



# Stochastic Modeling for System Remaining Life Prognosis

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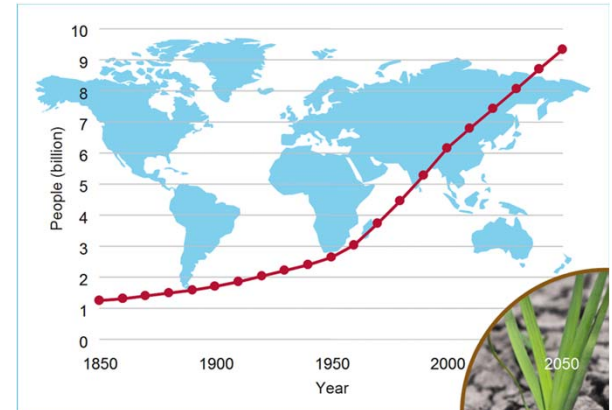
# Prognosis: Predictive Science

## ➤ Definition

to *forecast* the likely outcome of an situation ...

- ✓ Disease/Epidemiology
- ✓ Weather forecasting
- ✓ Economic development

— Oxford Dictionaries Online



## ➤ Originally a *medical* term back in the 19<sup>th</sup> century:

- ✓ Main aim was not to *cure* disease, but to give a medical *diagnosis* and *predict* the patient's chance of survival in terms of *remaining life*;
- ✓ Focus shifted only decades later to *curing* disease.

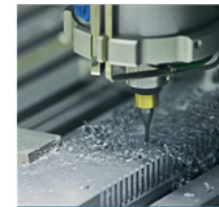
### Predictive Science



Epidemiology Prediction



Stock Prediction



Prognosis in Manufacturing

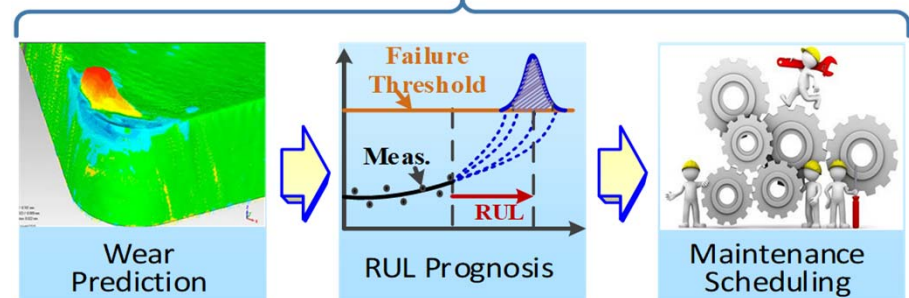
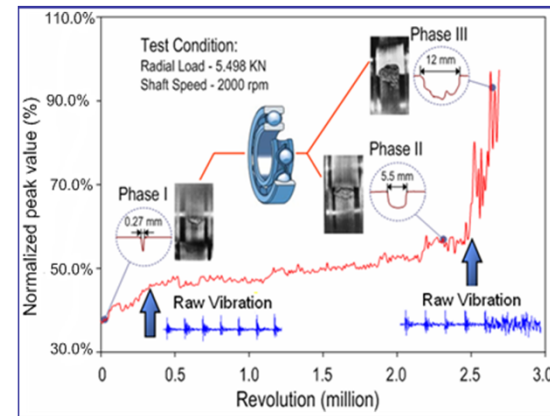
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Weather Forecast

# In the Context of Manufacturing

- Predict expected *progression* of degradation in a machine or its components from its *current* state to *future* functional *failure*, and the *confidence* associated with prediction;
- Identify short-term and long-term actions/decisions to improve *remaining useful life (RUL)* of a machine;
- Provide scientific and technical basis for *maintenance* scheduling, asset management, and effective *decision* making.

