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Bilateral comparison between NIST (USA) and NPLI (India) in the pneumatic pressure region 0.4 MPa to 4.0 MPa

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We report the results of a bilateral comparison of pressure measurement between NIST and NPLI using a piston gauge transfer standard (TS), designated as NPLI-4, over the range of nominal applied pressure 0.4 MPa to 4.0 MPa. This TS was cross-floated against the laboratory secondary standard designated as PG13 at NIST, USA and against NPLI-8 at NPLI, India. The nominal pressure points of the bilateral comparison were (0.4, 0.8, 1.2, 1.6, 2.0, 2.4, 2.8, 3.2, 3.6 and 4.0) MPa, respectively. The comparison was performed in both the institutes in identical pressure cycles in increasing pressures. The comparison data were analyzed in terms of the effective area [A_p (mm^2)] as a function pressure [p (MPa)] of the TS at the above-mentioned pressures. We have also estimated the zero pressure effective area [A_0 (mm^2)] and the pressure distortion coefficient [λ (MPa^{-1})] of the transfer standard. The consistency of the results at every pressure in the range indicates that the laboratory standards used in this comparison are compatible, uniform and can be considered traceable to each other. Finally, the degree of equivalence between NPLI and NIST is 11.4x10^{-6} or better, which is always less than the relative standard uncertainty of the difference (33.6x10^{-6}).

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1. Introduction

There has been considerable interest in international and bilateral key comparisons to ensure worldwide uniformity of measurements and their traceability to the International System of Units (SI). Recently, a regional comparison (identified as APMP.M.P-K1c) was organized by the Asia Pacific Metrology Programme (APMP) Secretariat under the guidance of the Technical Committee for Mass and Related Quantities (TCM) of APMP in the pressure region (0.4 to 4.0) MPa [1]. The linking of this regional comparison to the corresponding Consultative Committee for Mass and Related Quantities (CCM) key comparison (identified as CCM.P-K1c) was also carried out as per guidelines of the BIPM. A theoretical degree of equivalence was estimated between NIST and NPLI at two nominal pressures of 1 MPa and 4 MPa [1]. The bilateral comparison of the present work in the same pressure range serves three purposes: (1) to verify the validity of the model estimation of theoretical degree of equivalence as mentioned in [1]; (2) to explore directly the degree of equivalence between The National Institute of Standards and Technology (NIST, USA) and the National Physical Laboratory (NPLI, India); and (3), to provide a model for other bilateral/supplementary comparisons. Consistent with other TCM/RMO (Regional Metrology Organization) sponsored key comparisons in pressure metrology, we have evaluated equivalence by determining the effective area of a transfer standard (TS) at NIST and NPLI as a function of pressure using the conventional cross-float method [2]. The TS was cross-floated against the secondary laboratory standards (LS) of NIST (PG13) and NPLI (NPLI-8). The TS was a piston-cylinder assembly, without its base or masses. Both the laboratories provided bases, well-calibrated masses, temperature probes, and pressure balancing hardware for the TS as well as for their laboratory standards.

2. Apparatus

a. Transfer standard

The TS used in this comparison is designated as NPLI-4 provided by NPLI. It is a Ruska[1] 2465 piston-cylinder, serial number V-607, with a nominal effective area 8.40 mm^2 and a measuring range in pressure of (0.2 to 4.0) MPa. It has been in service since 1982. Table 1 lists the various metrological characteristics of this piston-cylinder assembly, including the effective area [A_o (m^2)] at atmosphere pressure and 23 °C, the pressure distortion coefficient [λ (MPa^{-1})], and the relative standard uncertainty in effective area. The effective area with pressure (A_p) and the subsequent A_o and λ of the TS was determined by cross floating against NPLI-P1, the primary pressure standard of NPLI. Details of NPLI-P1 are found in [3] and [4]. NPLI-4 was used as the NPLI laboratory standard during the recently concluded key comparison, APMP.M.P-K1c, from October 1998 to March 2001. Although the metrological characteristics of the TS

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[1] In order to describe materials and experimental procedure adequately, it is occasionally necessary to identify commercial products by manufacturer's name. In no instance does such identification imply endorsement by NIST, nor does it imply that the particular product or equipment is necessarily the best available for the purpose.

are listed in Table 1 for descriptive purposes, they were measured by NIST and NPLI again as part of the present comparison.

