The Election Integrity Audit

Kathy Dopp and Frank Stenger[1]

WHY AUDIT ELECTION RESULTS?

The elections industry is the only major industry not routinely subjected to independent manual audits. In any field, electronic mistakes or tampering[2] would go undetected in the absence of audits. Banks, churches and businesses are subjected to routine independent audits to detect and prevent errors and embezzlement. Virtually all U.S. votes will be counted electronically by 2008. Electronic voting systems make it possible for one person to rig a state-wide election and for electronic errors and failures to produce wrong results.[3]

Electronic vote counts determine who controls budgets of millions to trillions of dollars. Yet only perhaps fifteen states conduct any audits – manual checks - of vote count accuracy and their procedures are *not* sufficient to detect wrongfully altered outcomes in close races[4]. In some states audit procedures are not independent; audit results are not made public; audit results are not used to correct discrepancies found between manual and electronic vote counts; or the audits are performed *after* election results are certified.

This is especially alarming in light of the fact that U.S. jurisdictions publicly report their vote counts aggregated in a way that hides evidence of vote count errors, machine problems, and tampering.[5]

In other words, U.S. election outcomes are wide open to undiscovered fraud and innocent error.

The Accuracy of Any System Depends on the Ability to Detect and Correct Errors

The most accurate voting system employs both manual and electronic counts because manual counts can detect errors and most types of tampering in electronic tallies; and electronic tallies can detect some types of paper ballot box stuffing, tampering and voter errors. The more types of errors that can be prevented, detected, and corrected, the more accurate the system. The integrity of election results depends on "trusting no one" and verifying.

Requiring voter-verifiable paper ballot (VVPB) records with voting systems does little more than provide false assurance if the VVPB records are not used in recounts and audits. In audits 100% of the paper ballot records associated with randomly selected vote counts (precinct vote counts or digital electronic recording (DRE) machine vote counts) are manually counted using the VVPB records and compared exactly to their associated electronic vote counts.

Routine independent audits – manual counts of the voter-verifiable paper ballot records - of randomly selected[6] electronic vote counts could detect most types of electronic vote count error that could wrongfully

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[2] Any system is most at risk of tampering by insiders. In elections, insiders include election officials, poll workers, voting machine vendor technicians, programmers, and staff; and any county staff with access to voting machines or central tabulators.

[3] See the Brennan Center Report at http://brennancenter.org/programs/dem_vr_hava_machineryofdemocracy.html "The Machinery of Democracy: Protecting Elections in an Electronic World"

[4] Although some states require recounts when margins between candidates are under 1%, sometimes these states merely reconcile the electronic records during a "recount".

[5] See "What Election Data Can We Collect and Publicly Release to Monitor our Elections for Accuracy?" http://uscountvotes.org/ucvAnalysis/US/election_officials/ElectionArchive_advice.pdf

[6] "Random selection" is a scientific process designed by an expert. Having election officials select the electronic counts to audit does not qualify as "random" selection.

alter election outcomes, to any desired confidence level *if* audit sample sizes are properly calculated. Auditors must be independent. I.e. Auditors must *not* be insiders such as voting machine vendor staff or election officials.

WHAT TYPES OF ERRORS ARE *NOT* DETECTABLE VIA AUDITS?

Most types of electronic tampering and error are detectable by manually counting voter-verifiable paper ballot records. However, some types of deliberate vote tampering are not detectable. For instance:

- Paper ballot tampering before or after an election spoils ballots by marking too many candidates in one race, thus creating over-votes thus the voter's actual votes are not counted. Auditors should look for high or unusual ballot spoilage rates and treat them with suspicion.
- Paper ballot substitution – switches votes
- Paper ballot box stuffing – adds extra votes
- Programmers can make DRE voting machine electronic and paper records both wrong to subvert audits. Studies show that only about 30% of voters take the time to verify their DRE touch-screen paper roll ballot records[7], so about 2/3rds of votes for a candidate could be shifted to an opponent by making the paper and electronic record both wrong on the first try, and if the voter notices and cancels the ballot, get it right on the second try. Voters should be educated to take the time to verify that their paper ballot records match their selections and immediately notify officials if the paper does not match their touch-screen choices. A picture should be taken, the offending DRE machine should be removed from service for the remainder of the election, all votes previously cast on that machine included as possible discrepancies during an audit, and a forensic technical investigation done on that DRE machine.

Vote count error or fraud can only be completely identified if voters verify the accuracy of their paper ballot records and paper ballot frauds are prevented.[8] This paper focuses on manual audits to detect discrepancy between electronic and manual counts sufficient to alter election outcomes.

WHICH VOTING SYSTEMS ARE INDEPENDENTLY AUDITABLE?

A voting system is audit-able if it provides hand-countable, voter-verifiable paper ballot records that can be used to check the electronic counts. Auditable voting systems include

- Direct recording electronic (DRE) voting systems that provide voter-verifiable paper roll ballot records - Auditing paper roll ballot records requires special equipment, called "paper-roll advancers" to be able to conveniently read the paper rolls. If most voters do not bother to take the extra step to verify their paper roll ballot record, then not all electronic errors are detectable via manual audits. Automatically auditing paper roll ballot records using electronic bar codes is *not* an independent check of machine accuracy and does *not* qualify as an "independent audit" of DRE vote count accuracy.
- Paper ballot optical scan voting systems - Optical scan paper ballots are verified by most voters in the process of marking the ballots[9].

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[7] From a survey of voters conducted in Las Vegas, Nevada by Lombardo Consulting Group in conjunction with political science professor, Dr. Michael John Burton of Ohio University, Nevada http://e-grapevine.org/nvvotersurvey.pdf

[8] From Roy G. Saltman's **Independent Verification: Essential Action to Assure Integrity in the Voting Process** submitted to the National Institute of Standards and Technology Gaithersburg, MD 20899 Aug. 22, 2006 "…protections against ballot frauds. It was recommended that all of the official ballots printed and distributed must be accounted for, and there must be assurance against use of counterfeit ballots. At each precinct, the number of blank ballots received must equal the sum of ballots voted, unused, spoiled and otherwise employed (Saltman, 1975, pp. 41-45). This additional reconciliation is intended to prevent "ballot stuffing," i.e., the addition of extra voted ballots to a ballot box. Furthermore, if there are missing votes that do not complete the total, or if there are more votes cast than voters signed-in to vote, these inconsistencies can be investigated and resolved. (p 22) If reconciliations are thoroughly and correctly made, and prevention of counterfeit ballots is assured, then only the fraud of electronic miscount can be perpetrated. Good ballot-handling and tracking procedures are critical.

[9] The exception is with touch-screen ballot marking devices used by disabled or foreign-language voters. Ballot marking devices provide for electronic verifiability or read-back of the marked ballot.

Mechanical lever machines and paperless DRE voting systems are *not* independently audit-able. As such, both lever machines and paperless DRE voting systems are inherently *in*accurate and open to undetectable tampering since no method is provided for detecting and correcting errors.

ABSTRACT

This paper explains how to directly calculate a sufficient number of electronic vote counts (either precinct vote counts or DRE machine counts) to audit – count manually - in order to detect an amount of electronic miscount that could wrongfully alter an election outcome. Vote count audits of fixed 1 or 2% of electronic counts provide no confidence that outcome-altering errors would be detected in close races.

This new audit calculation method builds on the National Election Data Archive's (NEDA) prior work on the mathematics of vote count audits and the Brennan Justice Center's threat analysis which assumes a maximum amount of miscount that could occur in each electronic vote count without raising immediate suspicion and investigation.[10] Thus:

- The minimum number of corrupt electronic vote counts that could wrongfully alter an election outcome depends on the margin between candidates and the assumed maximum miscount rate in any vote count, and
- If we solve the equation for calculating the probability of detecting one or more vote miscounts for the audit sample size in terms of the desired probability, the total number of vote counts, and the number of corrupt counts sufficient to wrongfully alter outcomes, then we can determine the audit sample size that will ensure election outcome integrity.