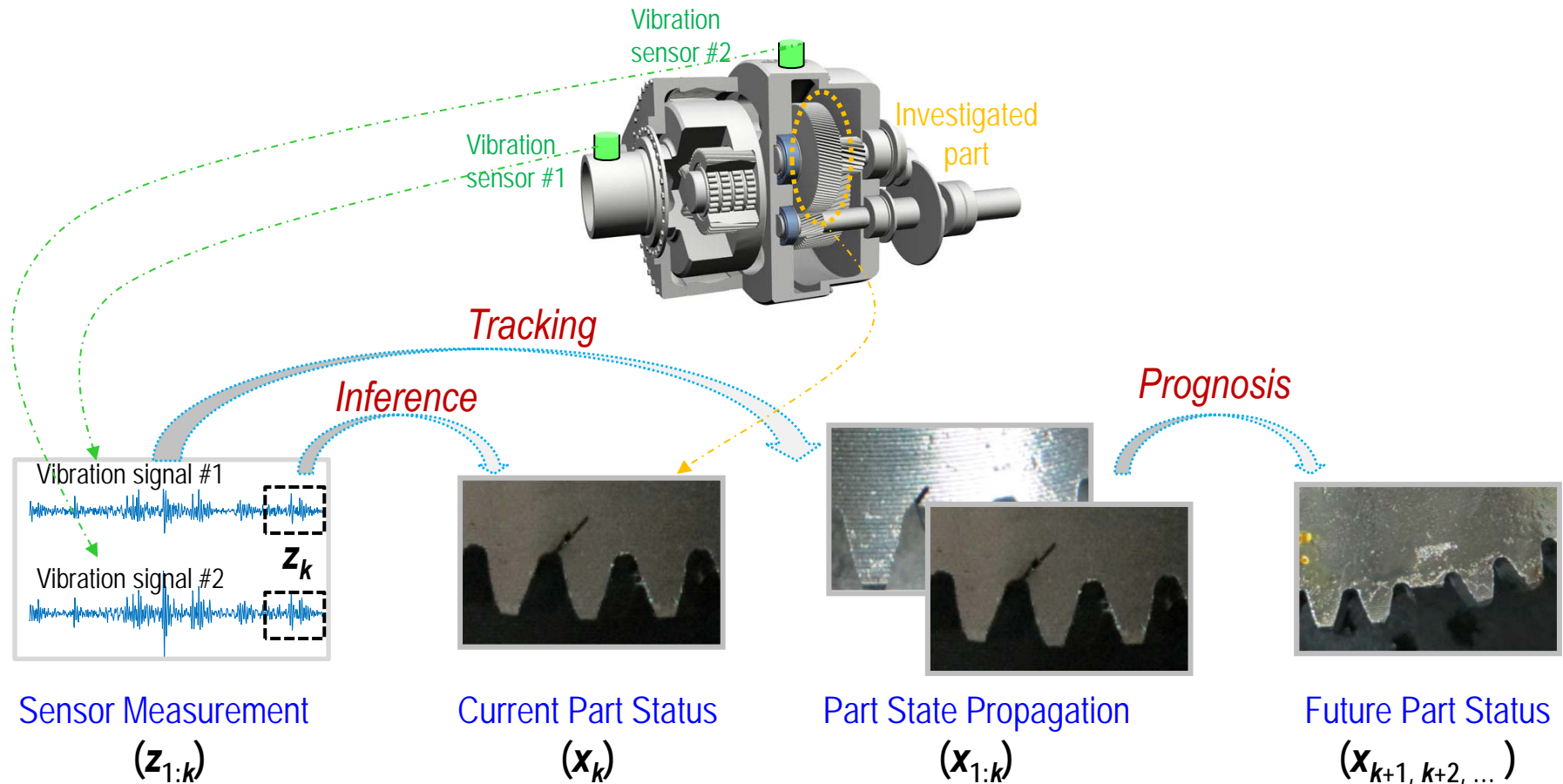


# Outline

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- ❑ Background
  - ✓ Basic concepts
  - ✓ Major modeling techniques
- ❑ Particle Filter (PF)
  - ✓ Gradual degradation and time-varying rates
  - ✓ Multi-mode PF
- ❑ Case Study
  - ✓ Rolling bearing remaining life prognosis
- ❑ Conclusion and Future Work

