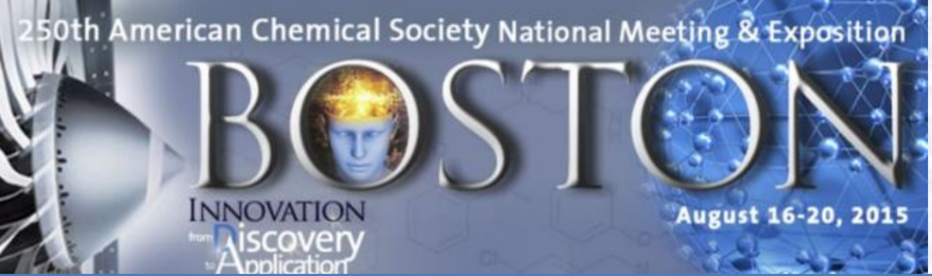


Boston, MA
August 19, 2015

Le Système international d'unités
The International System of Units
SI

Linking the International System of Units to Fundamental Constants

Prof Dr Joachim H. Ullrich

President of PTB, Physikalisch-technische Bundesanstalt
Vice President of the CIPM
President of the Consultative Committee of Units
Vice President of DIN, the German Standardisation Organisation

The Metre Convention

Bureau International des Poids et Mesures

Meter Convention: 1875

CGPM: General Conference of Weights and Measures

CIPM: International Committee of Weights and Measures

BIPM: International Bureau of Weights and Measures

Provides a global measurement infrastructure

Quantities and Measurement Units

Bureau International des Poids et Mesures

Measurement units

number

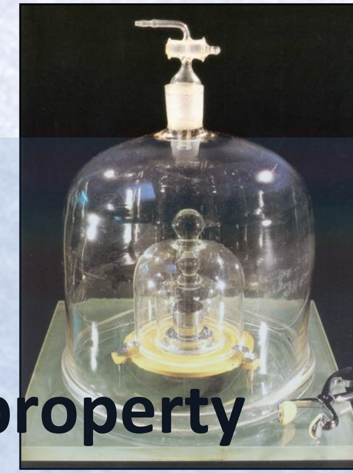
unit

quantity

$$Q = \{Q\}\ [Q]$$

$$m = 10.1(2)\ \mathrm{kg}$$

$$t = 55.4\ (1)\ \mathrm{s}$$



artefact

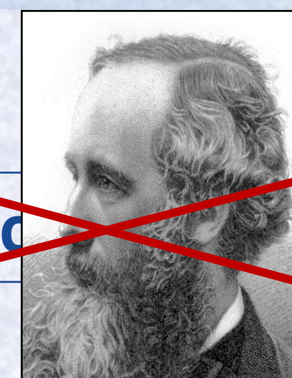
material property

measurem. prescription

defining constant

$$\Delta\nu(^{133}\mathrm{Cs})_{\mathrm{hfs}} = 9\ 192\ 631\ 770.\ 5\ (6)\ \mathrm{s}^{-1}$$

unc



J. C. Maxwell 1870

constant of nature

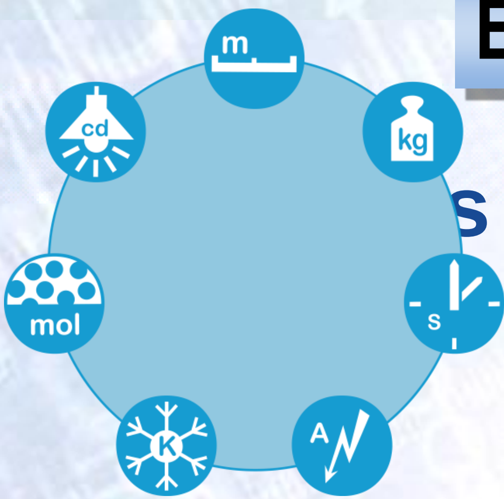
$$1\ \mathrm{s} = \frac{9\ 192\ 631\ 770}{\Delta\nu(^{133}\mathrm{Cs})_{\mathrm{hfs}}}$$

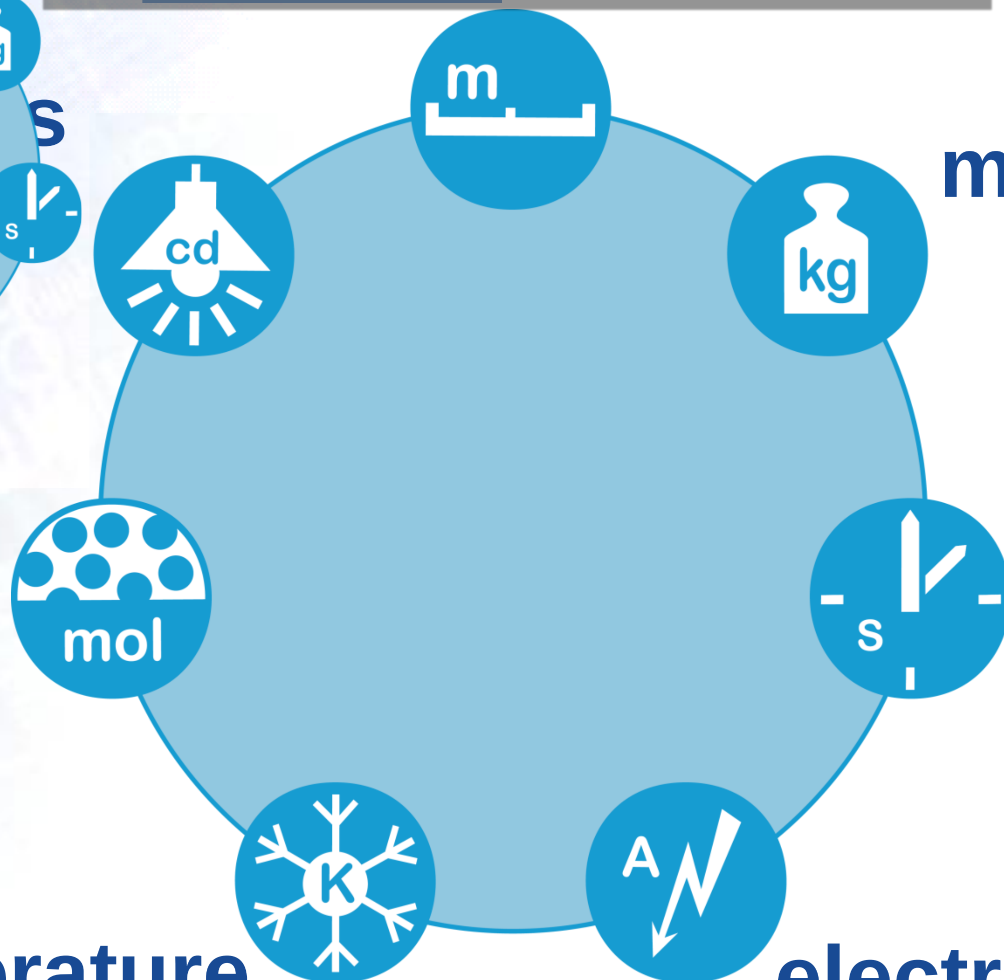
Define a unit by fixing the numerical value of a constant of nature

The International System of Units: SI

Bureau International des Poids et Mesures

Established in 1960: CGPM





mass

time

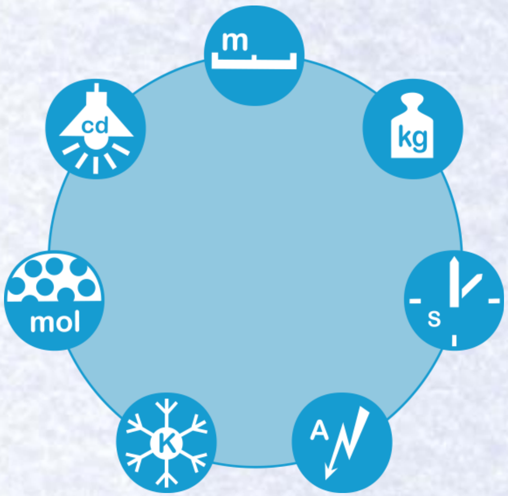
electric current

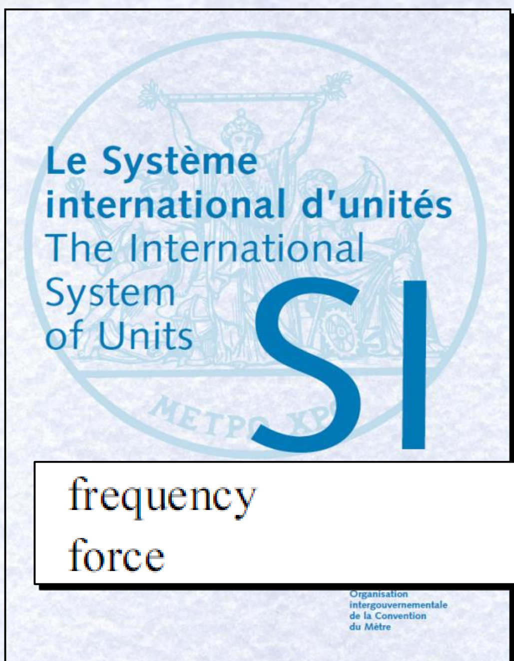
temperature

amount of substance

The International System of Units: SI

Bureau International des Poids et Mesures





Base units

SI base unit

Symbol

Derived units with special names

SI coherent derived unit (a)