The "election integrity" probability equation cannot be solved by hand, but it can be solved by computer. *In this paper, we provide a numerical (computer) solution to directly calculate vote count audit sample sizes to detect electronic errors to any desired probability that could have altered election outcomes.*[11]

Success in detecting evidence of outcome-altering electronic vote miscount via audits is influenced by the following relationships:

- As the margin between candidates decreases, the amount of vote miscount that could wrongfully alter an election decreases and the audit amount must increase.
- For a fixed rate of corruption, as the total number of vote counts to be audited decreases, the required audit amount does not similarly decrease. i.e. As the number of total vote counts decreases, the audit percentage required to keep the same probability of detecting errors increases. For example if one in ten electronic vote counts is corrupt, then at least ten vote counts must be audited in order to have a good chance of detecting one corrupt count, regardless of how many total counts there are.
- When auditing optical scan precinct counts (as opposed to DRE machine counts), if precinct sizes vary significantly, it becomes possible for miscounts to be targeted to a fewer number of large precincts, and larger audit sample sizes are needed.
- When auditing DRE machine counts variability of the number of ballots cast on DRE machines may have a similar effect (to the effect of precinct-size variation on optical scan audits) on the probability of detecting outcome-altering miscount. The conservative approach would be to use the number of ballots cast on DRE machines to adjust audit sample sizes but research is needed.
- The total number of electronic vote counts in each race, rather than the total number of vote counts in the entire jurisdiction, determines the sample size that must be used to audit that race outcome.

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[10] VoteTrustUSA's Howard Stanislevic suggests using 0.20 for the maximum vote shift per vote count that would not be suspicious based on historical trends. We use 0.15 here but 0.20 is good because it is more conservative.

[11] We make our new "election integrity" audit methodology publicly available on ElectionArchive.org and in Appendix B.

Our numerical method is for calculating the audit sample size for *actual* vote count audits to find *actual* discrepancies between machine and manual counts expected to be exactly equal.[12] In other words, 100% of paper ballot records associated with randomly selected vote counts are manually counted and compared exactly to their corresponding electronic vote counts.

Future research is needed to determine what calculations and methodology to employ when discrepancies between electronic and manual counts are uncovered during initial audits. Further research is needed to investigate the effects of variability of the total number of ballots cast on individual DRE machines on probabilities of detection. Systematic study is needed on the pros and cons of state-wide versus county-wide audits such as reduction of information, efficiency of the overall process, and reductions in the probability of detecting outcome-altering miscount in state-wide audits. State-level audit sample sizes could be easily calculated in states having uniform voting systems, but would be difficult to correctly calculate in states with mixed voting systems.[13]

Background

In 1975, Roy G. Saltman[14], then of the National Bureau of Standards (now called the National Institute of Standards and Technology) wrote a report[15] that, in Appendix B, recommends an almost identical calculation for determining the number of corrupt vote counts that audits should be designed to detect. Saltman also uses the margin between candidates, an assumed "maximum level of undetectability by observation"[16] and precinct sizes to calculate vote count audit sample sizes.

In June, 2005 NEDA publicly released a paper by Kathy Dopp and Ron Baiman, "How Can Independent Paper Audits Correct & Detect Vote Miscounts" which recommended vote count audits, demonstrated the basic equations, and made a spreadsheet available to calculate the probabilities for detecting one or more vote miscounts, given various amounts of corrupt vote counts.

Since June 2005, the Carter-Baker Election Reform Commission, the U.S. Government Accountability Office, the National Science Foundation's "A Center for Correct, Usable, Reliable, Auditable, and Transparent Elections" (Project ACCURATE), Verified Voting, the Brennan Justice Center Voting System Task Force, and the U.S. League of Women Voters have recommended routine vote count audits - manual counts of randomly-selected voter-verifiable paper ballot records.

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[12] This is *not* a ballot sampling procedure. Most voting systems today do not use unique anonymous identifiers on paper ballots that match a machine-tabulated record. Thus ballots are not individually auditable. Thus ballot sampling procedures are only capable of finding statistical anomalies rather than actual discrepancies – leaving the debate open to "sampling error"; and making scientific random sampling of ballots problematic. Scientific random ballot sampling of DRE machine paper roll ballot records would be impractical, even if there were anonymous identifiers.

[13] It may be difficult to correctly calculate state-wide audits when some counties use DRE machine counts and other counties use precinct optical scan counts because of the problem of not being able to randomly select comparable counts because DRE machine counts often include multiple precincts. The total number of ballots cast in each count would be required to adjust the calculation for size variation, and may cause some counties to be over-sampled. Even when states use uniform voting systems, state-level audits reduce the information obtained and the probability of detecting outcome-altering miscount, and prolong the audit process when discrepancies are found as compared to county-level audits as recommended here. More research is needed.

[14] Roy G. Saltman was instrumental in creating the first federal engineering and procedural performance standards for voting systems. See http://en.wikipedia.org/wiki/Certification_of_voting_machines#Work_of_Roy_Saltman

[15] Saltman, Roy G. (1975), "Appendix B: Mathematical Considerations and Implications in Selection of Recount Quantities," in Effective Use of Computing Technology in Vote-Tallying, Report NBSIR 75-687, (reprinted as SP 500-30, 1978), National Institute of Standards and Technology, Gaithersburg, MD 20899. The report was paid for by the US General Accounting Office, now called the US Government Accountability Office. http://csrc.nist.gov/publications/nistpubs/NBS_SP_500-30.pdf

[16] On p. 115 Saltman defines this as "…the largest percent switch of votes in any one precinct that will fail to make the opposition correctly suspicious that a switch has occurred in that precinct. The higher the maximum level of undetectability by observation, the higher the number of switched votes that can be packed into a single precinct, and the fewer the number of misreported precincts that are needed to reverse an election, the less likelihood there is of a vote-switching scheme being discovered by a partial recount."

On July 16 and 17, 2006, the National Election Data Archive (NEDA) publicly revised its 2005 paper and spreadsheet for calculating election audit amounts to explain how to calculate the number of corrupt vote counts required to wrongly alter an election outcome, and began working to develop a method to directly calculate audit amounts that would ensure election outcome integrity.

On August 9, 2006 Howard Stanislevic of the VoteTrustUSA E-Voter Education Project publicly released a paper independently confirming NEDA's calculation method and pointing out the necessity of adjusting calculations for precinct-size variation.[17] Stanislevic and Dopp had independently derived equivalent audit calculation methods, both taking precinct size into account, using slightly different assumed maximum miscount rates per vote count.

Saltman's, Stanislevic's and Dopp's previous calculations for calculating audit sample size relied partially on a trial and error, generate and test method that requires some expertise.

In September, 2006 Ron Rivest presented a method for estimating vote count audit sample sizes that can be performed easily on a calculator, is always conservative (because it is never less than the optimal value), and is a better estimate than the "sampling with replacement" estimate.[18]

The method presented in this paper by Dopp and Stenger makes it possible to automatically calculate the optimal vote count audit sample size that gives at or just above any desired probability for detecting outcome-altering amounts of vote miscount.

HOW TO INDEPENDENTLY AUDIT VOTE COUNTS?

Manual "hand-to-eye" counts of voter-verifiable paper ballots for randomly selected electronic vote counts (precinct or machine counts) can achieve any desired probability for detecting most electronic errors that could wrongly alter election outcomes, particularly if voters are encouraged to verify that their paper ballot records match their choices.[19]

An "auditable" vote count report must be publicly released prior to randomly selecting vote counts and assigning auditors randomly to each count.[20] After the auditable report is publicly released, then the calculated number of electronic vote counts must be transparently, scientifically, randomly selected and 100% of the paper records associated with those electronic counts manually counted. If discrepancies between the electronic and hand counts are found, then actions must be taken including investigations, expanded audits, 100% hand counts, correction of the official results, or even possibly a new election.

If the goal of elections is to ensure that voters decide who is sworn into office and what ballot measures pass or fail, then audit amounts must be sufficient to detect any error that could wrongfully alter election

[17] See Howard Stanislevic's "Random Auditing of E-Voting Systems: How Much is Enough?" http://www.votetrustusa.org/pdfs/VTTF/EVEPAuditing.pdf. The effect of precinct-size variation on vote count audit probabilities, and other important points related to manual audits were also brought up on the Open Voting Consortium email discussion list, a group of computer experts working on better open source voting system designs. See http://OpenVoting.org.

[18] September 2006, "Estimating the Size of a Statistical Audit" by Ronald L. Rivest PhD, Computer Science and Artificial Intelligence Laboratory Massachusetts Institute of Technology, Cambridge, MA 02139 http://theory.csail.mit.edu/~rivest/Rivest-OnEstimatingTheSizeOfAStatisticalAudit.pdf Ron Rivest is a member of the Technical Guidelines Development Committee which is advisory to the U.S. Election Assistance Commission.

