}\_CAL} = K\,\frac{Z_{10\,\mu\mathrm{F}\_LCR}}{Z_{1\,\mu\mathrm{F}\_LCR}}\,Z_{1\,\mu\mathrm{F}\_CAL} \tag{52}$$

$$u_c(C_{10\,\mu\mathrm{F}}) = \sqrt{u_c^{\,2}(K) + \left(s\left(\frac{Z_{10\,\mu\mathrm{F}\_LCR}}{Z_{1\,\mu\mathrm{F}\_LCR}}\right)\right)^2 + u_c^{\,2}(C_{1\,\mu\mathrm{F}/10\,\mu\mathrm{F}}) + u^2(LCR\_NL)} \tag{53}$$

The 10 μF LCR meter nonlinearity contribution was similar to that for the 1 μF capacitor. The data for 10 μF is given in Table 28. Note the large contribution at 100 kHz. The reason for this inflation is that the impedance of the measured capacitor is 0.16 Ω and the measured impedance for 10 μF and 1 μF differ by 2.2%. The optimal impedance to be measured by the LCR meter is 500 Ω. Table 29 provides data necessary to calculate of the uncertainty for the 10 μF capacitor. Table 30 gives the uncertainty contributors to the 10 μF capacitance measurement. The combined standard 10 μF capacitance uncertainty is given in Table 31. Note the discrepancy between the low standard deviation of measurements at 100 kHz and the high uncertainty.

Table 28. LCR meter deviations when measuring the 1 μF and 10 μF capacitors for the 10 μF calibration.

Frequency	LCR Meter Uncertainty $\frac{(A_E + A_{CAL})}{\sqrt{3}}$ [parts in 10^6]	LCR Meter Deviations $\frac{Z_{1\,\mu\mathrm{F}\_measured} - Z_{10\,\mu\mathrm{F}\_measured}}{Z_{10\,\mu\mathrm{F}\_measured}}$	LCR Meter Nonlinearity for 10 μF [parts in 10^6]
100 Hz	757	0.000030	0.0
1 kHz	651	0.000040	0.0
10 kHz	4133	0.000170	0.7
100 kHz	152493	0.022000	3354.9

Table 29. Uncertainty contributions for the 10 μF capacitor, in parts in 10^6. Note that there is a 10 nF calibration step as part of the 10 μF calibration.

Frequency	$s\left(\frac{Z_{100\,\mathrm{pF}\_LCR}}{Z_{1\,\mathrm{nF}\_LCR}}\right)$ data obtained from a set of measurements	$u_c(K_{10\,\mu\mathrm{F}})$	$s\left(\frac{Z_{10\,\mathrm{nF}\_LCR}}{Z_{1\,\mathrm{nF}\_LCR}}\right)$ data obtained from a set of measurements	$u_c(C_{10\,\mathrm{nF}/10\,\mu\mathrm{F}})$
100 Hz	12	12	3	13
1 kHz	35	35	13	37
10 kHz	43	44	3	44
100 kHz	15	20	15	28

Table 30. Uncertainty contributions for the 10 μF capacitor, in parts in 10^6. Note that there are 100 nF and 1 μF calibration steps as part of the 10 μF calibration.

Frequency	$s\left(\frac{Z_{100\,\mathrm{nF}\_LCR}}{Z_{10\,\mathrm{nF}\_LCR}}\right)$ data obtained from a set of measurements	$u_c(C_{100\,\mathrm{nF}/10\,\mu\mathrm{F}})$	$s\left(\frac{Z_{1\,\mu\mathrm{F}\_LCR}}{Z_{100\,\mathrm{nF}\_LCR}}\right)$ data obtained from a set of measurements	$u_c(C_{1\,\mu\mathrm{F}/10\,\mu\mathrm{F}})$	$s\left(\frac{Z_{10\,\mu\mathrm{F}\_LCR}}{Z_{1\,\mu\mathrm{F}\_LCR}}\right)$ data obtained from a set of measurements
100 Hz	22	28	2	31	3
1 kHz	7	51	4	62	2
10 kHz	10	63	7	77	26
100 kHz	8	35	25	416	679

Table 31. Combined standard uncertainty of 10 µF capacitance.