During a span of twenty years, it has been observed that the relative stability of A_o of the transfer standard is within $4x10^{-6}$. We take the A_o stability as the A_p stability uncertainty in the TS over the entire pressure range. During the present bi-lateral comparison, only the piston-cylinder assembly of the TS was transported. The respective laboratories provided the bases and mass sets.

b. Laboratory standards

NIST. The laboratory standard used at NIST is designated as PG13, also a Ruska 2465 piston-cylinder gauge, serial number V-373, which has been in service since 1972. The characterization and uncertainty of this standard comes from cross-float against two NIST twin piston gauges designated as PG34 and PG37. The details of the metrological characteristics of PG13 are listed in Table 1 and described in [5] and [6]. PG13 is rotated manually.

NPLI. The NPLI laboratory standard is designated as NPLI-8. It is an oil-lubricated, gas operated piston-cylinder system manufactured by Des Granges et Huot, France, Model 5303, serial number 2943. The nominal effective area is 49 mm^2. The piston is rotated by a pulley driven by a DC motor mounted remotely from the piston-cylinder assembly. The non-conductive pulley is used to avoid heat transmission to the system and hence good thermal stability is achieved. NPLI-8 was earlier used in a bilateral comparison between NPLI and Physikalisch-Technischen Bundesanstalt (PTB) (Germany) [7]. Table 1 also lists the metrological characteristics of this piston-cylinder assembly, which are described in more detail in [4].

3. Calibration procedure

The calibration procedure followed well-known methods for cross-float comparison between two piston gauges [2, 6]. At each pressure setting for the laboratory standard (PG13 or NPLI-8) and the TS (NPLI-4), the piston gauges were loaded with masses in ratios approximately the same as the ratios in effective area. The pressure was increased to float the pistons, and pressure equilibrium between the piston gauges was determined by either of two methods: the fall rate method at NPLI, or the differential pressure cell method at NIST. Both methods are commonly used for cross-float calibrations and are described in [6]. If the piston gauges were not in pressure equilibrium, small fractional masses were added or subtracted from the laboratory standard. Both the transfer and laboratory standards were housed in a room that provided a stable temperature to within ±0.5 K. The temperature of the piston gauges was measured with platinum resistance thermometers (PRTs) attached near the pistons, and their outputs were read with digital multimeters. Both piston gauges were mounted on a stainless steel base to minimize vibration and magnetic effects. A pressure head correction term was applied to compensate for the difference in the reference levels of the pistons. Before the measurement cycle, each piston was leveled to ensure the verticality of its axis and the system was checked for leaks to its full-scale pressure value of 4.0 MPa.

The comparison was performed at NIST, USA from July to August 2003, and at NPLI, India from March to April 2004. The pressures of the comparison were (0.4, 0.8, 1.2, 1.6, 2.0, 2.4, 2.8, 3.2, 3.6 and 4.0) MPa. A single observation was made at each pressure of the cycle, and the complete cycle was performed three times in ascending pressure. About 15 minutes time was adequate for changes in pressure to bring the piston gauges into equilibrium.

4. Results and discussion

a. Mathematical Model

The effective area (A_p) of the TS for each observation, referred to 23°C, is calculated using the equation

$$A_p = \frac{\sum_i m_i g \left(1 - \frac{\rho_{0\mathrm{a}}}{\rho_0} + \frac{\rho_{0\mathrm{a}} - \rho_\mathrm{a}}{\rho_i}\right)}{p\left[1 + \left(\alpha_p + \alpha_c\right)\left(t - t_0\right)\right]}, \tag{1}$$

where m_i are the conventional masses of the piston, the weight carrier, and the mass pieces placed on the weight carrier of the TS; ρ_i are the densities of the parts with masses m_i; ρ_a is the air density; $\rho_{0\mathrm{a}}$ is the conventional value of the air density ($\rho_{0\mathrm{a}} = 1.2$ kg/m^3); ρ_0 is the conventional value of the mass density ($\rho_0 = 8000$ kg/m^3); g is the local acceleration of gravity; p is the pressure generated by the laboratory standard at the TS reference level; α_p and α_c are the thermal expansion coefficients of the piston and cylinder materials, respectively; t is the temperature of the TS; and t_0 is the reference temperature ($t_0 = 23$ °C). Because the LS is also a piston gauge, the pressure it generates is determined by an equation very similar to (1) with p and A_p transposed. In the transposed equation for the LS, mass values and other parameters are those of the LS, and its A_p is determined from A_0 and λ given in Table 1.