[19] A random manual audit, in combination with voter education and mathematical analysis of detailed precinct-level by vote-type election results, would virtually ensure election integrity because mathematical analysis of election results could catch significant errors or patterns that indicate problems that may not be detected with a manual audit. NY State, Subtitle V of Title 9 of the Official Compilation of Codes, Rules and Regulations, Part 6209, Voting Systems Standards, Section 6209.2. F. (2) states: "There shall be instructions for performing the verification process made available to the voter in a location on the voting system."

[20] An "auditable report must enumerate votes cast for each race and measure on each DRE voting machine or optical scan machine in each precinct for each race and candidate, including under-votes and over-votes, along with unique machine identifiers. It might be helpful to include batch numbers and the dates votes are cast in case of possible investigations.

outcomes. The smaller the margins between the winners and losers, the smaller the amount of vote miscount that could wrongfully alter an outcome; and the larger the audit size must be to detect it.

Elections are administered by county, parish, and township election jurisdictions, so subjecting each jurisdiction to audits gives the highest assurance of integrity.

Procedures that are *Not* Independent Vote Count Audits include:

- Comparing the barcodes on touch-screen voting machine paper-roll ballot records with electronic counts. This checks whether two electronic records, neither of which can be verified by voters, match and is *not* a check of whether electronic vote counts reflect voter intent.
- Printing out the paper audit tape for each machine's memory card to see if the printout matches the machine counts. This is a test of whether print programs work.[21] They usually do.
- Randomly selecting ballots after the election and counting them by hand and by machine. This is post-election machine testing and is not a measure of Election Day vote count accuracy.

What is the Probability of Finding Corrupt Vote Counts with a 2% Audit?

To calculate the *probability* of detecting *at least one* corrupt vote count we need to know the:

N - number of total vote counts (DRE machine memory card or optical scan precinct counts) eligible to cast votes in a particular race or ballot measure

C – number of corrupt vote counts that we want to be able to detect[22]

S - amount of machine or precinct vote counts that are randomly selected for audits

The average number of precincts in a U.S. Congressional district is 440. The table below (rightmost column) shows probabilities that a 2% audit will find one or more corrupt vote counts, based on N=440 total vote counts with varying hypothetical number of corrupt vote counts.[23]

Total Number of Vote Counts	Hypothetical Number of Corrupt Vote Counts	Percent of Corrupt Vote Counts	Number of Audited Vote Counts for 2% Audit	Expected Value of Corrupt Counts in Audit	Standard Deviation	Probability of Finding a Corrupt Vote Count
440	220	50.00%	9	4.50	1.50	99.82%
440	198	45.00%	9	4.05	1.49	99.57%
440	176	40.00%	9	3.60	1.47	99.05%
440	154	35.00%	9	3.15	1.43	98.02%
440	132	30.00%	9	2.70	1.37	96.11%
440	110	25.00%	9	2.25	1.30	92.70%
440	88	20.00%	9	1.80	1.20	86.85%
440	66	15.00%	9	1.35	1.07	77.18%
440	44	10.00%	9	0.90	0.90	61.61%
440	22	5.00%	9	0.45	0.65	37.25%
440	18	4.00%	9	0.36	0.59	31.58%
440	14	3.00%	9	0.27	0.51	25.45%
440	5	1.00%	9	0.09	0.30	9.86%
440	3	0.50%	9	0.05	0.21	6.03%
440	1	0.10%	9	0.01	0.09	2.05%
440	1	0.05%	9	0.00	0.07	2.05%

[21] The NV 2004 election "audit" used this method in some counties according to independent investigations by several groups.

[22] A rule of thumb for determining the percentage of corrupt counts needed to alter an election result for a race is to divide the margin between two candidates by 0.30 (twice the assumed maximum vote shift that would not be considered immediately suspicious) to obtain an approximation of the smallest percentage of corrupted counts needed to alter the outcome of any race.

[23] Details of the basic formula are in Appendix A. USCountVotes has made a spreadsheet audit calculator available. http://ElectionArchive.org/ucvAnalysis/US/paper-audits/ and an "election integrity" audit calculator program.

The probability (rightmost column above) for finding at least one corrupt vote count decreases dramatically as the percentage of corrupt vote counts falls below 10%. Yet errors only need exist in a small percentage of vote counts to wrongfully alter outcomes of close races. A 2% audit is *not* likely to detect the small amount of vote miscount that could wrongfully alter outcomes of U.S. Congressional races with less than 5% margins between candidates.

Smaller jurisdictions and races with smaller number of total vote counts must audit approximately the same amount of vote counts as larger jurisdictions. In other words, smaller jurisdictions with fewer total counts must audit a larger percentage of their vote counts to have the same probability of detecting outcome-altering miscount.

Vote count audits of 1 or 2% provide no confidence that outcome-altering errors would be detected in close races. The right number to use in election audit legislation is the desired *probability of detecting outcome-altering* vote miscount.[24]

A nationwide 2% audit, such as that proposed in U.S. House of Representatives Bill 550 introduced in the 109th Congress is not only insufficient to ensure election outcome integrity, but worse, could practically interfere with more efficient local audits by taking the paper ballot records for a federal audit, making them unavailable for local audits.

How to Calculate Audit Amounts that will Detect "Outcome-Altering" Vote Miscount?

The purpose for auditing election results is ***to ensure that no outcome is wrongfully altered by error****.*

Therefore, we introduce a calculation to determine the number of vote counts to audit in order to achieve any desired probability of detecting outcome-altering error, for any margin between candidates. We begin with the basic probability equation for detecting one or more corrupt vote counts:

Eq. 1 $$P = 1 - \frac{\binom{C}{0}\binom{N-C}{S}}{\binom{N}{S}}$$

where

P is the probability of detecting one or more vote counts
C is the number of corrupt vote counts needed to wrongfully alter an election outcome
S is the number of vote counts that we randomly select for our hand counted audit, and
N is the total number of vote counts for a particular race or measure to be audited in our county

For the moment, we put off the discussion of how to correctly calculate the number of corrupt vote counts, C that we need to be able to detect in order to assure that no election outcomes are wrongly altered.

We want to solve our probability equation for S, the number of vote counts to audit, because we want to specify our desired probability, P, calculate C from our margins and other assumptions or facts, and we know N, the total number of vote counts for each race or outcome.

Attempting to solve for S, $$P = 1 - \frac{\binom{C}{0}\binom{N-C}{S}}{\binom{N}{S}}$$ reduces to

[24] That could be set at a constant 99% for state-wide audits or perhaps at 95% for county-wide audits.

Eq. 2: $\frac{(1-P)N!}{(N-C)!}-\frac{(N-S)!}{(N-S-C)!}=0$ and taking the natural log gives us:

Eq. 3: $\ln(1-P)+\ln((N-S-C)!)-\ln((N-S)!)+\ln(N!)-\ln((N-C)!)=0$

It would be wonderful if there was an easy solution for S that would allow election officials to simply plug numbers for N, P, and C into a simple formula. However due to the factorials, and real world values of the variables involved, no exact algebraic solution for S in terms of the other variables has been found for this equation. Hence we use the $LN\Gamma(x)=GammaLN$ function to approximate N! where $\Gamma(x)=\int_0^\infty e^{-u}u^{x-1}du$ where $\Gamma(x+1)=x!$ for any non-negative integer x.
We numerically determine a real number S to desired accuracy, which solves the equation

Eq. 4:
$$\ln(1-P)+gamma\ln(N-C-S+1)-gamma\ln(N-S+1)+gamma\ln(N+1)-gamma\ln(N-C+1)=0$$

for given values of C, P, and N. Then, since vote counts `S' are integers, we replace S with the integer value greater than our computed S. Our method thus calculates the correct number of vote counts to audit in order to achieve at least the desired probability of detecting at least one corrupt vote count given C corrupt counts out of a total of N vote counts..[25]

How To Estimate The Minimum Number Of Corrupt Vote Counts Required To Wrongfully Alter Election Outcomes

To calculate the correct audit sample size, we must first estimate the minimum number of corrupt vote counts, C, required to wrongfully alter an election outcome.

What Is The Maximum Number Of Votes That Might Be Wrongfully Switched From One Candidate To Another Within Any One Vote Count Without Raising Suspicion by the Opposition?

The Brennan Center Task Force, a group of security and voting system experts, in their threat analysis of U.S. voting systems, suggests a value of maximum wrongful vote shift of 15% of votes from one candidate to another. This results in a maximum wrongful margin shift between the two candidates of 30% (when 15% of votes are taken from one candidate and wrongfully given to another) in any one vote count. We assign, therefore, a "MaxShift" value of 0.30 as the maximum wrongful margin shift we might expect within any one vote count when the maximum vote shift is 0.15. Discrepancy data obtained from future audits should be publicly released and used to adjust the assumed value of the maximum rate of margin shift per vote count that was uncovered that was not noticed to be suspicious prior to the audit.