Frequency	Applied voltage [mV]	$u_c(C_{10\,\mu\text{F}})$ [parts in 10^6]	$\frac{s(C_{10\,\mu\text{F}})}{C_{10\,\mu\text{F}}}$ [parts in 10^6] data obtained from a set of measurements
100 Hz	600	33	44
1 kHz	200	71	116
10 kHz	25	92	190
100 kHz	5	3448	645

6.5. 100 µF Capacitance

The 100 µF capacitor is characterized using equations (54) and (55).

$$Z_{100\,\mu\text{F}\_CAL} = K\,\frac{Z_{100\,\mu\text{F}\_LCR}}{Z_{10\,\mu\text{F}\_LCR}}\,Z_{10\,\mu\text{F}\_CAL} \tag{54}$$

$$u_c(C_{10\,\mu\text{F}}) = \sqrt{u_c^{\,2}(K) + \left(s\left(\frac{Z_{100\,\mu\text{F}\_LCR}}{Z_{10\,\mu\text{F}\_LCR}}\right)\right)^2 + u_c^{\,2}(C_{10\,\mu\text{F}}) + u^2(LCR\_NL)} \tag{55}$$

The LCR meter nonlinearity contribution was similar to the data presented for the 10 µF capacitor. The data for the 100 µF capacitor is given in Table 32. Note the large contribution at 10 kHz. The reason for this inflation is the fact that the impedance of the measured capacitor is 0.16 Ω and the measured impedance for 100 µF and 10 µF differ by 4.5%. The optimal impedance to be measured by the LCR meter is 500 Ω. Table 33 provides data necessary for the calculation of the uncertainty for the 100 µF capacitor. Table 34 gives the uncertainty contributors to the 100 µF capacitor calibration. Table 35 shows the 100 µF combined standard uncertainty. Table 36 gives a summary of the expanded capacitance uncertainties (*k* = 2) for the scaling procedure. Note that the expanded uncertainties are approximately twice the combined standard uncertainties reported in Tables 18, 23, 27, 31, and 35.

Table 32. LCR meter deviations when measuring the 100 μF and 10 μF capacitors for the 100 μF calibration.

Frequency	LCR Meter Uncertainty $\frac{(A_E + A_{CAL})}{\sqrt{3}}$ [parts in 10^6]	LCR Meter Deviations $\frac{Z_{10\ \mu\text{F}\_measured} - Z_{100\ \mu\text{F}\_measured}}{Z_{100\ \mu\text{F}\_measured}}$	LCR Meter Nonlinearity for 100 μF [parts in 10^6]
100 Hz	1055	0.000259	0.3
1 kHz	4016	0.000062	0.2
10 kHz	152486	0.004489	684.5

Table 33. Uncertainty contributions for the 100 μF capacitor, in parts in 10^6. Note that there are 10 nF and 100 nF calibration steps as part of the 100 μF calibration.

Frequency	$s\left(\frac{Z_{100\ \text{pF}\_LCR}}{Z_{1\ \text{nF}\_LCR}}\right)$ data obtained from a set of measurements	$u_c\left(K_{100\ \mu\text{F}}\right)$	$s\left(\frac{Z_{10\ \text{nF}\_LCR}}{Z_{1\ \text{nF}\_LCR}}\right)$ data obtained from a set of measurements	$u_c\left(C_{10\ \text{nF}/100\ \mu\text{F}}\right)$	$s\left(\frac{Z_{100\ \text{nF}\_LCR}}{Z_{10\ \text{nF}\_LCR}}\right)$ data obtained from a set of measurements	$u_c\left(C_{100\ \text{nF}/100\ \mu\text{F}}\right)$
100 Hz	15	15	41	44	105	115
1 kHz	114	114	189	221	82	261
10 kHz	262	262	214	339	73	435

Table 34. Uncertainty contributions for the 100 μF capacitor, in parts in 10^6, cont'd. Note that there are 1 μF and 10 μF calibration steps and as part of the 100 μF calibration.