We estimate the average value of the effective area, $A_{p,av}$, for the TS at each pressure point from $n = 3$ observations, that is:

$$A_{p,av} = \left(\sum_{k=1}^{n} A_{p,k}\right) / n\,. \tag{2}$$

The uncertainties in the measurement of effective area arise from Type A and Type B sources. All uncertainties stated in this paper are standard uncertainties (k=1). Type B uncertainties come from the uncertainties in the parameters of the measurement equations (eq. (1) for the TS and its companion for the LS), which are summed in quadrature according to the methods in [8]. The major source of Type B uncertainty is the uncertainty in effective area of the laboratory standard. The Type A standard uncertainty is taken as the standard deviation of the average, $u_A(A_{p,av})$:

$$u_A(A_{p,av}) = \left[\frac{1}{n(n-1)} \sum_{k=1}^{n} \left(A_{p,k} - A_{p,av} \right)^2 \right]^{1/2} . \tag{3}$$

The uncertainty given by Eq. (3) is added in quadrature with the Type B uncertainty discussed above to give the combined standard uncertainty in the average effective area, or

$$u_C(A_{p,av}) = \left[u_A(A_{p,av})^2 + u_B(A_{p,av})^2 \right]^{1/2} . \tag{4}$$

The relative uncertainty is the uncertainty in eq (4) divided by $A_{p,av}$. Uncertainties are evaluated at each pressure. The average effective area and the combined relative standard uncertainty for the TS obtained from NIST and NPLI are listed in Table 2. We have also fit the average effective area data for the TS from each laboratory standard by least squares regression to the linear distortion model:

$$A_{p,fit} = A_0 (1 + \lambda p) \quad . \tag{5}$$

Results of the fit parameters are listed in Table 3. The standard uncertainty of the effective area calculated from the linear distortion model, $u(A_{p,fit})$, is taken as the standard deviation of the linear fit added in quadrature with $u_c(A_{p,av})$. This uncertainty is listed in Table 3.

b. Degree of equivalence

The degree of equivalence is evaluated using the standard method of a CCM comparison [1] by calculating the difference in average effective area in the transfer standard, as found by NIST and NPLI, at similar pressure points. The difference is made dimensionless by dividing it by the "reference effective area" of the transfer standard, defined as the average of the effective areas determined by NIST and NPLI at each pressure. Or,

$$D = \frac{\left(A_{p,av}^{NIST} - A_{p,av}^{NPLI} \right)}{A_{p,ref}}, \text{ with } A_{p,ref} = \left(A_{p,av}^{NIST} + A_{p,av}^{NPLI} \right) / 2 . \tag{6}$$

The associated standard uncertainty in the difference is determined from:

$$u(D) = \left[u_C(A_{p,av}^{NIST})^2 + u_C(A_{p,av}^{NPLI})^2 + u_{tr.std}^2 \right]^{1/2} / A_{p,ref} \quad , \tag{7}$$

where $u_{tr.std}$ is the uncertainty of the transfer standard ($4x10^{-6}$, in the present case). If *D* is less than or equal to *u*(*D*) at a pressure point, then there is equivalence between the laboratory standards of NIST and NPLI at that pressure.

c. Results of the comparison

Results for average effective area, $A_{p,av}$, of the TS as measured by NIST and NPLI are listed in Table 2 and plotted in Figure 1 from 0.4 MPa to 4.0 MP. The standard uncertainty, $u_c(A_{p,av})$ is shown as error bars. Except at the lowest pressure point (0.4 MPa), the TS exhibited linear change of effective area with pressure at both NIST and NPLI. For all pressures of the comparison, the agreement in $A_{p,av}$ is better than the standard uncertainty from either LS. The zero pressure effective area (A_0) for the TS, as determined by linear least squares fitting, differs by $10.1x10^{-6}$ (relative) between NIST and NPLI, which is also less than the relative standard uncertainty of the effective area from the fit ($21.2x10^{-6}$ at NIST, $26.3x10^{-6}$ at NPLI). The distortion coefficient (λ) differs by $2.61x10^{-6}$ MPa^{-1} between NIST and NPLI.