From the minimum margin between any two candidates[26] and the maximum margin shift on any one vote count, we can estimate the amount of vote counts which must be corrupted to wrongfully alter the election outcome for any race. Dividing the candidate margins by the 0.3 assumed maximum vote margin shift per vote count gives $C=\frac{mN}{MaxShift}$ where m = margin between candidates in the race or ballot measure being audited, N = #total vote counts in the for the particular race being audited, and MaxShift = the maximum assumed margin shift per vote count that would not be immediately suspicious. To obtain at least the

[25] This numerical method solution to Dopp's "election integrity" equation was derived by Professor Frank Stenger, PhD expert in numerical methods, University of Utah.
[26] The margin between candidates that should be used for county-wide audits is the minimum of the county-wide or the state-wide margin between candidates in any one race.

desired probability of detecting outcome-altering vote miscount, the integer floor values for C are used to calculate integer ceiling values for S.

The smaller the margin between candidates, the smaller the percentage of corrupt vote counts that is needed to wrongfully alter an election outcome and the larger the audit percentage that is needed to detect at least one corrupt vote count.

Should State-Wide Or County-Wide Margins Between The Candidates Be Used? The state-wide margin determines the percentage of votes that could be wrongfully switched, on average, in each county to wrongfully alter the state-wide outcome. Therefore the amount of vote counts audited in each county needs to be sufficient to detect at least the amount of vote miscount that could have altered the state-wide margin. However, in order to determine if the outcome in each county is correct and to reduce the need for expanded audits when discrepancies are uncovered in other counties, each county should audit sufficient vote counts to validate the outcome in their county. Therefore, *whichever margin is less, the state-wide or the county-wide margin, should be used to calculate the audit amount in each county* in order to make the audit process most efficient and informative.[27]

Should State-Wide Or County-Wide Total Number Of Vote Counts Be Used? A*udits should be conducted for any race in each county or township using N = the total #vote counts within the county*. Although audits could be conducted with smaller sample sizes using the state-wide total number of vote counts, this could prolong the audit process by requiring expanded audits in all counties whenever discrepancies are found in any one county. If county-level audits are conducted initially, more information is obtained from the first round of audits to make determinations that could avoid unnecessary expanded audits in counties where no discrepancies are found during initial audits. County-wide audits could enable the election process to be more quickly completed to announce final election winners.

When auditing precinct vote count audits (as opposed to machine DRE machine count audits) an adjustment in the calculation of the number of corrupt counts that could wrongfully alter the outcome is required whenever precinct sizes vary significantly.

When auditing precinct vote counts, the probability of detecting outcome-altering vote miscount is reduced if vote miscount is targeted in fewer large-sized precincts. To adjust for precinct-size variation we directly calculate the minimum number of corrupt vote counts that could alter outcomes using a list of the total number of ballots cast in precincts.[28] The following information is needed to adjust for precinct-size variation:

PrecinctSize = (n0, n1,…,) the total number of ballots cast in each precinct in which votes were cast for a race.
T = total number of ballots cast in the county for the particular race to be audited (T should be equal to the sum of the precinct sizes input above).

On the other hand, DRE machine audits do not need adjustment for precinct-size variation because there are more DRE machines in larger precincts, increasing the probability of selecting counts in larger precincts[29]. At an average ten minutes per voter, 6 voters per hour could use one DRE times a 14 hour Election Day would be 84 voters. Responsible election officials provide adequate DRE machines for voters, and provide backup optical scan paper ballots. To be conservative DRE machine count audit sizes could be adjusted using the number of total ballots cast on each DRE.

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[27] Email discussions with Howard Stanislevic led to the realization that the state-wide margin should be used whenever the state-wide margin is smaller than the county-wide margin.

[28] The method for calculating the number of corrupt precincts needed to wrongly alter an outcome is described in Appendix B. The effect of precinct-size variation affects precinct count (not DRE machine count) audits. When auditing precinct counts, a decrease in probability is avoided by calculating precisely the number of corrupt vote counts required to alter election outcomes as shown in Appendix B. Because more DRE machine counts exist in larger counties, the audit amount does not need adjustment for precinct-size when auditing DRE machine counts. DRE machine count size variability still needs further study.

[29] Although there might be certain errors that kick in after a certain number of ballots are cast on a DRE machine.

Some Real Life Examples – Election Integrity Vote Count Audit Amounts

Here are examples of calculated audit sample sizes using Lake County, OH and Multnomah County, OR.

INPUTS		
wrong margin shift per vote count	0.3	<< constant - let expert analysts set
total# of vote counts	217	
probability of detecting miscounts	0.95	<<set as desired

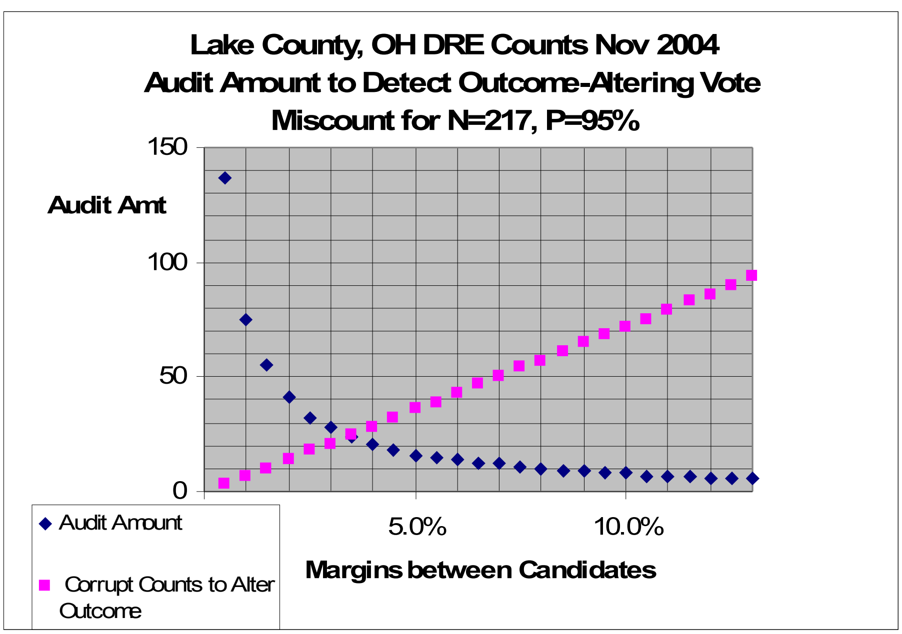
OUTPUTS					
Candidate Margins	Number of Vote Counts to Audit	Percentage of Vote Counts to Audit	Number Corrupt Counts to Alter Outcome	Percent Corrupt Counts to Alter Outcome	Probability Check
0.5%	137	63.1%	3	1.4%	95.11%
1.0%	75	34.6%	7	3.2%	95.12%
1.5%	55	25.3%	10	4.6%	95.00%
2.0%	41	18.9%	14	6.5%	95.19%
2.5%	32	14.7%	18	8.3%	95.03%
3.0%	28	12.9%	21	9.7%	95.29%
3.5%	24	11.1%	25	11.5%	95.57%
4.0%	21	9.7%	28	12.9%	95.29%
4.5%	18	8.3%	32	14.7%	95.03%
5.0%	16	7.4%	36	16.6%	95.11%
5.5%	15	6.9%	39	18.0%	95.42%
6.0%	14	6.5%	43	19.8%	95.93%
6.5%	12	5.5%	47	21.7%	95.10%
7.0%	12	5.5%	50	23.0%	96.07%
7.5%	11	5.1%	54	24.9%	96.06%
8.0%	10	4.6%	57	26.3%	95.60%
8.5%	9	4.1%	61	28.1%	95.20%
9.0%	9	4.1%	65	30.0%	96.23%
9.5%	8	3.7%	68	31.3%	95.35%
10.0%	8	3.7%	72	33.2%	96.28%
10.5%	7	3.2%	75	34.6%	95.12%
11.0%	7	3.2%	79	36.4%	96.03%
11.5%	7	3.2%	83	38.2%	96.78%
12.0%	6	2.8%	86	39.6%	95.38%
12.5%	6	2.8%	90	41.5%	96.18%

Lake County, OH November, 2004 has N=217 total county-wide DRE machine vote counts.