$s\left(\frac{Z_{1\ \mu\text{F}\_LCR}}{Z_{100\ \text{nF}\_LCR}}\right)$ data obtained from a set of measurements	$u_c\left(C_{1\ \mu\text{F}/100\ \mu\text{F}}\right)$	$s\left(\frac{Z_{10\ \mu\text{F}\_LCR}}{Z_{1\ \mu\text{F}\_LCR}}\right)$ data obtained from a set of measurements	$u_c\left(C_{10\ \mu\text{F}/100\ \mu\text{F}}\right)$	$s\left(\frac{Z_{100\ \mu\text{F}\_LCR}}{Z_{10\ \mu\text{F}\_LCR}}\right)$ data obtained from a set of measurements
4	116	23	119	257
12	285	42	310	62
513	722	455	893	962

Table 35. Combined standard uncertainty of 100 μF capacitance.

Frequency	Applied voltage [mV]	$u_c\left(C_{100\,\mu\text{F}}\right)$ [parts in 10^6]	$\frac{s\left(C_{100\,\mu\text{F}}\right)}{C_{100\,\mu\text{F}}}$ [parts in 10^6] data obtained from a set of measurements
100 Hz	200	284	101
1 kHz	25	336	577
10 kHz	5	1503	806

Table 36. Summary of expanded capacitance uncertainty (*k* = 2), in parts in 10^6.

Frequency	Capacitance				
	10 nF	100 nF	1 μF	10 μF	100 μF
100 Hz	25	20	35	70	570
1 kHz	25	43	40	150	680
10 kHz	30	40	90	200	3000
100 kHz	35	43	200	7000	n/a

7. Dissipation Factor Uncertainty

7.1. 10 nF Dissipation Factor

The phase uncertainty of the scaling ratio is calculated using the uncertainty of the capacitance bridge and the LCR meter as shown in Eq. (56).

$$\partial K = \left(\frac{Z_{1\,\text{nF}\_CB}}{Z_{100\,\text{pF}\_CB}}\right)\Delta\left(\frac{Z_{100\,\text{pF}\_LCR}}{Z_{1\,\text{nF}\_LCR}}\right) + \left(\frac{Z_{100\,\text{pF}\_LCR}}{Z_{1\,\text{nF}\_LCR}}\right)\Delta\left(\frac{Z_{1\,\text{nF}\_CB}}{Z_{100\,\text{pF}\_CB}}\right) \tag{56}$$

As a reminder, Eqs. (7) and (8) showed that the scaling ratio is represented in terms of magnitude and phase. Since the phase uncertainty is presented in μrad, the contribution from the LCR meter to the scaling ratio phase uncertainty is multiplied by the factor $\frac{Z_{1\,\text{nF}\_CB}}{Z_{100\,\text{pF}\_CB}}$ that has a nominal value of 0.1. The factor $\frac{Z_{100\,\text{pF}\_LCR}}{Z_{1\,\text{nF}\_LCR}}$ has a nominal value of 1.

Equation (57) presents the scaling ratio phase uncertainty calculation.

$$u_c(K_\varphi)=\sqrt{\begin{array}{l}\left(0.1\,s\left(\varphi_{\frac{Z_{100\,\mathrm{pF}\_LCR}}{Z_{1\,\mathrm{nF}\_LCR}}}\right)\right)^2+u^2\left(DF_{100\,\mathrm{pF}\_CB}\right)+u^2\left(DF_{1\,\mathrm{nF}\_CB}\right)+\cdots\\ \cdots\left(\dfrac{0.1\,u\left(LCR\_NL\right)}{100}\right)^2+u^2\left(CB\_NL\right)\end{array}} \qquad (57)$$