Dimensions of quantities

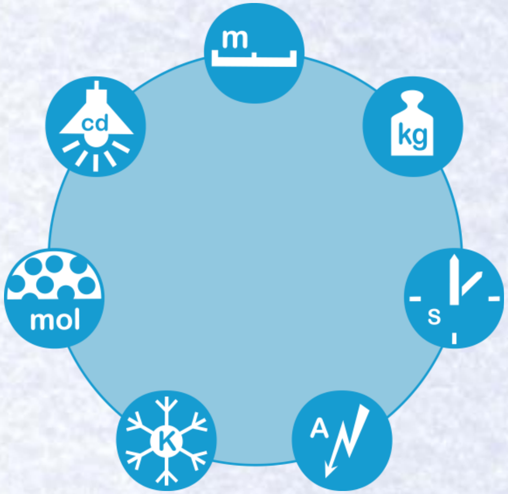
→ A set of coherent SI units

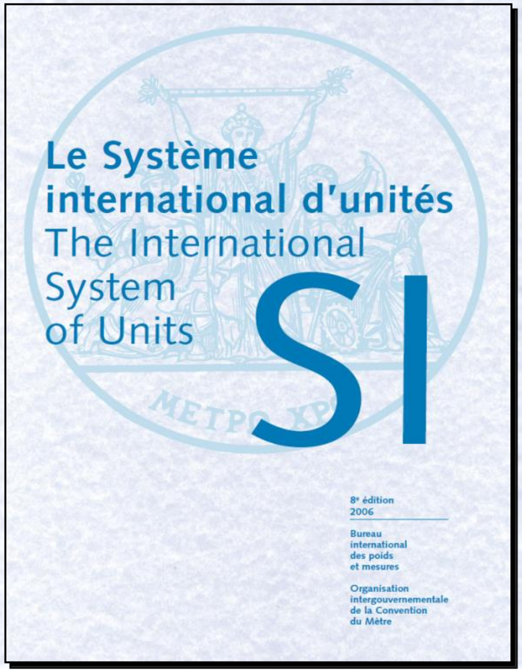
Derived quantity	Name	Symbol	Expressed in terms of other SI units	Expressed in terms of SI base units
				m/m
solid angle	steradian (b)	sr (c)	1 (b)	m^2/m^2
frequency	hertz (d)	Hz		s^{-1}
force	newton	N		$m\ kg\ s^{-2}$
pressure, stress	pascal	Pa	N/m^2	$m^{-1}\ kg\ s^{-2}$
energy, work, amount of heat	joule	J	N m	$m^2\ kg\ s^{-2}$
power, radiant flux	watt	W	J/s	$m^2\ kg\ s^{-3}$
electric charge, amount of electricity	coulomb	C		s A
electric potential difference, electromotive force	volt	V	W/A	$m^2\ kg\ s^{-3}\ A^{-1}$
capacitance	farad	F	C/V	$m^{-2}\ kg^{-1}\ s^4\ A^2$
magnetic flux density	tesla	T	Wb/m^2	$kg\ s^{-2}\ A^{-1}$
inductance	henry	H	Wb/A	$m^2\ kg\ s^{-2}\ A^{-2}$

frequency	hertz (d)	Hz	s^{-1}
force	newton	N	$m\ kg\ s^{-2}$

The International System of Units: SI

Bureau International des Poids et Mesures





Base units

Derived units with special names

Dimensions of qunatities

→ A set of coherent SI units

- A global measurement infrastructure
- Valid world wide
  - Intensity of light from an LED
  - CO_2 concentration in the air
  - Creatinine concentration in blood serum
  - Dose equivalent outside nuclear reactors
  - ….

CCU: Consultative Committee for Units

Bureau International des Poids et Mesures

- provide advice about **units of measurement**
- develop the **International System of Units (SI)**
- prepare the ***SI Brochure***

President:

Prof. Dr Joachim Ullrich

Former President:

Prof. Dr Ian Mills

Executive Secretary:

Dr Estefania de Mirandes

Date established:

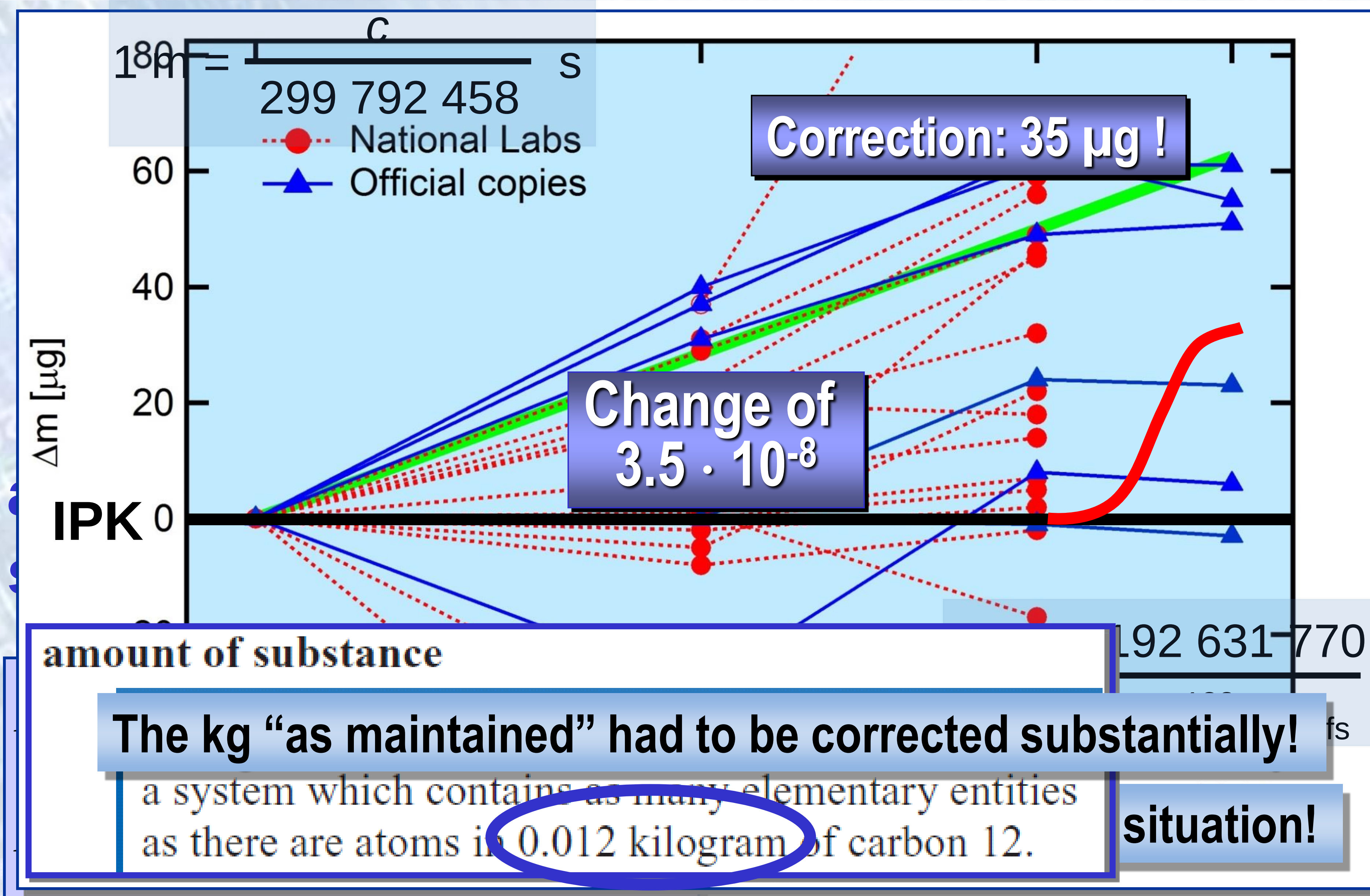
1964, to replace the "Commission for the System of Units", set up by the CIPM in 1954

CCU: Consultative Committee for Units

Bureau International des Poids et Mesures

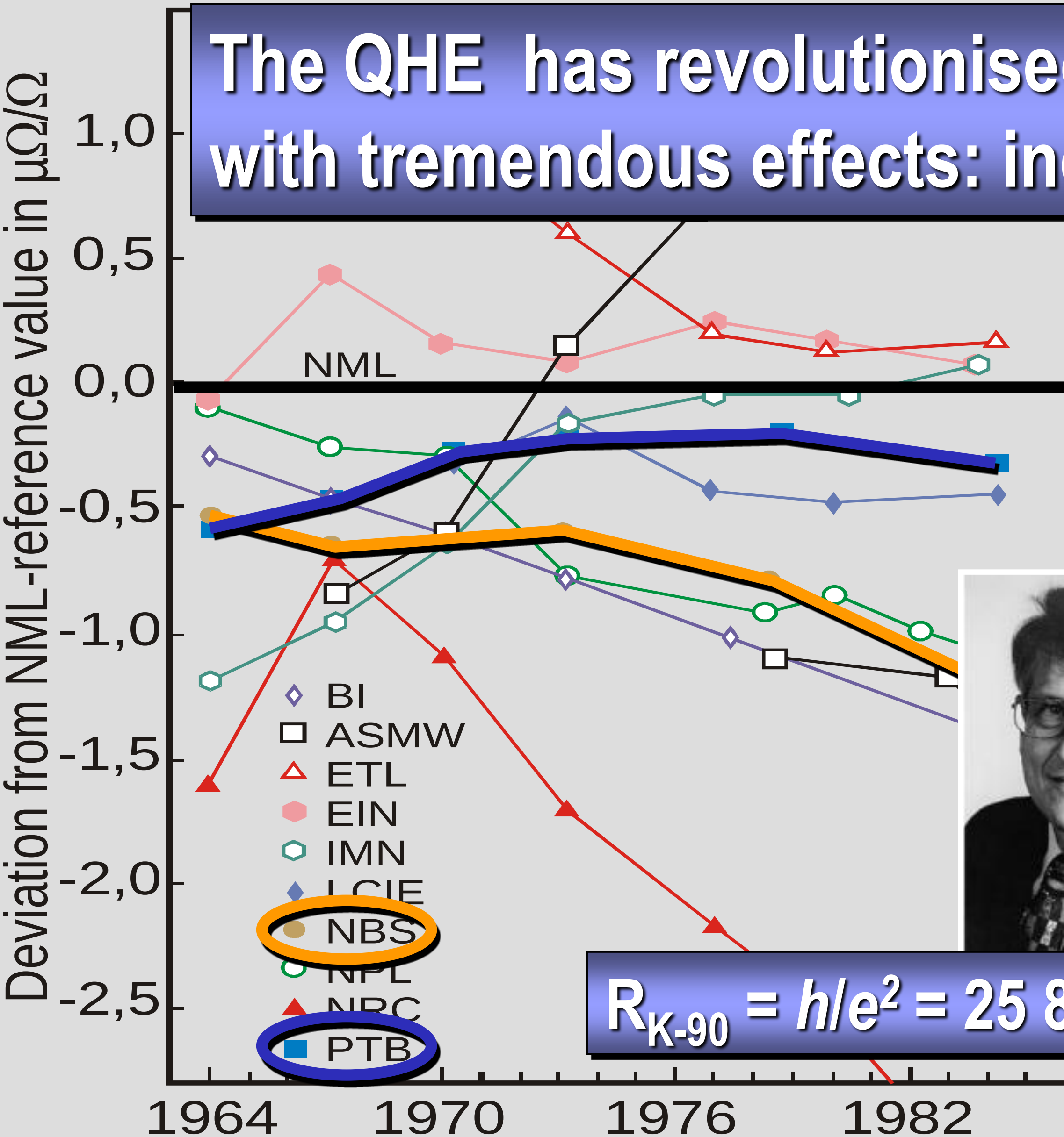
21st meeting: 11 and 12 June 2013: 43 participants

The New International System of Units



The New International System of Units

The QHE has revolutionised electrical metrology with tremendous effects: industry & innovation



Some 10^{-8} do matter!