The table to the left and the chart below show the audit amounts required for a county-wide race for selected margins between candidate (leftmost column in table and horizontal axis in chart) in order to have at least 95% probability for detecting at least one corrupt count if there is outcome-altering vote miscount, assuming that at most 15% of vote counts are wrongly shifted per vote count (i.e. assuming that candidate margins are wrongly shifted by at most 30% per vote count). Lake County uses DRE voting machines so no adjustment is needed for precinct-size variation.

We check the resulting probabilities of detection using the calculated audit sample sizes. The probabilities are at or above our desired 95%. If we increase the desired probability to 99%, then the audited number of vote counts increases.

The smaller the margin between candidates, the fewer corrupt counts could wrongly alter outcomes, the harder is it to detect, so the larger the required audit size. Whereas the larger the margin between candidates, the larger the number of corrupt vote counts needed to alter the race, and the smaller the audit required to detect it.



Multnomah County, OR in its 2006, May primary election had N=128 total number of precincts using paper ballots counted with optical scan machines.

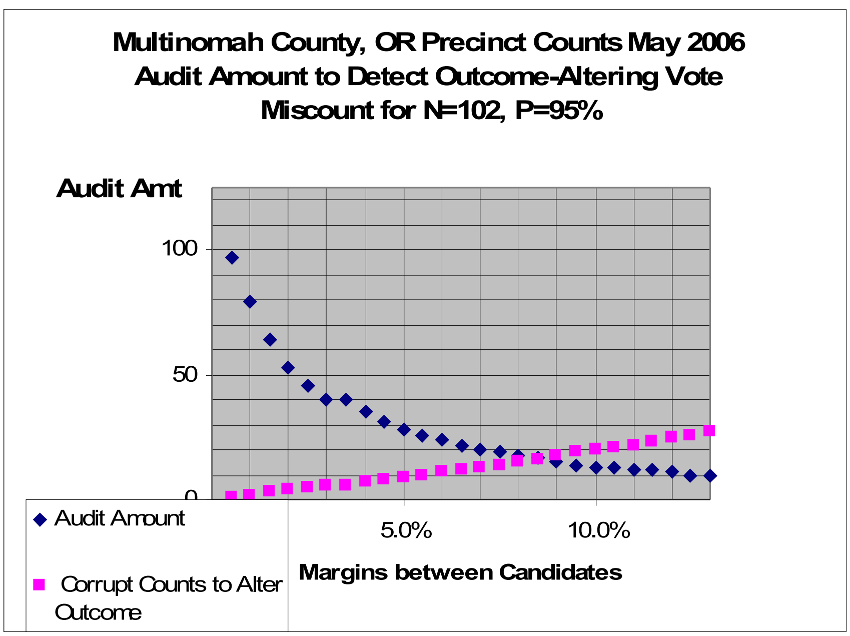
The Democratic US Representative primary election had N=102 total precincts with ballots cast in that race[30], with T=62,651 total ballots cast and the total number of ballots cast in the largest precincts were (1292, 1267, 1239, 1192, 1188, 1157, 1144, 1109, 1060, 1054, 1053, 1047, 1028, 1028, 983, 964, 960, 959, 939, 921, 889, 888, 832, 832, 831, 826, 812, 793, 791, 783, 750, 740, 727, 723, 722, 717, 703, 691, 667, 666, 663, 652, 647, 639, 637, 631, 630, 614, 613, 600, 583, 580, 579, 569). For desired probability P=95%, assuming a 15% maximum wrongful vote shift per vote count (i.e. 30% margin shift) the table on the left shows the minimum amount of vote counts to audit for selected margins between candidates, adjusted for precinct-size variation.

The table at right and the chart below shows the audit sample sizes for Multnomah County's Democratic primary for the U.S. House of Representatives. However, the actual margin between the two leading candidates in Multnomah County in the May 2006 election was about 80% (not shown on our chart) so that a 1% minimum audit rate is sufficient.

The table below (lower right) shows how the probability of detecting at least one corrupt count decreases if vote miscounts are targeted to the largest precincts yet audit size is *not* adjusted for precinct-size variation. Counties with more divergent precinct sizes would have lower probabilities.

		INPUTS
max wrong margin shift per vote count	0.3	<< constant - let experts set
total# of vote counts	102	<< total number of counts in race
probability of detecting at least one miscount	0.95	<<set as desired

OUTPUTS					
Margin between candidates	Audit this many vote counts	Audit percent	Number corrupt counts to alter outcome	Percent corrupt counts to alter outcome	Probability Check
0.5%	97	95.1%	1	1.0%	95.10%
1.0%	79	77.5%	2	2.0%	95.09%
1.5%	64	62.7%	3	2.9%	95.09%
2.0%	53	52.0%	4	3.9%	95.01%
2.5%	46	45.1%	5	4.9%	95.41%
3.0%	40	39.2%	6	5.9%	95.43%
3.5%	40	39.2%	6	5.9%	95.43%
4.0%	35	34.3%	7	6.9%	95.29%
4.5%	31	30.4%	8	7.8%	95.15%
5.0%	28	27.5%	9	8.8%	95.17%
5.5%	26	25.5%	10	9.8%	95.52%
6.0%	24	23.5%	11	10.8%	95.63%
6.5%	22	21.6%	12	11.8%	95.54%
7.0%	20	19.6%	13	12.7%	95.23%
7.5%	19	18.6%	14	13.7%	95.55%
8.0%	18	17.6%	15	14.7%	95.75%
8.5%	17	16.7%	16	15.7%	95.85%
9.0%	15	14.7%	18	17.6%	95.75%
9.5%	14	13.7%	19	18.6%	95.55%
10.0%	13	12.7%	20	19.6%	95.23%
10.5%	13	12.7%	21	20.6%	95.99%
11.0%	12	11.8%	22	21.6%	95.54%
11.5%	12	11.8%	23	22.5%	96.21%
12.0%	11	10.8%	25	24.5%	96.25%



Margins between candidates	Probability of detection without adjusting for precinct-size variation	Audit Amount if NOT adjusted for precinct-size variation
0.5%	95.1%	97
1.0%	86.4%	64
1.5%	83.9%	46
2.0%	86.9%	40
2.5%	84.4%	31
3.0%	83.8%	26
3.5%	80.9%	24
4.0%	79.4%	20
4.5%	80.1%	18
5.0%	79.9%	16
5.5%	81.2%	15
6.0%	79.5%	13
6.5%	79.7%	12
7.0%	82.4%	12
7.5%	82.0%	11
8.0%	81.2%	10
8.5%	79.9%	9
9.0%	83.9%	9
9.5%	82.0%	8
10.0%	83.7%	8

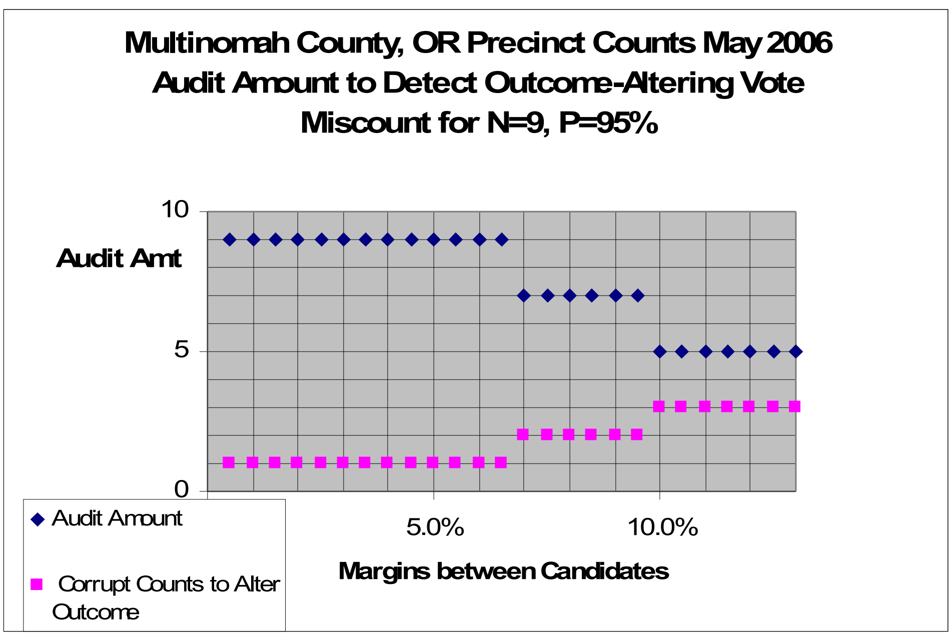
[30] There were 104 precincts eligible to vote in this primary race, but in two precincts no ballots were cast.