The uncertainty in measuring the dissipation factor using the capacitance bridge is derived from the bridge manufacturer specifications [8]. The uncertainty in measuring the dissipation factor using the LCR meter is produced using the meter manufacturer specifications [7]. The meter specifications define dissipation factor uncertainty as $\frac{A_E}{100}$ where A_E is the relative uncertainty, as given in Eq. (37). Since we are measuring a ratio of impedances using the LCR meter, the factor $\frac{u\left(LCR\_NL\right)}{100}$ is the appropriate estimate of the contribution. The Type A contribution from the LCR meter measurements is given using the standard deviations of the measurements, $0.1\,s\left(\varphi_{\frac{Z_{100\,\mathrm{pF}\_LCR}}{Z_{1\,\mathrm{nF}\_LCR}}}\right)$. The results are presented in Table 37. Note that since the measured angles are small, dissipation factor and phase are used interchangeably. In the discussions below, phase (φ) is used to represent the imaginary part of the scaling ratio and dissipation factor (*DF*) is used for capacitive impedance.

Table 37. Uncertainty contributions for the scaling ratio phase, in µrad.

Frequency	Measured *DF* for 100pF and 1 nF capacitors	$u\left(C_{100\,\mathrm{pF}\_CB}\right)$ data obtained from the manufacturer manual	$u\left(C_{1\,\mathrm{nF}\_CB}\right)$ data obtained from the manufacturer manual	$0.1\,s\left(\varphi_{\frac{Z_{100\mathrm{pF}\_LCR}}{Z_{1\mathrm{nF}\_LCR}}}\right)$ data obtained from a set of measurements	$u_c\left(K_{\varphi\,10\,\mathrm{nF}}\right)$
100 Hz	2	3.7	3.7	1	5
1 kHz	2	1.9	0.3	1	2
10 kHz	10	10.5	10.3	1	15
100 kHz	*Air capacitor dissipation factor negligible			0	18

*At 100 kHz the measurements are performed using air capacitors. The measurement procedure is slightly different compared to the rest of the frequencies. The air capacitors are measured at 1 kHz and their impedance is extrapolated to 100 kHz. The discussion in section 6.1 argued the assumption that the air capacitors have negligible dissipation factor. The uncertainty in measuring the dissipation factor of the 1 nF and 100 pF capacitors at 100 kHz is 12 µrad. These contributions are labeled in Eq. (58) as $u\left(DF_{100\,\mathrm{pF}\_CB\_extrapolated\_to\_100\,\mathrm{kHz}}\right)$ and $u\left(DF_{1\,\mathrm{nF}\_CB\_extrapolated\_to\_100\,\mathrm{kHz}}\right)$.

At 100 kHz, the uncertainty of measuring the scaling ratio phase, K_φ, is

$$u_c(K_\varphi) = \sqrt{\begin{aligned} & s\left(0.1\varphi_{\frac{Z_{100\,\mathrm{pF}\_LCR}}{Z_{1\,\mathrm{nF}\_LCR}}}\right)^2 + u^2\left(DF_{100\ \mathrm{pF}\_CB\_extrapolated\_to\_100\ \mathrm{kHz}}\right) + \cdots \\ & \cdots u^2\left(DF_{1\ \mathrm{nF}\_CB\_extrapolated\_to\_100\ \mathrm{kHz}}\right) + \left(\frac{0.1\,u\left(LCR\_NL\right)}{100}\right)^2 + u^2\left(CB\_NL\right) \end{aligned}} \tag{58}$$

The uncertainty of measuring the dissipation factor of a 10 nF capacitor is given as

$$u_c\left(DF_{10\,\mathrm{nF}}\right) = \sqrt{u_c^{\,2}\left(K_\varphi\right) + \left(s\left(DF_{\frac{Z_{10\,\mathrm{nF}\_LCR}}{Z_{1\,\mathrm{nF}\_LCR}}}\right)\right)^2 + u^2\left(DF_{1\,\mathrm{nF}\_CB}\right) + \left(\frac{u\left(LCR\_NL\right)}{100}\right)^2 + u^2\left(CB\_NL\right)} \tag{59}$$

Table 38 shows the calculations of the dissipation factor uncertainty for the 10 nF capacitor. All of the contributions from Equation 59 are previously given except for the Type A factor $s\left(\varphi_{\frac{Z_{10\,\mathrm{nF}\_LCR}}{Z_{1\,\mathrm{nF}\_LCR}}}\right)$. This factor has a dominant role in determining the uncertainty.