Results for *D* are listed in Table 4 and plotted in Figure 2. The figure shows the combined standard uncertainty of *D* plotted as an error bar, which is $33.6x10^{-6}$. For all pressures of the comparison, *D* is less than *u*(*D*), demonstrating the equivalence between NIST and NPLI for realizing pressure over the range of 0.4 to 4.0 MPa. The degree of equivalence between NIST and NPLI, from APMP.M.P-K1c [1], is shown at the pressures of 1 MPa and 4 MPa. Because NIST did not participate in that comparison, the "linking method" as described in [1] was used to calculate the theoretical degree of equivalence. The agreement between the direct method of the present comparison and the theoretical estimation of APMP.M.P-K1c gives us confidence in the formulation of that theoretical method

5. Conclusion

We have determined the effective area of a transfer standard (NPLI-4) against the pneumatic laboratory standards of NIST (PG13) and NPLI (NPLI-8) over the pressure range 0.4 to 4.0 MPa. The piston cylinder assembly only was exchanged between the two institutes, while the bases, masses, and temperature instrumentation were provided at NIST and NPLI. We observed that the relative agreement in the effective area of the TS over the pressure range is within $11.4x10^{-6}$, which is always less than the relative standard uncertainty of the difference ($33.6x10^{-6}$). The results validate the theoretical linking method employed in APMP.M.P-K1c. The results also demonstrate the degree of equivalence between NPLI and NIST for pressure measurement in the pneumatic region up to 4.0 MPa, which is a region of industrial importance for both countries.

6. References

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Table 1. Description of the piston cylinder assemblies used in NIST – NPLI bilateral pressure comparison.

	Laboratory Standards		**Transfer Standard**
	NPLI NPLI-8 [3-4, 7]	**NIST PG13 [5-6]**	**NPLI-4[1,4,7]**
Manufacturer	Desgranges et Huot, France	Ruska Instrument Corporation, USA	Ruska Instrument Corporation, USA
Range in pressure (MPa)	0.4 to 8.0	0.3 to 7.0	0.2 to 4.0
Effective area at atmosphere pressure and at 23°C [A_o (m^2)]	49.02598 x 10^{-6}	8.389264 x 10^{-6}	8.392422 x 10^{-6}
Relative standard uncertainty of A_o (x10^{-6})	24.0	20.0	20.0
Piston material	Tungsten carbide	Tungsten carbide	Tungsten carbide
Cylinder material	Tungsten carbide	Tungsten carbide	Tungsten carbide
Piston and cylinder serial number	DH-2943	V-373	V-607
Thermal expansion coefficient of piston	4.55 x 10^{-6}	4.55 x 10^{-6}	4.55 x 10^{-6}
Thermal expansion coefficient of cylinder	4.55 x 10^{-6}	4.55 x 10^{-6}	4.55 x 10^{-6}
Pressure distortion coefficient [λ (MPa^{-1})]	2.88 x 10^{-6}	0	7.52 x 10^{-6}
Relative standard uncert. in A_p (x10^{-6}) produced by standard uncertainty in λ at 4 MPa	5.0	Note 1	5.0
Combined relative standard uncertainty of A_p (x10^{-6})	**25.0**	**20.0**	**20.6**

[*] Reference numbers indicate the source of information of this Table 1.

Note 1. NIST includes the uncertainty in λ with the uncertainty in A_0.

Table 2. **Average effective area, $A_{p,av}$, and relative standard uncertainty of the transfer standard measured at NIST and NPLI.**

Nominal P (MPa)	NIST			NPLI		
	P (MPa)	$A_{p,av}$ (mm^2)	$u_c(A_{p,av})/A_{p,av}$ (x10^{-6})	P (MPa)	$A_{p,av}$ (mm^2)	$u_c(A_{p,av})/A_{p,av}$ (x10^{-6})
0.4	0.399982	8.392583	21.0	0.413341	8.392487	25.9
0.8	0.813321	8.392447	21.0	0.812979	8.392415	25.9
1.2	1.213063	8.392473	21.0	1.212589	8.392424	25.9
1.6	1.605919	8.392506	21.0	1.612212	8.392452	25.9
2.0	2.012553	8.392464	21.0	2.011844	8.392425	25.9
2.4	2.405437	8.392478	21.0	2.411468	8.392438	25.9
2.8	2.805192	8.392486	21.0	2.811089	8.392471	25.9
3.2	3.218728	8.392497	21.0	3.210713	8.392466	25.9
3.6	3.604690	8.392513	21.0	3.610328	8.392493	25.9
4.0	4.011325	8.392498	21.0	4.009940	8.392530	25.9

Table 3. **Fit coefficients of TS using linear model of effective area, data from NIST and NPLI. Relative standard uncertainty in the TS using the fit coefficients also listed.**

Laboratory	A_o (mm^2)	λ (MPa^{-1})	$u(A_{p,fit})/A_{p,fit}$ ($x10^{-6}$)
NIST	8.392502	-0.42x10^{-6}	21.2
NPLI	8.392419	2.19x10^{-6}	26.3

Table 4: Degree of equivalence, D, between NIST and NPLI from difference in effective area of transfer standard at measured pressures. Standard uncertainty in difference, $u(D)$, given for same pressure. D and $u(D)$ also listed from APMP.M.P-K1c [1] for comparison.