Candidate Margins	Number of Vote Counts to Audit	Percentage of Vote Counts to Audit	Number Corrupt Counts to Alter Outcome	Percent Corrupt Counts to Alter Outcome
0.5%	9	100.0%	1	11.1%
1.0%	9	100.0%	1	11.1%
1.5%	9	100.0%	1	11.1%
2.0%	9	100.0%	1	11.1%
2.5%	9	100.0%	1	11.1%
3.0%	9	100.0%	1	11.1%
3.5%	9	100.0%	1	11.1%
4.0%	9	100.0%	1	11.1%
4.5%	9	100.0%	1	11.1%
5.0%	9	100.0%	1	11.1%
5.5%	9	100.0%	1	11.1%
6.0%	9	100.0%	1	11.1%
6.5%	9	100.0%	1	11.1%
7.0%	7	77.8%	2	22.2%
7.5%	7	77.8%	2	22.2%
8.0%	7	77.8%	2	22.2%
8.5%	7	77.8%	2	22.2%
9.0%	7	77.8%	2	22.2%
9.5%	7	77.8%	2	22.2%
10.0%	5	55.6%	3	33.3%
10.5%	5	55.6%	3	33.3%
11.0%	5	55.6%	3	33.3%
11.5%	5	55.6%	3	33.3%
12.0%	5	55.6%	3	33.3%

Now consider the **Multnomah County, OR May 2005 primary race for State Senate District #17 where there were only N = 9 precinct counts eligible to vote in that race**. To the left are the audit sample sizes required for various margins. Without even adjusting for precinct size variation, a 100% audit is required for margins up to 6.5% in order to achieve a 95% probability for detecting outcome-altering vote miscount when assuming a 15% maximum vote counts shift per vote count (i.e. 30% margin shift).

The margin between the two OR State Senate candidates was approximately 6% so that all nine counts must be audited – hand counted - to detect outcome-altering electronic vote miscount for this race.

Because only one corrupt count in nine could wrongly alter the race outcome, all nine precinct counts must be manually counted to have a good chance to detect one corrupt count.



SUMMARY

The National Election Data Archive (NEDA)'s Frank Stenger and Kathy Dopp developed a numerical solution for calculating vote count audit sample sizes to ensure electronic vote count integrity in all auditable voting systems, particularly when voters verify that their paper ballot records are correct. Properly calculated and conducted audits – hand counts of paper ballots performed to check the accuracy of electronic vote counts - can be performed in jurisdictions using audit-able voting systems.[31]

To assure electronic election outcome accuracy in auditable voting systems, the following information is required to calculate vote count audit sample sizes:

1. The desired probability for detecting outcome-altering miscount.[32]
2. The minimum of the state-wide or county-wide margin between the winning and losing candidates or ballot measures.
3. The assumed maximum vote miscount per electronic vote count not considered suspicious, usually set by an expert analyst.[33]
4. The number of total (DRE machine or precinct) electronic vote counts in the jurisdiction for which votes were cast for the race being audited.[34]
5. In the case of auditing precinct counts, the total number of ballots cast in each precinct in which votes were cast for the race. (Also possibly, in the case of auditing DRE machine counts, the total number of ballots cast on each DRE eligible to be voted on in a particular race.)

NEDA's new "election integrity" audit calculation method is shown in Appendix B.

To calculate vote count audit size for a particular race requires knowing the margin between candidates, the total number of electronic vote counts in a race for the jurisdiction being audited, an assumed maximum wrongful rate of vote miscount per vote count that would not otherwise raise suspicion and action[35], the desired probability for detecting errors, and, in the case of optical scan precinct count audits, the total number of ballots cast in each precinct.

Combined with procedures to secure paper ballots and reconcile precinct counts, vote count integrity can be assured by auditing sufficient vote counts of auditable voting systems to detect any miscount that could wrongfully alter election outcomes and by educating voters to verify paper ballot records.

The National Election Data Archive is a national, not-for-profit nonpartisan organization devoted to promoting scientific means to ensure election-outcome integrity. NEDA's "Election Integrity Audit" paper can be found at http://electionarchive.org/ucvAnalysis/US/paper-audits/ElectionIntegrityAudit.pdf. A program that can be used to calculate election audit amounts is available on ElectionArchive.org: http://electionarchive.org/auditcalculator/eic.cgi

[31] Paperless electronic voting machines and lever machines provide no method to independently check electronic vote count accuracy. Some states, like Maryland, Georgia, and parts of Florida, Pennsylvania, and several other states use voting systems that are not auditable. Any voting system that provides no routine measures to detect and correct errors is fundamentally inaccurate and error-prone.

[32] We tentatively suggest a minimum of 95% for county-wide audits.

[33] We recommend initially using 0.15=15% or 0.20=20% (i.e. a 30% or 40% margin shift respectively). *If an election official uses 20% (i.e. a 40% margin shift) for the maximum vote shift considered not to raise suspicion, then any amount of shift in a vote count that is more than 20% in comparison to prior comparable election results could be considered suspicious and its count automatically included in an audit or investigation in addition to the randomly selected vote counts.*

[34] Assuming a county-wide audit, or alternatively for a state-wide audit the total number of vote counts in the state in a race.

[35] The Brennan Justice Center has suggested a rate of 15% to 30% (i.e. a 30% to 60% wrongful margin shift) and Howard Stanislevic of VoteTrustUSA suggests a rate of 20% (i.e. a 40% wrongful margin shift). NEDA adopts 15% (a 30% margin shift) in this paper, but encourages people to use a larger number to produce more conservative (larger) audit samples.

AUDIT PROCEDURES & WHAT IF DISCREPANCIES ARE FOUND?

Audits of electronic vote counts would be useless without consequences and procedures to follow when discrepancies are found. This paper does not go into detailed audit procedures. However we suggest:

- If the "election integrity audit" calculation results in an audit amount of less than 1% of vote counts, then a minimum audit rate of 1% should be used. If 1% of vote counts is an integer plus a fraction, then the next larger integer value should be used.[36]
- Audits performed on a county level[37] are most conservative and practical because elections are administered separately in each county and errors may be specific to particular counties. State-wide audits would prolong the audit process in all counties whenever discrepancies are found in any county due to the reduced information they provide of each county's election integrity.
- Only after an auditable report of unofficial vote counts is publicly released by election officials should vote counts be selected and auditors randomly assigned to count each.
- After randomly selecting vote counts, extra vote counts could be selected either by candidates or by criteria (such as whenever absolute increases in vote count margins from comparable previous elections exceed the assumed maximum non-suspicious margin shift).
- States and county vote count audit committees comprised of *independent*[38] PhD mathematicians and statisticians, PhD computer scientists, gaming experts with masters' degrees, election integrity activists and non-voting members representing local election officials; all representing several political parties, could be appointed to determine audit policies and procedures specific to local election systems. These committees could oversee audits, make decisions on what to do when discrepancies are found, produce vote count audit reports, and work with local election officials and state legislators to suggest improvements to voting systems.[39]
- Audit procedures should be transparently open to public observation.
- Audits must be performed by *independent auditors* who do not work for election offices or for voting machine equipment or service companies or otherwise have access to voting equipment. Auditors could consist of state audit officers; election judges or poll workers who did not work during the election to be audited; persons whose names are supplied by candidates and authors of ballot measures; or independent accounting firms.
- At least two political parties should be present to verify the hand-count of each voter-verifiable paper ballot record.
- Audit results should be made publicly available before vote counts are made official.
- Any discrepancy between electronic and paper ballots must be evaluated to determine:
    - Can the discrepancy be explained or not?

---

[36] Any audit of less than 1% of vote counts could potentially not detect outcome-altering miscount in state-wide contests even in cases where calculations show less than 1% to be adequate.

[37] Some states like MA have townships administer elections and audits should be designed per each township. State-wide audits would reduce the sample size per county and may result in reduced and varying probabilities of detecting vote miscount in different counties due to differing margins, number of vote counts, and other factors specific to each county. State-wide designed audits could be less efficient when some counties use optical scan precinct counts and others use DRE machine counts because precinct counts would have to be audited in order to pool counts together for random selection.

[38] Independent means not related to any election officials or voting vendors.

[39] For instance, a Vote Count Audit Committee may recommend the implementation of unique anonymous identifiers that match paper ballot to machine tabulated records, in order to facilitate audit investigations and corrections.

- Could the amount of discrepancy have altered the outcome or not?
- Can the vote count be corrected using existing available records or not?