Table 38. Combined standard uncertainty of 10 nF capacitor dissipation factor, in μrad.

Frequency	$s\left(\varphi_{\frac{Z_{10\,\mathrm{nF}\_LCR}}{Z_{1\,\mathrm{nF}\_LCR}}}\right)$ data obtained from a set of measurements	$u_c\left(DF_{10\,\mathrm{nF}}\right)$
100 Hz	13	15
1 kHz	6	6
10 kHz	12	21
100 kHz	17	27

7. 2. 100 nF Dissipation Factor

The dissipation factor uncertainty of the 100 nF capacitor is given in Eq. (60) and Table 39 provides the uncertainty component values and the combined standard uncertainty.

$$u_c\left(DF_{100\,\mathrm{nF}}\right) = \sqrt{u_c^2\left(K_\varphi\right) + \left(s\left(DF_{\frac{Z_{100\,\mathrm{nF}\_LCR}}{Z_{10\,\mathrm{nF}\_LCR}}}\right)\right)^2 + u_c^2\left(DF_{10\,\mathrm{nF}}\right) + \left(\frac{u\left(LCR\_NL\right)}{100}\right)^2} \tag{60}$$

Table 39. Combined standard uncertainty of 100 nF capacitor dissipation factor, in μrad.

Frequency	$u(K_{\varphi 100\,\mathrm{nF}})$	$u(DF_{10\,\mathrm{nF}/100\,\mathrm{nF}})$	$s\left(DF_{\frac{Z_{100\,\mathrm{nF}\_LCR}}{Z_{10\,\mathrm{nF}\_LCR}}}\right)$ data obtained from a set of measurements	$u_c(DF_{100\,\mathrm{nF}})$
100 Hz	5	13	25	29
1 kHz	2	4	8	9
10 kHz	15	22	23	35
100 kHz	18	33	25	45

The fourth column presents the standard deviations of the measured dissipation factor for the 100 nF capacitor. The same approximation will be used throughout the report. This is a conservative estimate.

7.3. 1 μF Dissipation Factor

The dissipation factor uncertainty for the 1 μF capacitor is given in Eq. (61) and Table 40 provides the uncertainty component values and the combined standard uncertainty.

$$u_c(DF_{1\,\mu\mathrm{F}}) = \sqrt{u_c^2(K_\varphi) + \left(s\left(DF_{\frac{Z_{1\,\mu\mathrm{F}\_LCR}}{Z_{100\,\mathrm{nF}\_LCR}}}\right)\right)^2 + u^2(DF_{100\,\mathrm{nF}}) + \left(\frac{u(LCR\_NL)}{100}\right)^2} \tag{61}$$

Table 40. Combined standard uncertainty of 1 μF capacitor dissipation factor, in μrad.

Frequency	$u_c(K_{\varphi 1\,\mu\mathrm{F}})$	$u(DF_{10\,\mathrm{nF}/1\,\mu\mathrm{F}})$	$u(DF_{100\,\mathrm{nF}/1\,\mu\mathrm{F}})$	$s\left(DF_{\frac{Z_{1\,\mu\mathrm{F}\_LCR}}{Z_{100\,\mathrm{nF}\_LCR}}}\right)$ data obtained from a set of measurements	$u_c(DF_{1\,\mu\mathrm{F}})$
100 Hz	5	14	31	47	57
1 kHz	2	6	13	13	19
10 kHz	15	22	39	38	57
100 kHz	18	27	26	47	57

7.4. 10 μF Dissipation Factor

The dissipation factor uncertainty of the 10 μF capacitor is calculated using (62) and is presented in Table 41.

$$u_c\left(DF_{10\,\mu\mathrm{F}}\right) = \sqrt{u_c^2\left(K_\varphi\right) + \left(s\left(DF_{\frac{Z_{10\,\mu\mathrm{F}\_LCR}}{Z_{1\,\mu\mathrm{F}\_LCR}}}\right)\right)^2 + u_c^2\left(DF_{1\,\mu\mathrm{F}}\right) + \left(\frac{u\left(LCR\_NL\right)}{100}\right)^2} \qquad (62)$$

Table 41. Combined standard uncertainty of 10 μF capacitor dissipation factor, in μrad.