$R_{K-90} = h/e^2 = 25\,812{,}807\ \Omega$

Reproducibility: 10^{-9}

1900. № 1.

ANNALEN DER PHYSIK.

VIERTE FOLGE. BAND 1.

irreversible Strahlungsvorgänge;

von Max Planck.

Dem gegenüber dürfte es nicht ohne Interesse sein zu bemerken, dass mit Zuhülfenahme der beiden in dem Ausdruck (41) der Strahlungsentropie auftretenden Constanten a

…with the help of fundamental constants we have the possibility of establishing units of length, time, mass, and temperature, which necessarily retain their validity for all times and cultures, even extraterrestrial and nonhuman…

. 1900. №1.

ANNALEN DER PHYSIK.

VIERTE FOLGE. BAND 1.

Member of PTR Advisory Board

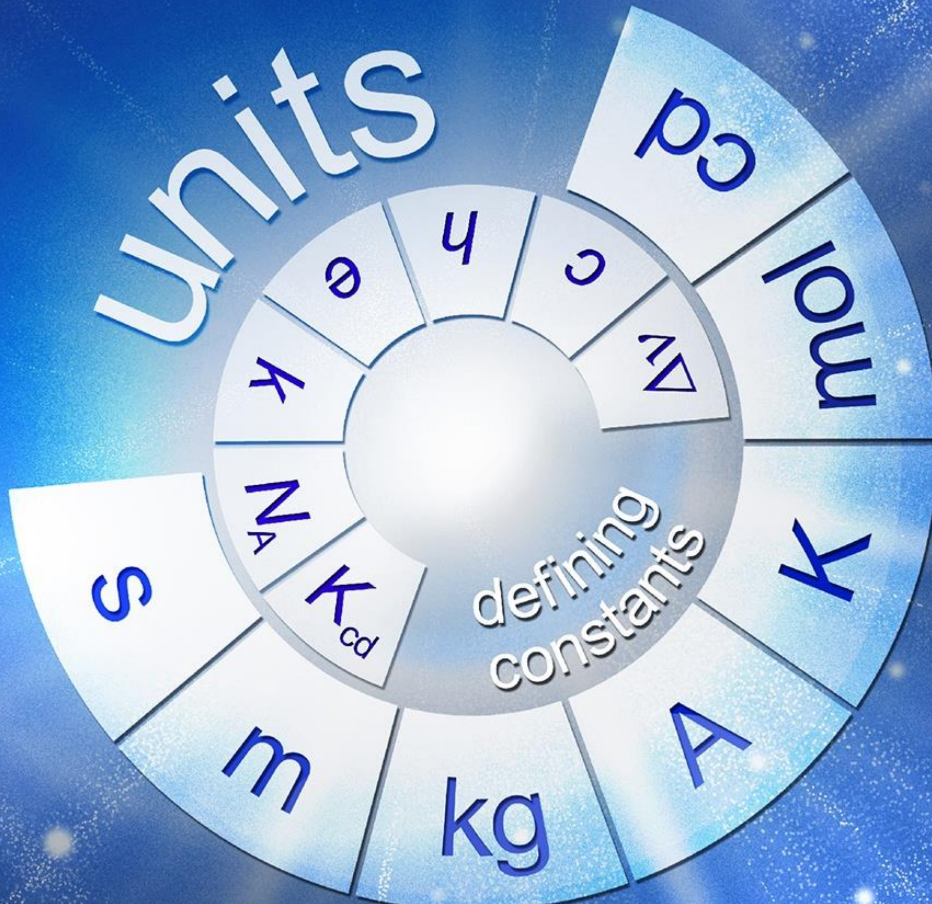
irreversible Strahlungsvorgänge;
von Max Planck.

Dem gegenüber dürfte es nicht ohne Interesse sein zu bemerken, dass mit Zuhülfenahme der beiden in dem Ausdruck (41) der Strahlungsentropie auftretenden Constanten *a*

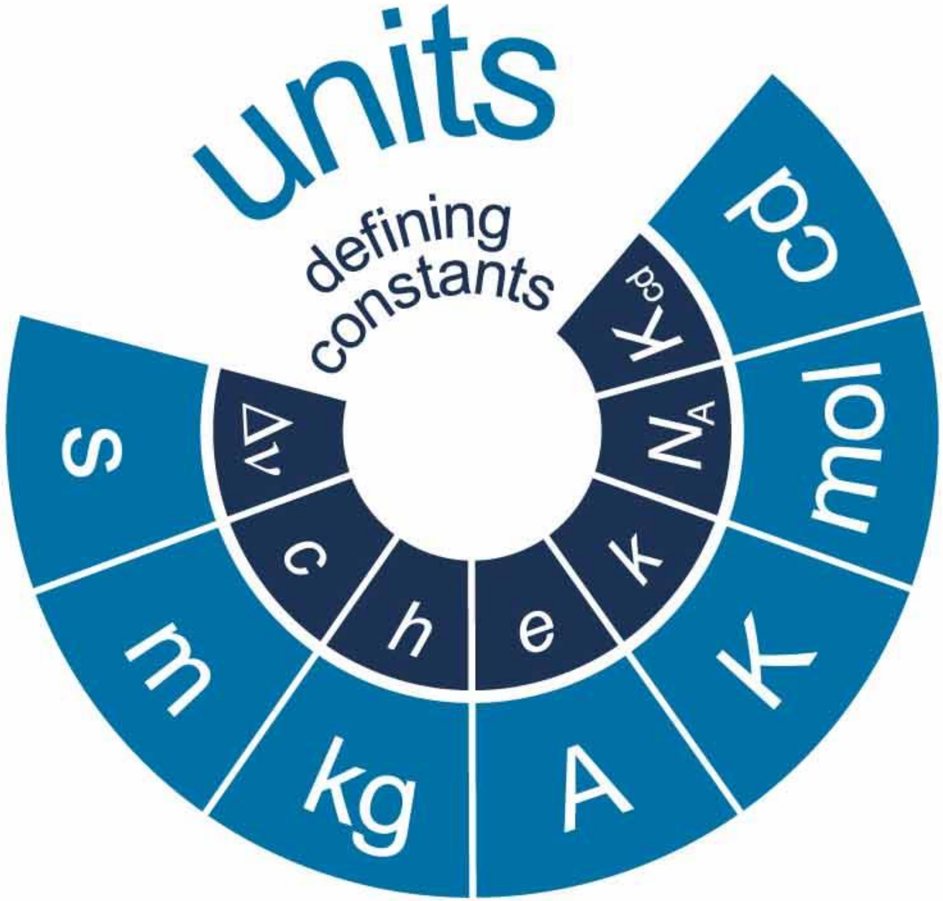
und ... e,
Zei ... n
spe ... e
Zei ... e
Cul ... -
lich

…with the help of fundamental constants we have the possibility of establishing units of length, time, mass, and temperature, which necessarily retain their validity for all times and cultures, even extraterrestrial and nonhuman…