- The audit should be expanded, perhaps to a 100% manual count in some counties, to search for additional errors whenever it is possible that errors could have wrongfully altered an outcome.
- There may be cases when it can be determined that vote miscount may have wrongfully altered election outcomes, yet vote counts cannot be corrected using available records. Electronic ballot voting systems are susceptible to election tampering in ways that are not detectable by audits, or sufficient paper ballot tampering may be detected to wrongfully alter outcomes. In such cases elections may have to be re-held.
- Election results should not be certified official until after the independent vote count audit report is publicly released.
- The National Election Data Archive stresses that vote count data must be broken out by precinct and by vote type (Election Day, early, early- provisional, Election Day – provisional, absentee, overseas, military, etc.) both for statistical analysis and for audits. Otherwise errors could be hidden in aggregated data. Different audit procedures may need to be followed for different types of votes and different races within one county, parish, or township.
- Random selection must be done scientifically. An informative paper on the subject of random vote count selection is "The Role of Dice in Election Audits – Extended Abstract".[40]

More research by teams of computer scientists, election officials, mathematicians, and others are needed to establish best practices and procedures for vote count audits.

[40] June 16, 2006 "The Role of Dice in Election Audits – Extended Abstract" by Arel Cordero, David Wagner, and David Dill. http://www.cs.berkeley.edu/~daw/papers/dice-wote06.pdf

Appendix A:

The probability estimates are based on a "Hypergeometric" distribution which determines the probability of finding:

a) **x** (target corrupt electronic vote counts) We let x = 0 to find the probability of detecting no corrupted vote counts.

b) in an overall sample of **n** (Number of Audited Precincts) which in this case is 20 (for 2% audit) or 50 (for 5% audit),

c) when there are **X** (Hypothetical Number of Corrupted Precincts) which in two tables above range from 50% to 0.05% of 1000. However, the best way to calculate X is [the smallest margin between any candidates in any race] divided by [the maximum percentage of corruption that would not raise immediate suspicion in any one electronic vote count] to give the number of corrupt vote counts that could wrongly alter an election outcome. When there are differences in the size of precincts, X can be calculated using the total number of ballots cast in each precinct where votes are casts in the audited race.

d) out of **N** (Total Number of Precincts) which in this case is 1000.

This distribution is calculated using the Excel Function:

$$\text{HYPGEOMDIST(x, n, X, N)} = \frac{\binom{X}{x}\binom{N-X}{n-x}}{\binom{N}{n}}$$

The hypergeometric function assumes all individual "picks" are random but adjusts this random probability for each pick. The first row in Table 1 for example, assumes a 50% probability that the first of 20 picks will be corrupted, a 499/999 probability that the second precinct chosen will be corrupted, and so on for all 20 picks.

The probability that one or more of the 20 vote counts will be corrupt is 1 or 100% minus the probability that none of the 20 precincts will be corrupted. So, the probability that *at least one* of the 20 is corrupted which equals:

$$P = 1 - \frac{\binom{X}{0}\binom{N-X}{n}}{\binom{N}{n}}$$

This equation can be solved using numerical methods for the sample size n.

For any county and race or ballot measure, the probability distribution will depend on the exact values of n, X, and N.

A spreadsheet calculator for determining the probabilities of detecting one or more miscounted precincts is available on the USCV web site: http://ElectionArchive.org/ucvAnalysis/US/paper-audits/

Appendix B: Numerical Calculations

The following Matlab program shows the basic "election integrity" numerical algorithm for vote count audit amounts. It needs revision to be fully general and usable in all situations.

```
%
% `k_d2_revised5.m'
%
% This routine computes an S, via method of bisections to an accuracy EP, given
% a probability P , 0 < P < 1, positive integers  N, and C, with 0 < C < N,
% and with 0 <= S <= N-C, such that dopp(P,N,C,S) = 0, with (*) dopp(P,N,C,S) =
% ln(1-P)-ln[(N-C)! (N-S)!/(N! (N-C-S)!)].
clear all
format long e
%
% Initialization
%
% Set this to 1 for precinct count audits
%
PrecinctCounts = 0;
%
% These are input values supplied by the user.
%
P = .95;
N = 211;
Maxshift = 0.3;
marginmin = .005;
marginmax = .20;
marginstep = .005;
EP = .005;
%
% These input values are supplied for precinct count audits only
%
TotBallots = 16900;
Precincts = ...
[921, 755, 731, 719, 687, 679, 676, 675, 675, 672, 670, 668, 668, 667, 660, 660, 660, 658, 657, 656, 656, 639, 639, 638, 636,
627, 621, 620, 620, 617, 617, 611, 609, 609, 608, 606, 605, 604, 596, 596, 594, 593, 593, 592, 591, 589, 588, 588, 585, 585,
585, 580, 579, 574, 572, 569, 568, 567, 565, 560, 554, 552, 552, 551, 551, 547, 547, 545, 544, 541, 541, 540, 539, 538, 537,
536, 536, 533, 532, 530, 528, 526, 526, 525, 524, 522, 519, 519, 518, 518, 517, 517, 514, 509, 509, 499, 497, 496, 495, 495,
495, 494, 494, 488, 488, 486, 486, 484, 481, 481, 480, 479, 479, 479, 478, 478, 478, 478, 477, 477, 476, 476, 475, 475, 474,
470, 468, 465, 465, 461, 460, 456, 455, 455, 455, 455, 454, 454, 453, 451, 450, 449, 448, 446, 444, 444, 443, 443, 443, 443,
438, 435, 435, 435, 434, 431, 429, 429, 429, 428, 427, 425, 425, 424, 424, 422, 422, 419, 417, 416, 415, 415, 412, 411, 407,
407, 400, 399, 398, 396, 393, 392, 391, 383, 380, 374, 372, 369, 366, 365, 365, 363, 362, 352, 349, 347, 346, 346, 340, 330,
327, 320, 320, 316, 316, 314, 314, 314, 311, 310, 287, 261, 247, 241, 223, 114, 94];
ArraySize =size(Precincts,2);
%
% Compute CumulativeShift
%
if(PrecinctCounts == 1)
  CumulativeShift(1)=Precincts(1);
  for i=2:ArraySize
        CumulativeShift(i) = Precincts(i)+CumulativeShift(i-1);
  end
  for i=1:ArraySize
        CumulativeShift(i) = CumulativeShift(i)*Maxshift/TotBallots;
  end
end
%
% Compute max j
%
maxj = (marginmax - marginmin + .005)/marginstep; % = 40
```

```
%
% Compute C values % C = margin*N/Maxshift
%
for j=1:maxj
   margin(j) = j * marginstep;
      if(Precinctcount==1)
      i=1;
      while (margin(j) > CumulativeShift(i))
         i=i+1;
      end
         c(j)=i;
   else
         c(j) = floor(margin(j)*N/Maxshift);
   end
end
%
% Computing S via method of bisections
%
% First, the lower and upper end values of S
%
for j=1:maxj
   C(j) = c(j);
% First, the lower and upper end values
% of S
%
   A=0;
   B = N-C(j);
%
   a = dopp(P,N,C(j),A);
   b = dopp(P,N,C(j),B);
   while(abs(A-B) > EP)
      S = (A+B)/2;
      cc = dopp(P,N,C(j),S);
      if a*cc > 0
         A = S;
         a = cc;
      else
         B = S;
         b = cc;
      end
   end
   s(j) = S;
   S0 = ceil(S);
   c0 = dopp(P,N,C(j),S0);
   if(c0 >= 0)
      ss(j) = S0;
   else
      ss(j) = S0+1;
   end
end
%
plot(margin,s,'-',margin,ss,'.', margin,C,'x')
pause
title('Plot of C & S VS margin, N = 211, .005 < margin < .2, P = .95')
xlabel(' margin  = values = C*Maxshift/N')
ylabel('S (--) Continuous & SS (.) integer values')
print -dps2 dopp_plot211.ps
%
% Data Files
for j=1:maxj
   TOT(j,:) = [margin(j), C(j), ss(j)];
```

```
end
save('tot','TOT', '-ASCII');
```

The following subroutine, dopp.m, is used in the above program. It evaluates ln(1-P) - ln[(N-C)!(N-S)!/(N!(N-C-S)!)] for input values of P, N, C, and S.:

```
%
% `dopp.m'
%
function y = dopp(P,N,C,S)
%
QQ = log(1-P);
AA = gammaln(N+1);
BB = gammaln(N-C+1);
CC = gammaln(N-S+1);
DD = gammaln(N-C-S+1);
y = QQ + DD-CC +AA-BB;
```

The algorithm for the adjustment for precinct-size variation in audits of precinct counts, in words, is:

- Sort the precinct size array in descending order (largest to smallest)
- Find the cumulative total number ballots cast for the largest precinct, the two largest precincts, the three largest precincts, etc. CumulativePrecinct =($PrecinctSize(0)$, $\sum_{i=0}^{i=1} PrecinctSize(i)$, $\sum_{i=0}^{i=2} PrecinctSize(i)$,…)
- For each margin between candidates (beginning with the smallest margin) find the smallest value of i for which (margin) < (CumulativePrecinct(i)*Maxshift/T)
- Compare i to (margin*N/Maxshift) and use the smaller value as the amount of corrupt precincts to input into the calculation of audit size, S.[41]
- Once we find that a value of margin for which the calculation is not needed, we do not perform the calculation for larger margins where it will not be needed.