Freq.	$u_c\left(K_{\varphi\,10\,\mu\mathrm{F}}\right)$	$u\left(DF_{10\,\mathrm{nF}/10\,\mu\mathrm{F}}\right)$	$u\left(DF_{100\,\mathrm{nF}/10\,\mu\mathrm{F}}\right)$	$u\left(DF_{1\,\mu\mathrm{F}/10\,\mu\mathrm{F}}\right)$	$s\left(DF_{\frac{Z_{1\,\mu\mathrm{F}\_LCR}}{Z_{100\,\mathrm{nF}\_LCR}}}\right)$ data obtained from a set of measurements	$u_c\left(DF_{10\,\mu\mathrm{F}}\right)$
100 Hz	5	8	14	199	29	201
1 kHz	3	18	51	87	86	122
10 kHz	9	80	95	158	86	180
100 kHz	18	61	138	234	545	593

7.5. 100 μF Dissipation Factor

The dissipation factor uncertainty for the 100 μF capacitor is calculated using (63) and is presented in Table 42. A summary of expanded uncertainties (*k* = 2) is shown in Table 43.

$$u_c\left(DF_{100\,\mu\mathrm{F}}\right) = \sqrt{u_c^2\left(K_\varphi\right) + \left(s\left(DF_{\frac{Z_{100\,\mu\mathrm{F}\_LCR}}{Z_{10\,\mu\mathrm{F}\_LCR}}}\right)\right)^2 + u_c^2\left(DF_{10\,\mu\mathrm{F}}\right) + \left(\frac{u\left(LCR\_NL\right)}{100}\right)^2} \qquad (63)$$

Table 42. Combined standard uncertainty of 100 μF capacitor dissipation factor, in μrad.

Freq.	$u_c\left(K_{\varphi\,100\,\mu\mathrm{F}}\right)$	$u\left(DF_{10\,\mathrm{nF}/100\,\mu\mathrm{F}}\right)$	$u\left(DF_{100\,\mathrm{nF}/100\,\mu\mathrm{F}}\right)$	$u\left(DF_{1\,\mu\mathrm{F}/100\,\mu\mathrm{F}}\right)$	$u\left(DF_{10\,\mu\mathrm{F}/100\,\mu\mathrm{F}}\right)$	$s\left(DF_{\frac{Z_{1\,\mu\mathrm{F}\_LCR}}{Z_{100\,\mathrm{nF}\_LCR}}}\right)$ data obtained from a set of measurements	$u_c\left(DF_{100\,\mu\mathrm{F}}\right)$
100 Hz	6	33	66	105	149	131	198
1 kHz	14	134	291	484	712	636	955
10 kHz	17	48	72	124	172	1049	1063

Table 43. Summary of expanded dissipation factor uncertainties (*k* = 2), in μrad.

Frequency	Capacitor Value				
	10 nF	100 nF	1 μF	10 μF	100 μF
100 Hz	30	60	120	400	200
1 kHz	15	20	50	250	2000
10 kHz	45	70	120	380	2200
100 kHz	55	90	120	1200	n/a

8. Summary

Throughout this document, the Type A components of the uncertainties were derived from step-up impedance measurements using the LCR meter. The Type B components were derived from nonlinearity coefficients from the measurement instruments used. The standard deviations of the measured capacitances were compared with the uncertainty bounds (see Tables 45 and 36). In some instances, the standard deviations in the measured capacitances were higher than the uncertainty bounds. This discrepancy may be due to environmental effects on the measured reference capacitors.