SI International System of Units

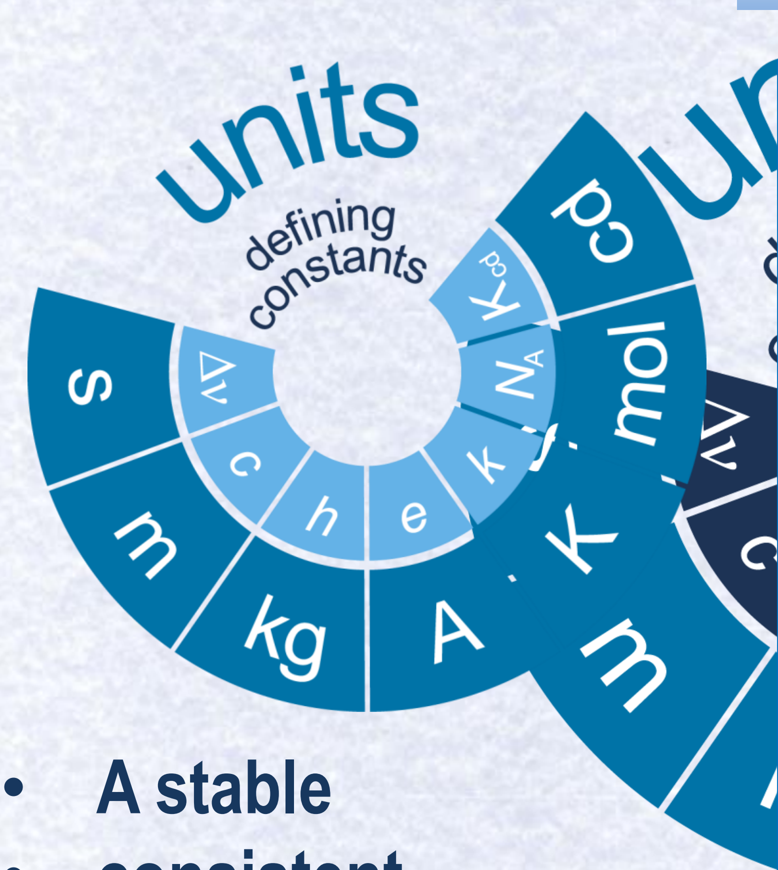


…for all times and cultures…



…for all times and cultures…

The New International System of Units



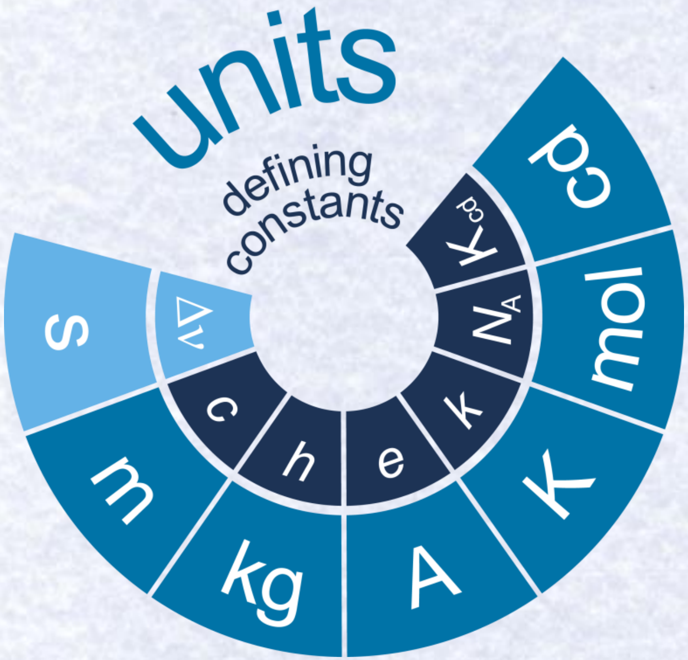
The defining constants

hyperfine splitting of Cs	$\Delta\nu(^{133}\mathrm{Cs})_{\mathrm{hfs}}$
speed of light in vacuum	c
Planck constant	h
elementary charge	e
Boltzman constant	k
Avogadro constant	N_{A}
luminous efficacy	K_{cd}

- A stable
- consistent
- and coherent set of units
- based on our present theoretical understanding of nature.

The New International System of Units

The SI unit of time, the second



The second, symbol s, is the SI unit of time. It is defined by taking the fixed numerical value of the caesium frequency $\Delta\nu_{Cs}$, the unperturbed ground-state hyperfine splitting frequency of the caesium 133 atom, to be 9 192 631 770 when expressed in the unit Hz, which is equal to s^{-1}.

Thus we have the exact relation $\Delta\nu(^{133}\mathrm{Cs})_{\mathrm{hfs}} = 9\,192\,631\,770$ Hz. Inverting this relation gives an expression for the unit second in terms of the value of the defining constant $\Delta\nu(^{133}\mathrm{Cs})_{\mathrm{hfs}}$:

$$1\ \mathrm{Hz} = \frac{\Delta\nu\left(^{133}\mathrm{Cs}\right)_{\mathrm{hfs}}}{9\,192\,631\,770} \quad \text{or} \quad 1\ \mathrm{s} = \frac{9\,192\,631\,770}{\Delta\nu(^{133}\mathrm{Cs})_{\mathrm{hfs}}}$$

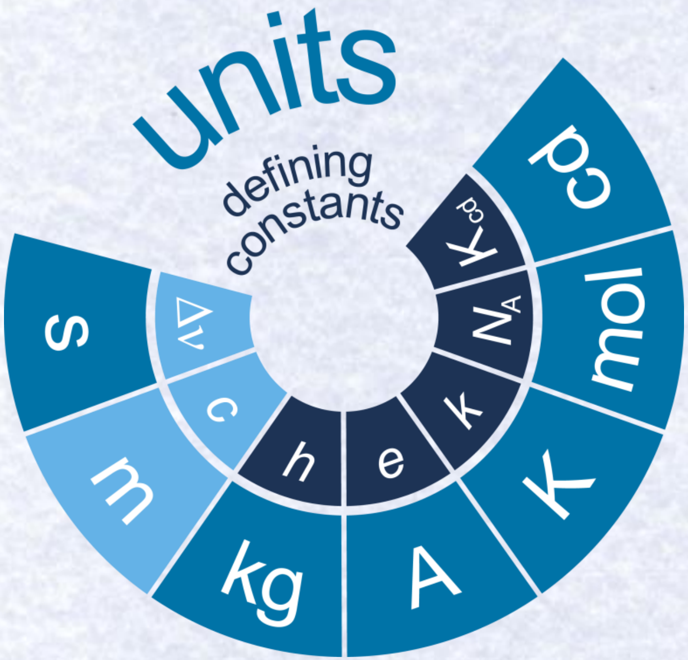
The effect of this definition is that the second is the duration of 9 192 631 770 periods of the radiation corresponding to the transition between the two hyperfine levels of the unperturbed ground state of the caesium 133 atom.

* From the Draft *Brochure*

The second, symbol s, is the SI unit of time. It is defined by taking the fixed numerical value of the caesium frequency $\Delta\nu_{Cs}$, the unperturbed ground-state hyperfine splitting frequency of the caesium 133 atom, to be 9 192 631 770 when expressed in the unit Hz, which is equal to s^{-1}.

The New International System of Units

The SI unit of length, the meter



The metre, symbol m, is the SI unit of length. It is defined by taking, in addition to the fixed numerical value of the caesium frequency ΔνCs, that of the speed of light in vacuum c to be 299 792 458 when expressed in the unit m s^{-1}.

Thus we have the exact relation $c = 299\ 792\ 458$ m/s. Inverting this relation gives an exact expression for the unit metre in terms of the defining constants c and $\Delta\nu(^{133}\text{Cs})_{\text{hfs}}$:

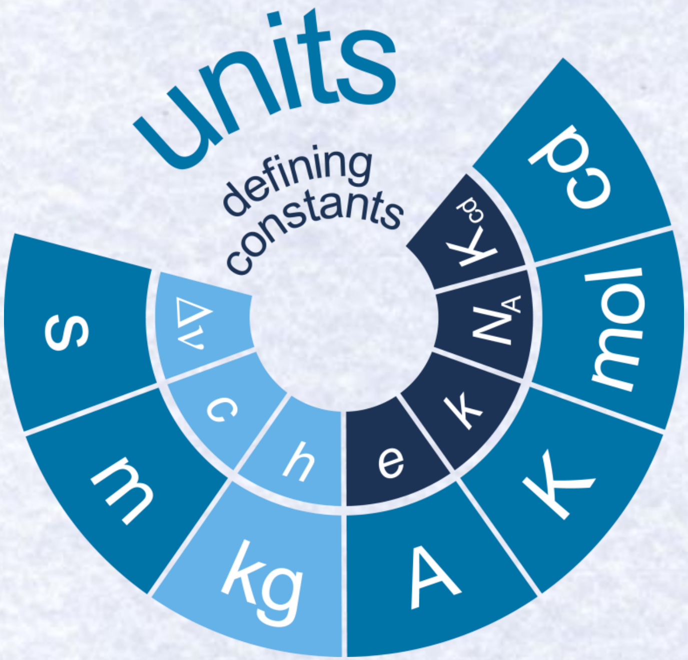
$$1\ \text{m} = \left(\frac{c}{299\ 792\ 458}\right)\text{s} = 30.663\ 318\ldots\ \frac{c}{\Delta\nu(^{133}\text{Cs})_{\text{hfs}}}$$

The effect of this definition is that the metre is the length of the path travelled by light in vacuum during a time interval of 1/299 792 458 of a second.

* From the Draft *Brochure*

The metre, symbol m, is the SI unit of length. It is defined by taking the fixed numerical value of the speed of light in vacuum *c* to be 299 792 458 when expressed in the unit m s^{-1}, where the second is already defined in terms of $\Delta\nu_{\text{Cs}}$.

The New International System of Units



The SI unit of mass, the kilogram

The kilogram, symbol kg, is the SI unit of mass. It is defined by taking, in addition to the fixed numerical values of the caesium frequency $\Delta\nu_{Cs}$, and the speed of light, c, that of the Planck constant h to be 6.626 069 XX $\times 10^{-34}$ when expressed in the unit J s, which is equal to kg m^2 s^{-1}.

Thus we have the exact relation $h = 6.626\,069\,57 \times 10^{-34}$ kg m^2 s^{-1} $= 6.626\,069\,57 \times 10^{-34}$ J s. Inverting this equation gives an exact expression for the kilogram in terms of the three defining constants h, $\Delta\nu(^{133}\mathrm{Cs})_{\mathrm{hfs}}$ and c:

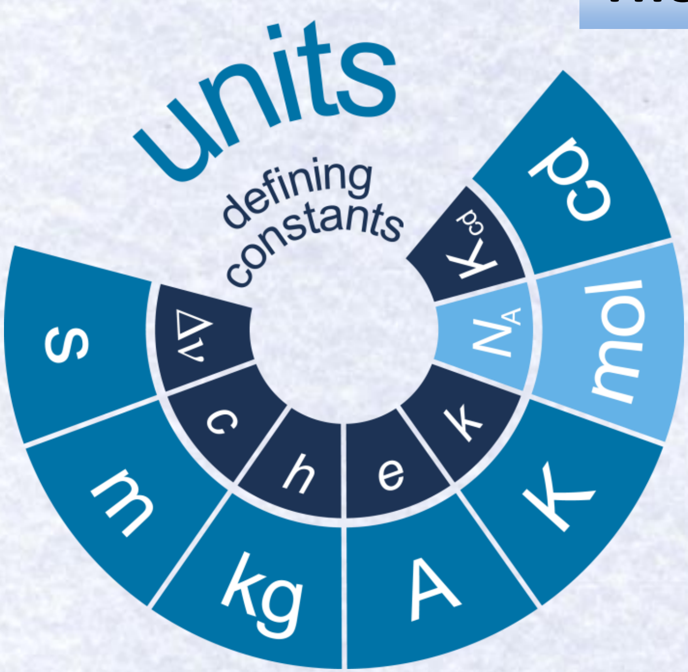
$$1 \quad \mathrm{kg} = \left(\frac{h}{6.626\,069\,57 \times 10^{-34}}\right) \mathrm{m}^{-2}\,\mathrm{s} = 1.475\,521... \times 10^{40} \frac{h\,\Delta\nu\left(^{133}\mathrm{Cs}\right)_{\mathrm{hfs}}}{c^2}$$

* From the Draft *Brochure*

The kilogram, symbol kg, is the SI unit of mass. It is defined by taking the fixed numerical value of the Planck constant h to be 6.626 069 XX $\times 10^{-34}$ when expressed in the unit J s, which is equal to kg m^2 s^{-1}, where the metre and second are defined in terms of c and $\Delta\nu_{Cs}$.