Present Comparison			APMP.M.P-K1c		
Nominal P (MPa)	D (x10^{-6})	$u(D)$ (x10^{-6})	Nominal P (MPa)	D (x10^{-6})	$u(D)$ (x10^{-6})
0.4	11.4	33.6	1.0	12	25
0.8	3.8	33.6	4.0	16	25
1.2	5.8	33.6			
1.6	6.4	33.6			
2.0	4.6	33.6			
2.4	4.8	33.6			
2.8	1.8	33.6			
3.2	3.7	33.6			
3.6	2.4	33.6			
4.0	-3.8	33.6			

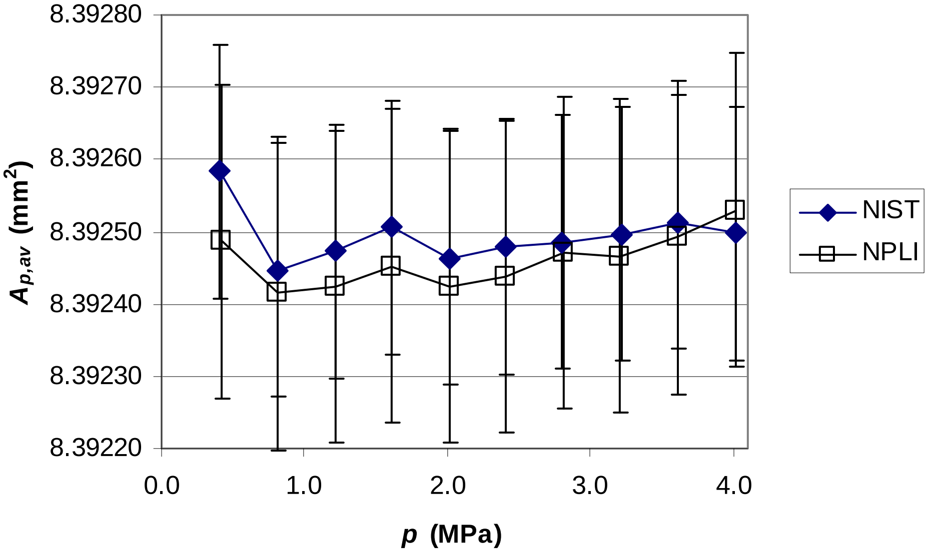

Figure 1. **$A_{p,av}$ of TS as a function of pressure as measured by the laboratory standards at NIST and NPLI. Standard uncertainty shown as error bars.**

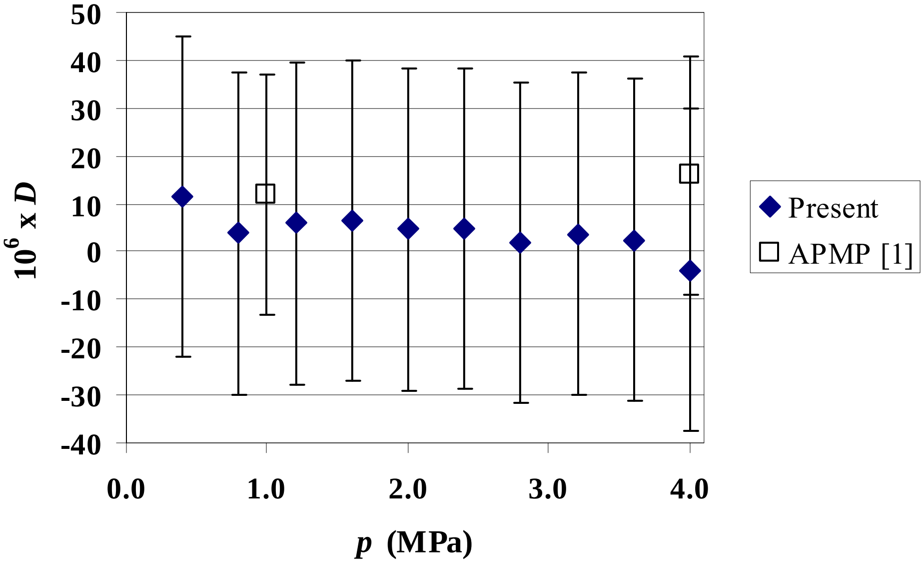

Figure 2. **Degree of equivalence, *D*, between NIST and NPLI from present comparison and APMP.M.P-K1c. Standard uncertainty in *D* shown as error bars.**