When the standard deviations of the measured reference capacitors are used as the Type A uncertainty contributions, the resulting expanded uncertainties are given in Table 44. Table 45 shows the standard deviations of the measured reference capacitors. It is important to note that Table 45 shows only standard deviations of the final measurement results. In calculating the values in Table 44, all of the described uncertainty components in the step up measurement procedures are taken into account. Table 36 is duplicated here for convenient comparison.

Table 44. Expanded capacitance uncertainties ($k = 2$), when the standard deviations of the measured reference capacitors are used as the Type A component, in parts in 10^6.

Frequency	Capacitor Value				
	10 nF	100 nF	1 μF	10 μF	100 μF
100 Hz	30	33	70	140	300
1 kHz	30	50	80	340	1810
10 kHz	30	63	175	540	4030
100 kHz	40	60	200	7000	n/a

Table 45. Standard deviations of the measured reference capacitors, in parts in 10^6.

Frequency	Capacitor Value				
	10 nF	100 nF	1 μF	10 μF	100 μF
100 Hz	12	12	25	44	101
1 kHz	13	19	31	116	577
10 kHz	10	23	63	190	806
100 kHz	9	18	9	645	n/a

Table 36. Summary of expanded capacitance uncertainty ($k = 2$), in parts in 10^6.

Frequency	Capacitor Value				
	10 nF	100 nF	1 μF	10 μF	100 μF
100 Hz	25	20	33	70	570
1 kHz	25	43	37	150	680
10 kHz	30	40	80	200	3000
100 kHz	37	47	200	7000	n/a

In the case of dissipation factor, the LCR meter measures the phase so that the standard deviations used in the analysis are the standard deviations of the measured dissipation factor of the reference standards.

Reported uncertainties for Special Tests of customer standard capacitors will reflect the values from Table 44, using the standard deviations of the measured reference capacitors as the Type A component.

9. Conclusion

The capacitance scaling system is described and calculations of uncertainties for each capacitor are presented in detail. Where appropriate, both Type A and Type B uncertainty components are considered. The capacitance scaling method uses ceramic capacitors with temperature coefficients on the order of 35 parts in 10^6 per °C (values 10 nF, 100 nF, 1 μF and 10 μF). Precautions were made to minimize the temperature changes during measurements, but the uncertainty of the capacitor was not separated from that of the capacitance scaling measurement system due to a lack of more stable capacitance standards. The presented analysis of the capacitance scaling method includes capacitor instability and, therefore, provides conservative uncertainties.

This measurement system will allow NIST to provide customers with measurement services for four-terminal-pair capacitance and dissipation factor. The capacitance range covered is from 10 nF to 100 μF. The frequencies measured are 100 Hz, 1 kHz, 10 kHz, and 100 kHz.

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Appendix A

Measurement of Reference Standard Capacitors

A set of Agilent Technologies[1] 16380C ceramic capacitors (including a specially-made 100 µF standard) was measured several times over the course of approximately one month and Tables 46 and 47 present the average values obtained.

Table 46. Average measured capacitance values for the set of ceramic standards.

Frequency	Capacitor Value				
	10 nF	100 nF	1 µF	10 µF	100 µF
100 Hz	9.99991	100.0111	1.000124	10.00133	99.9827
1 kHz	9.99909	99.9992	0.999988	9.99970	99.9841
10 kHz	9.99879	100.0037	1.000546	9.99940	100.2650
100 kHz	10.00057	100.2269	1.024391	10.42616	n/a

Table 47. Average measured dissipation factor values for the set of ceramic standards, in µrad.

Frequency	Capacitor Value				
	10 nF	100 nF	1 µF	10 µF	100 µF
100 Hz	16	6	36	31	153
1 kHz	33	33	55	21	438
10 kHz	62	135	797	232	93
100 kHz	305	2154	20268	1035	n/a

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[1] Identification of commercial products does not imply endorsement by the U.S. Government, nor does it imply that such products are necessarily the best available.