The New International System of Units

The SI unit of amount of substance, the mole



The mole, symbol mol, is the SI unit of amount of substance of a specified elementary entity, which may be an atom, molecule, ion, electron, any other particle or a specified group of such particles. It is defined by taking the numerical value of the Avogadro constant N_A to be 6.022 141 XX ×10[23] when expressed in the unit mol[–1].

Thus we have the exact relation $N_A = 6.022\ 141\ 29 \times 10^{23}\ \text{mol}^{-1}$. Inverting this equation gives an exact expression for the mole in terms of the defining constant N_A:

$$1\ \text{mol} = \frac{6.022\ 141\ 29 \times 10^{23}}{N_A}$$

The effect of this definition is that the mole is the amount of substance of a system that contains $6.022\ 141\ 29 \times 10^{23}$ specified elementary entities.

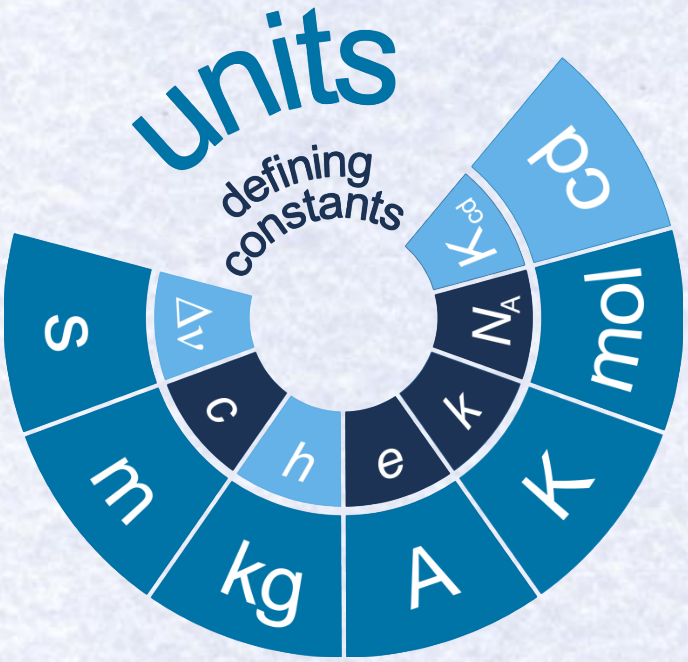
* From the Draft *Brochure*

The mole, symbol mol, is the SI unit of amount of substance of a specified elementary entity, which may be an atom, molecule, ion, electron, any other particle or a specified group of such particles. It is defined by taking the numerical value of the Avogadro constant N_A to be 6.022 141 XX ×10[23] when expressed in the unit mol[–1].

The New International System of Units

A fundamentally improved concept!

- Guarantees long-time stability
- A set of "defining constants" establish the units in general



The Silicon Route: Avogadro Collaboration

$$N_A = \frac{M_{Si} \cdot V_{sphere}}{\sqrt{8} \cdot d_{220}^3 m_{sphere}}$$

$$N_A \cdot h = \frac{\alpha^2 M(e^-) c}{2R_\infty}$$

$$u_{rel}(N_A \cdot h) = 4.5 \cdot 10^{-10}$$
$(CODATA\ 2014)$

BIPM
INRiM ISTITUTO NAZIONALE DI RICERCA METROLOGICA
irmm Institute for Reference Materials and Measurements
NIST
Australian Government
National Measurement Institute
NMIJ
NPL National Physical Laboratory
PTB

Realizing the kilogram and the mol!

One mol of ^{28}Si atoms is equivalent to the number of ^{28}Si atoms that is contained in a sample
12.05867069 cm^3 at 2
uncertainty of this vol

..count the number of atoms in a crystal sphere of enriched ^{28}Si

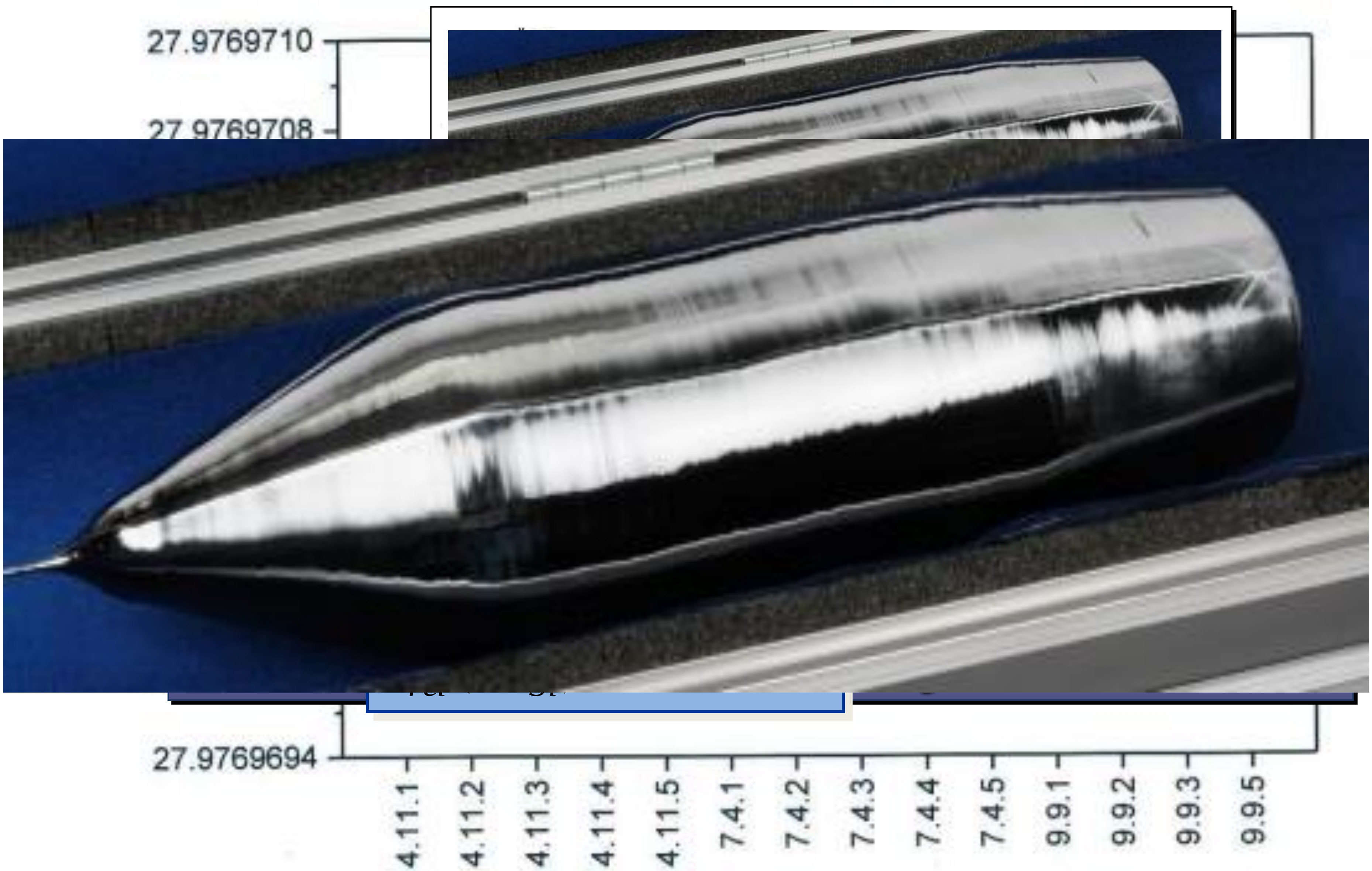
The Silicon Route: Avogadro Collaboration

$$N_A = \frac{M_{Si} \cdot V_{sphere}}{\sqrt{8} \cdot d_{220}^3 m_{sphere}}$$

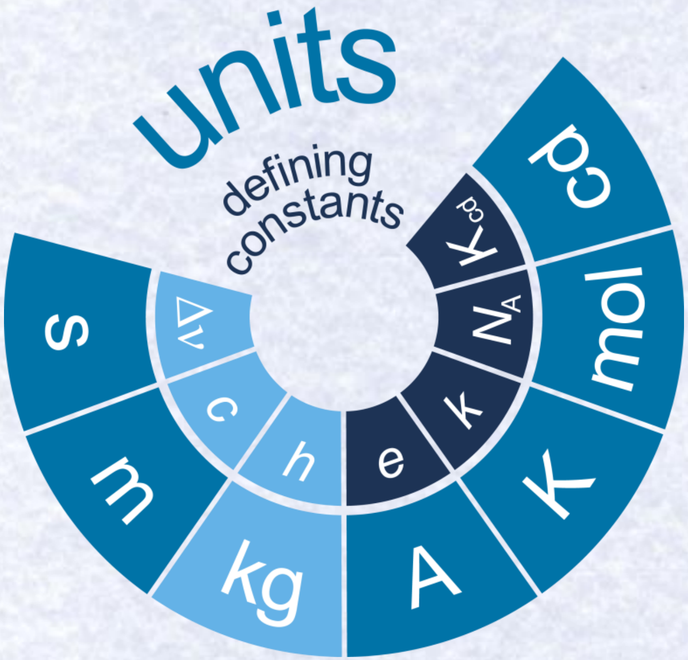
See talk of B. Güttler

BIPM
INRiM Istituto Nazionale di Ricerca Metrologica
irmm Institute for Reference Materials and Measurements
NIST
Australian Government National Measurement Institute
NMIJ
NPL National Physical Laboratory
PTB