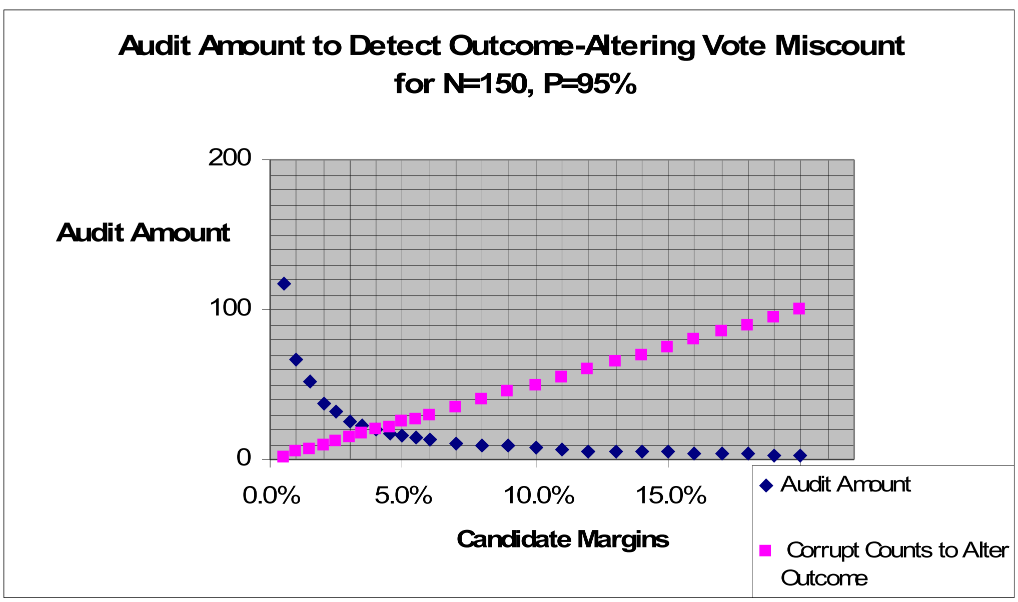
[41] We recognize that we are using a calculation which compares integer with decimal. We may want to bring the floor integer value for corrupt counts, C, into the calculation prior to this point. Little details still need review.

Appendix C: Example Results with N=150 and N=1000

Notice that the number of electronic vote counts that must be audited to detect a particular rate of miscount, does not change as significantly as one might expect when the total number of vote counts changes. Thus the percentage of vote counts to audit is much higher when the total number of counts is less.

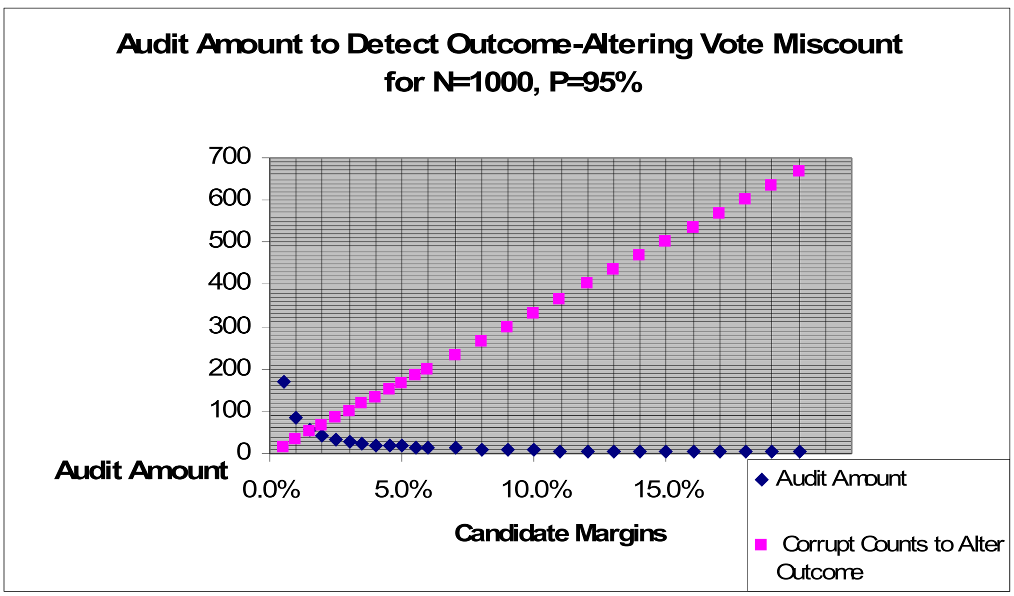
The first example is for races with N=150 total vote counts when assuming a 15% maximum vote counts shift per vote count (i.e. 30% margin shift).

Candidate Margins	Number of Vote Counts to Audit	Percentage of Vote Counts to Audit	Number Corrupt Counts to Alter Outcome	Percent Corrupt Counts to Alter Outcome	Probability Check
0.5%	117	78.0%	2.0	1.3%	95.28%
1.0%	67	44.7%	5.0	3.3%	95.09%
1.5%	52	34.7%	7.0	4.7%	95.30%
2.0%	38	25.3%	10.0	6.7%	95.16%
2.5%	32	21.3%	12.0	8.0%	95.05%
3.0%	26	17.3%	15.0	10.0%	95.09%
3.5%	23	15.3%	17.0	11.3%	95.06%
4.0%	20	13.3%	20.0	13.3%	95.39%
4.5%	18	12.0%	22.0	14.7%	95.25%
5.0%	16	10.7%	25.0	16.7%	95.45%
5.5%	15	10.0%	27.0	18.0%	95.68%
6.0%	13	8.7%	30.0	20.0%	95.21%
7.0%	11	7.3%	35.0	23.3%	95.22%
8.0%	10	6.7%	40.0	26.7%	95.99%
9.0%	9	6.0%	45.0	30.0%	96.38%
10.0%	8	5.3%	50.0	33.3%	96.46%



This next example is for a race with N=1000 total vote counts, much larger than average, except for state-wide races, assuming a 15% maximum vote counts shift per vote count (i.e. 30% margin shift).

Candidate Margins	Number of Vote Counts to Audit	Percentage of Vote Counts to Audit	Number Corrupt Counts to Alter Outcome	Percent Corrupt Counts to Alter Outcome	Probability Check
0.5%	170	17.0%	16.0	1.6%	95.05%
1.0%	86	8.6%	33.0	3.3%	95.11%
1.5%	57	5.7%	50.0	5.0%	95.08%
2.0%	43	4.3%	66.0	6.6%	95.03%
2.5%	34	3.4%	83.0	8.3%	95.01%
3.0%	29	2.9%	100.0	10.0%	95.50%
3.5%	24	2.4%	116.0	11.6%	95.00%
4.0%	21	2.1%	133.0	13.3%	95.17%
4.5%	19	1.9%	150.0	15.0%	95.58%
5.0%	17	1.7%	166.0	16.6%	95.55%
5.5%	15	1.5%	183.0	18.3%	95.29%
6.0%	14	1.4%	200.0	20.0%	95.70%
7.0%	12	1.2%	233.0	23.3%	95.94%
8.0%	10	1.0%	266.0	26.6%	95.53%
9.0%	9	0.9%	300.0	30.0%	96.03%
10.0%	8	0.8%	333.0	33.3%	96.14%



Prepared by Kathy Dopp of

National Election Data Archive Project (NEDA) - ElectionArchive.org

501(c)(3) status approved

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The National Election Data Archive is a scientific research project whose mission is to ensure the accuracy of elections in America through development and promotion of mathematical means to ensure accurate vote counts and the creation and analysis of a database containing precinct-level election data broken out by vote type for the entire United States. NEDA is seeking funds to complete a nationwide system for mathematically monitoring election results.