The Molar Mass

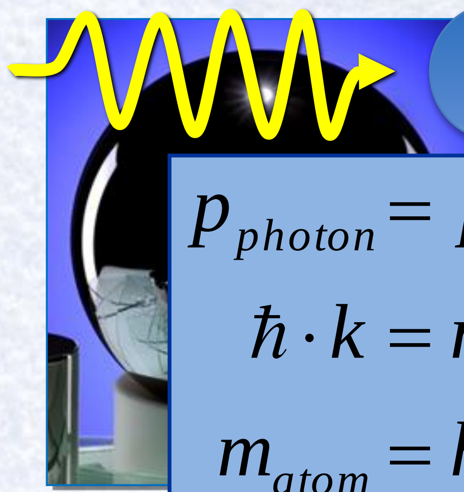


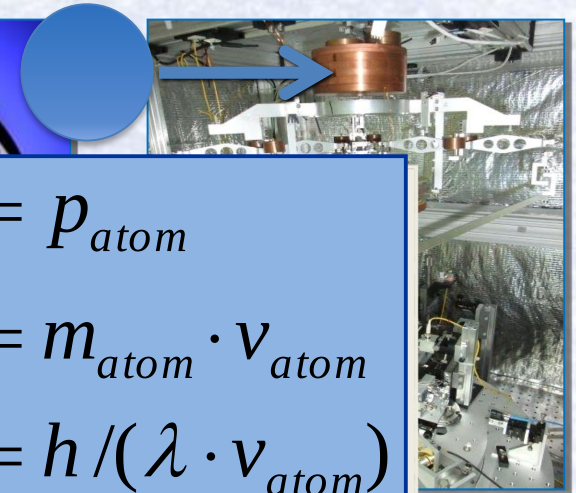
The New International System of Units



A fundamentally improved concept!

- Guarantees long-time stability
- <u>A set</u> of "defining constants" establish the units in general
- Different realisations
  - Key comparisons: safer, less correlation
- Realisation everywhere (Universe…)
  - Atomic masses



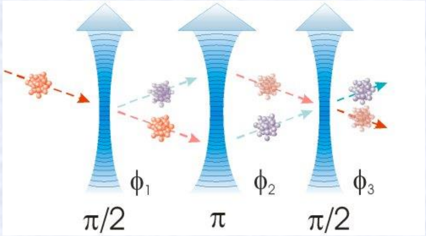


$$p_{photon} = p_{atom}$$

$$\hbar \cdot k = m_{atom} \cdot v_{atom}$$

$$m_{atom} = h/(\lambda \cdot v_{atom})$$

"De Broglie"

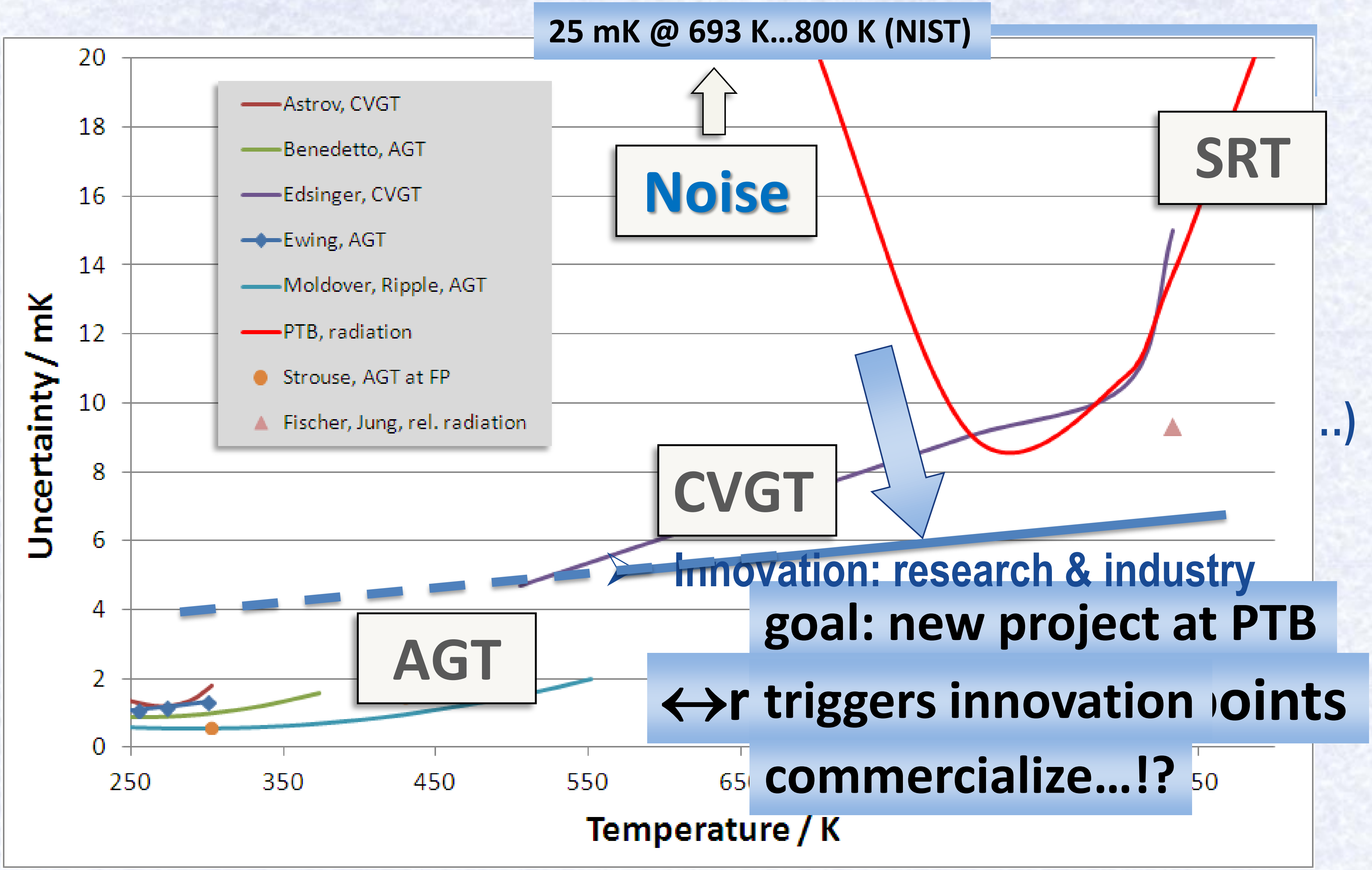


"Photon recoil"

"Silicon crystal"

"Watt balance"

The New International System of Units

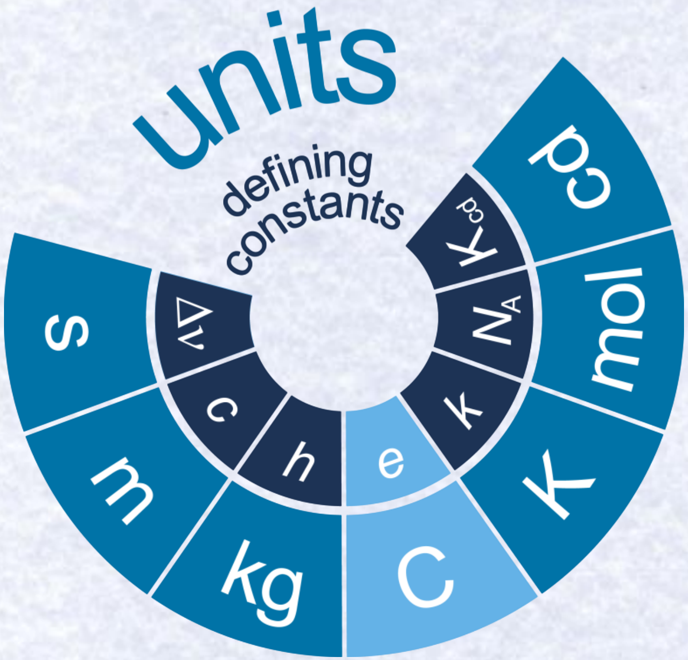


The New International System of Units

The International System of Units SI

A fundamentally improved concept!

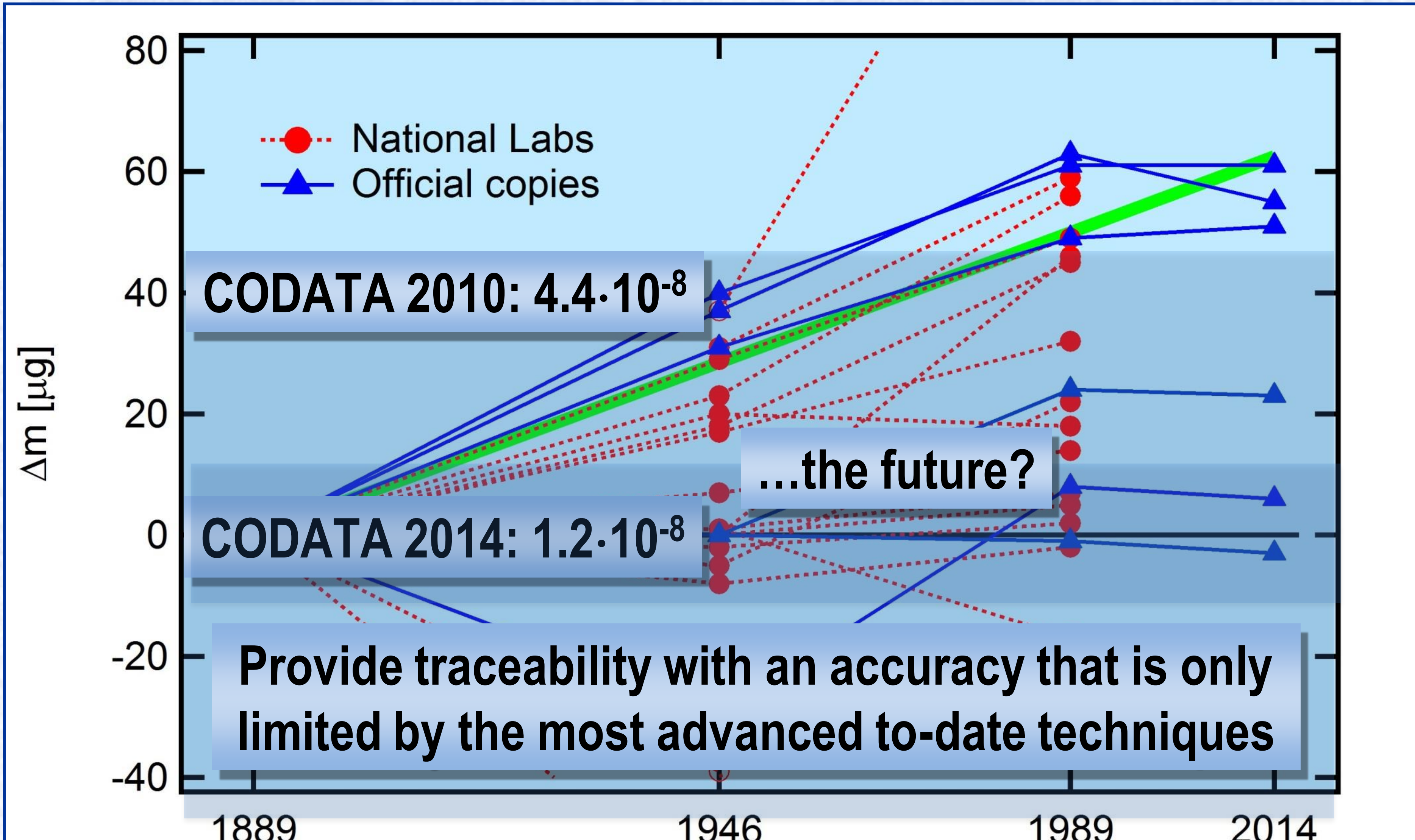
- Guarantees long-time stability
- A set of "defining constants" establish the units in general
- Different realisations
- Realisation everywhere (Universe…)
- Throughout the entire scale
- Innovation: research & industry
- Base units are only a convention
- Electric units are "back in the SI"
  - Consistency check of the whole system:
    - Si-kilogram ←→ Watt balance
    - Quantum metrological triangle, …



$$R_K = \frac{h}{e^2} \approx 25\,813\,\Omega \Leftrightarrow R_{K-90}$$

$$K_J = \frac{2e}{h} \approx 483\,598\,GHz/V \Leftrightarrow K_{J-90}$$

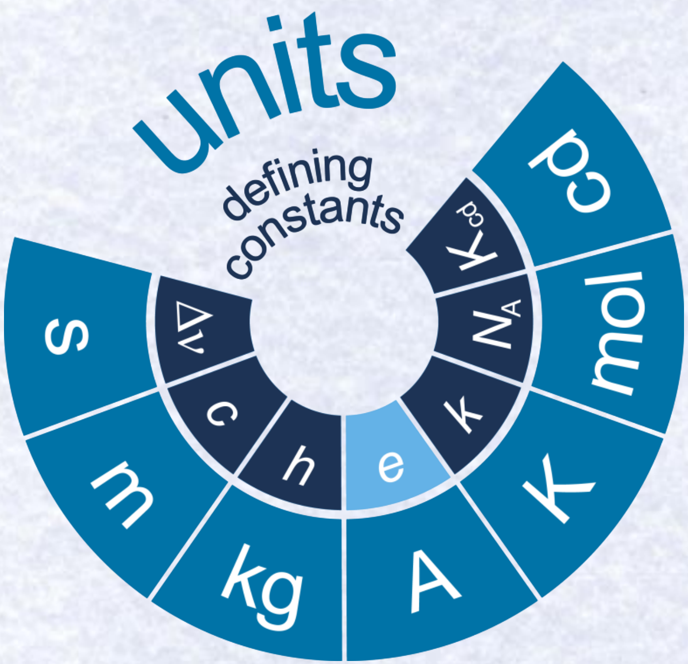
The New International System of Units



Ensure continuity and practicability

The New International System of Units

Establish the constants



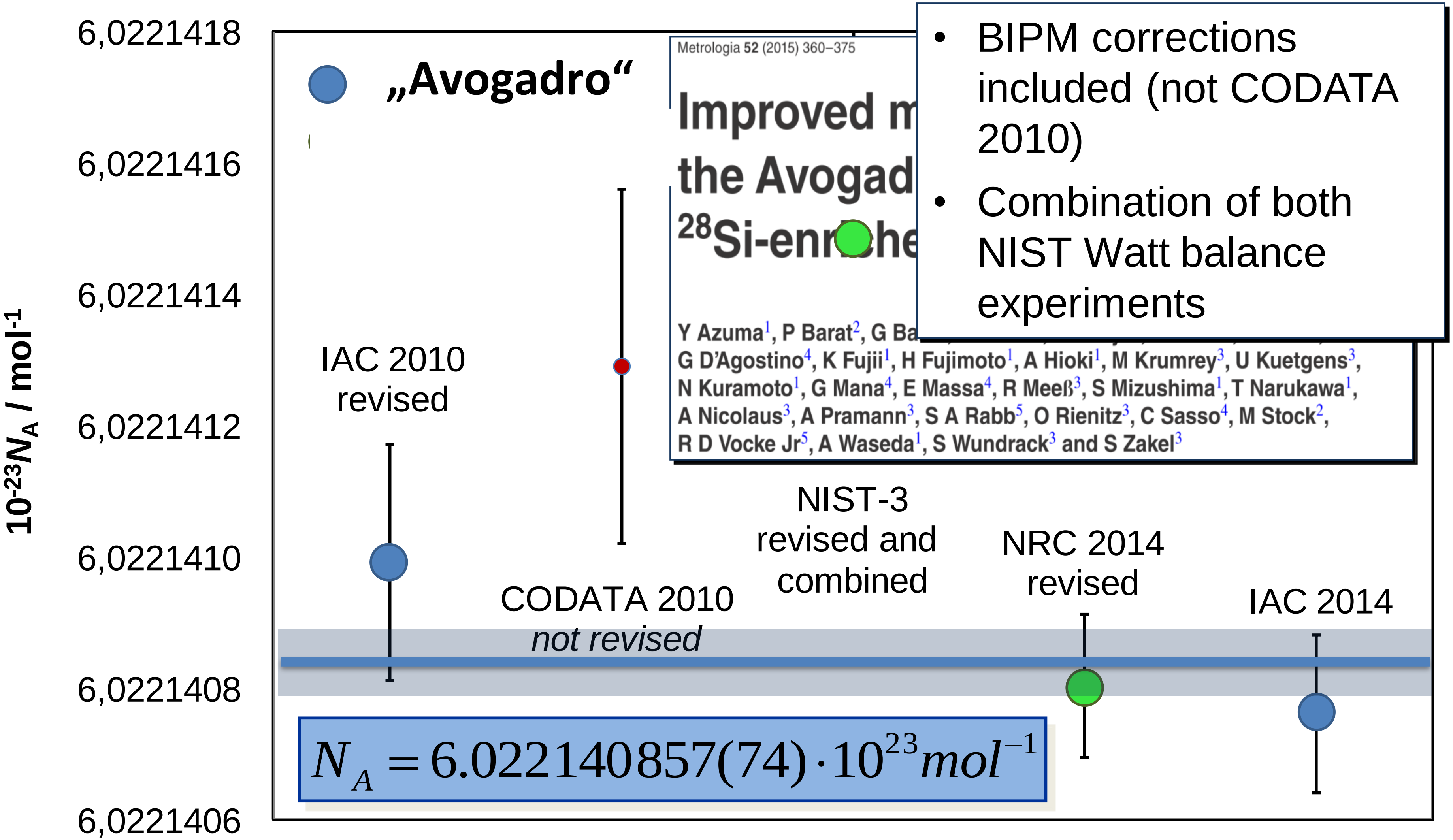
$$u_{rel}(e) = 1.2 \cdot 10^{-8}$$

A fundamentally improved concept!

- Guarantees long-time stability
- A set of "defining constants" establish the units in general
- Different realisations
- Realisation everywhere (Universe…)
- Throughout the entire scale
- Innovation: research & industry
- Base units are only a convention
- Electric units are "back in the SI"
- Better experiment → better realization

Ensure continuity and practicability

The New International System of Units



- BIPM corrections included (not CODATA 2010)
- Combination of both NIST Watt balance experiments

$$N_A = 6.022140857(74) \cdot 10^{23} mol^{-1}$$

SI International System of Units

Guaranteeing:

- long-time stability
- realization everywhere
- with ever-increasing accuracy as technology proceeds
- thus triggering innovation in science, industry and technology

for all times and cultures

End

Bureau
International des
Poids et
Mesures

CCU: Consultative Committee for Units

Why using the Planck-Constant

- One of the **most fundamental constants** in physics (see also R_K, K_J).
- Together with the electron charge and the speed of light, both fixed as well in the new SI, they form **the fine-structure constant**:

$$\alpha = \frac{e^2}{2\varepsilon_0 \cdot h \cdot c}$$

→ QED coupling constant, QED is the most precise, generic theory in physics!
→ Describes all our everyday macro- and microscopic world apart from gravity.

- Brings **electrical units** consistently "back to the SI".
- Determined by **macroscopic experiments** → direct connection to the kg.
- Two very different macroscopic experiments

  → consistency check for R_K, K_J

  → key comparisons with essentially no correlation.

CCU: Consultative Committee for Units

Bureau International des Poids et Mesures

Le Système international d'unités
The International System of Units
SI

Why not using an atomic mass

- **No fundamental constant** (depends e.g. on gravity, many masses)
- You need to **relate the macroscopic world:** only the Avogardo experiment!
  → No key comparisons
  → Correlations, danger of unknown systematic uncertainties
- But: better to understand?

$$m_{sphere} = \frac{8V}{a_{220}^3} \cdot \frac{2hR_\infty}{c\alpha^2} \cdot \frac{\sum_i f_i A_r^i}{A_r^e}$$

relative mass of Si / relative electron mass

number of atoms | electron mass

$$u_{rel}(\alpha) = 3.2 \cdot 10^{-10} \quad (CODATA\ 2010)$$

$$u_{rel}(R_\infty) = 5.5 \cdot 10^{-12} \quad (CODATA\ 2010)$$

→ **Not really!**

CCU: Consultative Committee for Units

Bureau International des Poids et Mesures

Le Système international d'unités
The International System of Units
SI

Why not using an atomic mass

- **No fundamental constant** (depends e.g. on gravity, many masses)
- You need to **relate the macroscopic world:** only the Avogardo experiment!
  → No key comparisons
  → Correlations, danger of unknown systematic uncertainties
- But: better to understand?

- **Metrology serves high-tech industry, economy and society**

- **More than 60 % of economy depends on quantum mechanics**

→The new SI, sometimes called "quantum SI" perfectly serves these needs !

The New International System of Units

The International System of Units SI

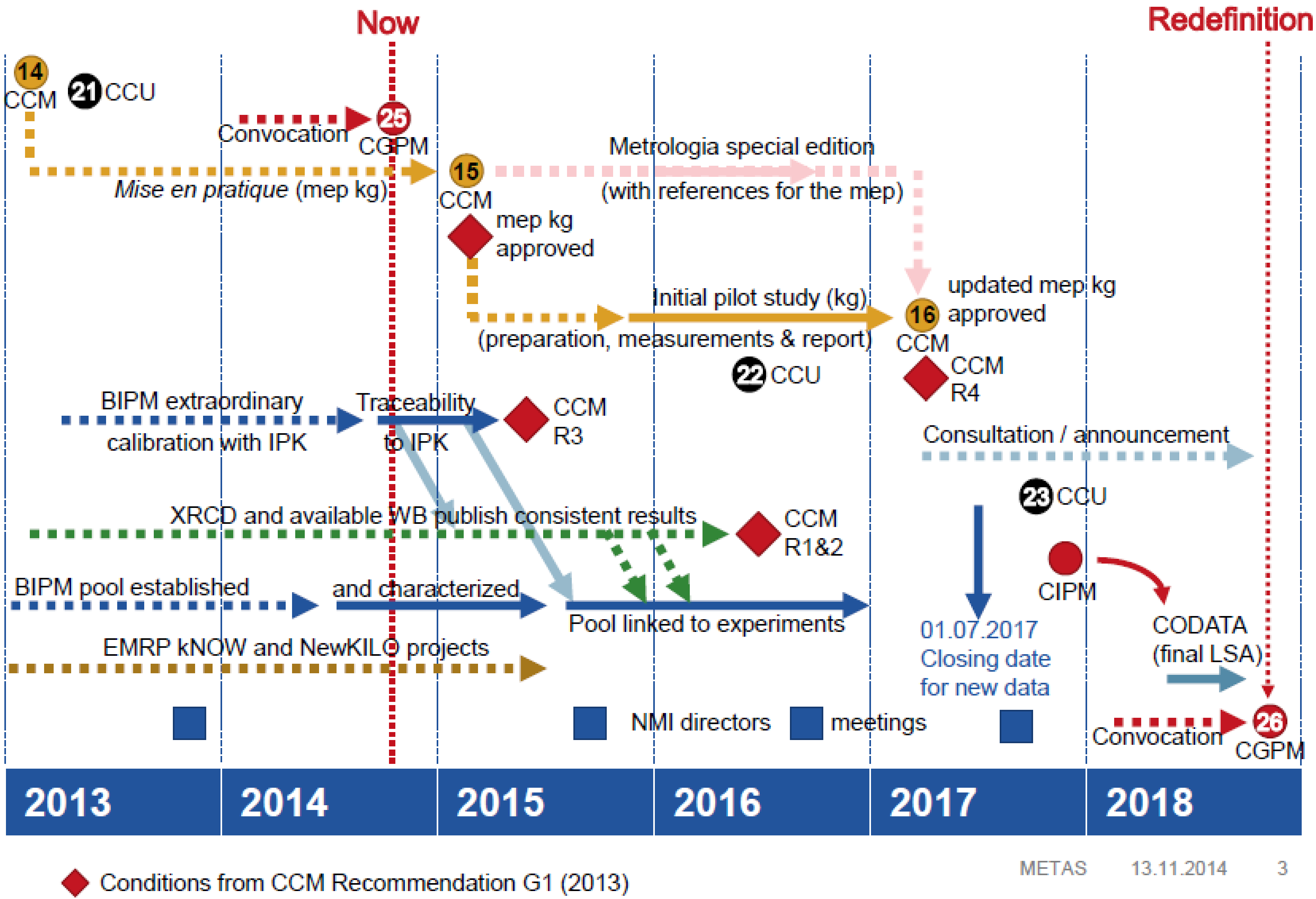
Relative uncertainties of constants in the new SI in parts in 10^{-8}

constant	now	new	constant	now	new
$m(\mathrm{K})$	0	4,4	R	91	0
T_{TPW}	0	91	F	2,2	0
$M(^{12}\mathrm{C})$	0	0,07	σ	360	0
μ_0	0	0,032	K_{J}	2,2	0
ε_0	0	0,032	R_{K}	0,032	0
Z_0	0	0,032	$N_{\mathrm{A}}h$	0,07	0
$\Delta\nu(^{133}\mathrm{Cs})_{\mathrm{hfs}}$	0	0	m_{e}	4,4	0,064
c	0	0	m_{u}	4,4	0,07
K_{cd}	0	0	$m(^{12}\mathrm{C})$	4,4	0,07
h	4,4	0	α	0,032	0,032
e	2,2	0	$\mathrm{J} \leftrightarrow \mathrm{m}^{-1}$	4,4	0,0
k_{B}	91	0	$\mathrm{J} \leftrightarrow \mathrm{Hz}$	4,4	0,0
N_{A}	4,4	0	$\mathrm{J} \leftrightarrow \mathrm{K}$	91	0,0

Towards the New SI: CCM roadmap

Bureau International des Poids et Mesures

Joint CCM and CCU roadmap for the new SI



CCU: Consultative Committee for Units

- **Present 9th edition of *SI Brochure* in French and English to the CGPM in 2018.**

March 2014: Suggest to the CIPM to invite **CCs to read and comment** on the present version of the first three chapters of the *SI Brochure* by **March 2015**.

February 25th to 26th, 2015: Meeting of CCU WG on "dimensionless quantities"

June 14th to 15th, 2015: Comments and CCU WG suggestions will be considered during a meeting of the *Brochure* Drafting Group,

June 14th to 16th, 2016: CCU meeting to **approve a close-to-final** version of the 9th edition of the *SI Brochure,* including units for dimensionless quantities, the policy for truncating the numerical values of the defining constants, the texts of the mises en pratique, new developments, etc.

September 5th to 7th 2017: CCU meeting to **approve the final draft.**

1) deciding about the fixed numerical values of the defining constants (rounding),
2) deciding upon the final version of the complete 9th SI Brochure,
3) creating the wording of a Draft Resolution to be provided to the CIPM

November 2018: 9th edition of the *SI Brochure* is ready in French and English

Towards the New SI

Bureau International des Poids et Mesures

- 31 August 2015 and 1st September 2015: CODATA TGFC meeting for 2015 (BIPM)
- **14 - 16 June 2016**: **22nd CCU meeting** with the aim of discussing the content of the complete 9th *SI Brochure* including units for dimensionless quantities, the policy for truncating the numerical values of the defining constants, the texts of the *mises en pratique*, etc.
- 16 July 2016: CODATA TGFC meeting for 2016 (CPEM'2016 Ottawa)
- **1st July 2017**: closing date for data to be adjusted by TGFC in order to prepare the redefinition
- **1st September 2017**: TGFC provides the adjustment to CCU
- **5 - 7 September 2017**: **23rd meeting of the CCU** with the aim of 1) deciding about the fixed numerical values of the defining constants (rounding), 2) deciding upon the final version of the complete 9th *SI Brochure*, and 3) creating the wording of a Draft Resolution to be provided to the CIPM (for the Convocation of the CGPM 2018) on the redefinition.
- **Mid-October 2017**: **CIPM meeting**, discussion of the CCU proposals and approval of the Draft Resolution for the Convocation
- **1st July 2018**: TGFC closes for the first fit of the constants under the New SI, this fit will be the official CODATA fit for 2018 having occurred 6 months earlier than usual
- **CGPM 2018**: redefinition with the vote of the Resolution, push-button for publication of the definitive 9th *SI Brochure*, and for the publication of the corresponding CODATA TGFC adjustment

CCU: Consultative Committee for Units

Bureau International des Poids et Mesures

Future scan (2013-2023)

→ **Please see the poster!**

Le Système international d'unités
The International System of Units
SI

- Convince the **academic, scientific** and **economic communities** of the need to depart from artefacts and prototypes.
- Review the **membership composition of CCU** with the CIPM in order to better involve its stakeholders in the discussions, and attain their support.
- Advice on the **accuracy and consistency of the defining constants** to ensure a smooth transition from the present to the "New SI".
- Providing advise on the **numerical values of the defining constants**.

$$\Delta\nu\,(^{133}\mathrm{Cs})_{\mathrm{hfs}} = 9\;192\;631\;770.\;\cancel{5(6)}\;\mathrm{s}^{-1}$$