# Modeling: Inference, Tracking, and Prognosis

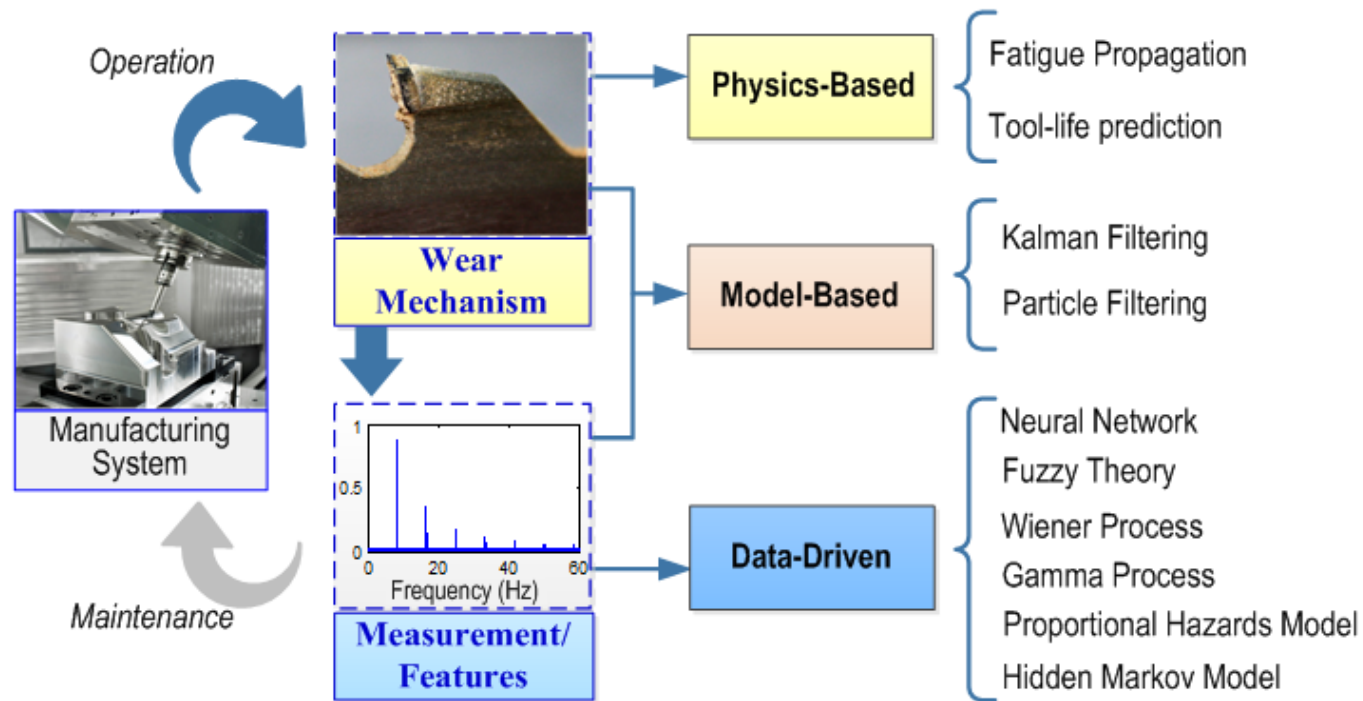


**Inference:** estimate system/part current state, based on current measurement

**Tracking:** identify propagation of state using historical measurements (*recursive* inference)

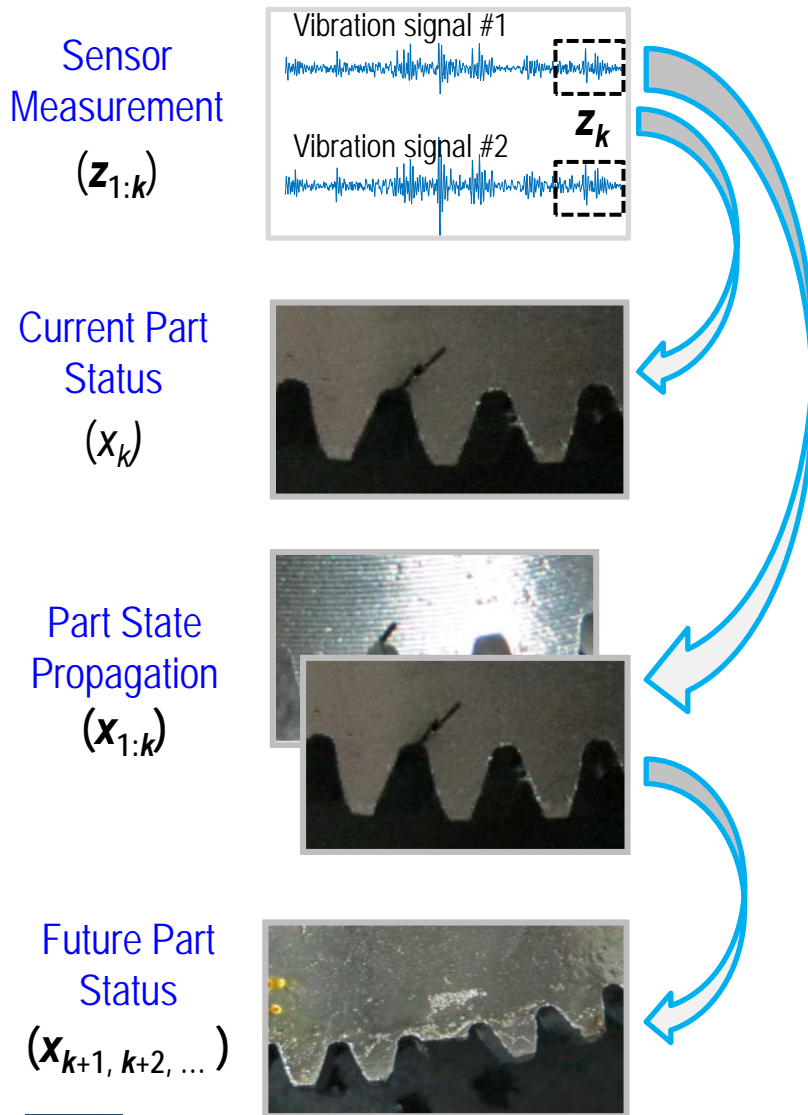
**Prognosis:** predict state propagation in the future, without available measurement

# Prognostic Modeling Methods: Classification



- **Physics-based:** describe system behavior **analytically**, parameters **experimentally** determined
- **Data-driven:** rely on measured data, **numerically** determine relation between current and future states
- **Model-based:** **combine** the two methods for improved robustness and prediction accuracy
- Alternative classification scheme: depending on how **uncertainty** is handled in the prediction process:
  - ✓ **Deterministic:** machine health as defined **value**
  - ✓ **Stochastic (probabilistic):** machine health as probability **distribution**, degradation as evolution of distributions

# Analytical Representation (1)



## Inference

- ✓ Estimate the current part status through a *posterior* PDF <sup>(1)</sup>, realized Bayes' Rule <sup>(2)</sup>

$$P(x_k | z_k) = \frac{P(x_k | x_{k-1}) P(z_k | x_k)}{P(z_k)}$$

Posterior PDF    Prior PDF    Constant    Likelihood

## Tracking

- ✓ Identify the state propagation model (or recursively update the *prior* PDF) through *a series of posterior* PDF, following a Markov process <sup>(3)</sup>

$$P(x_1, x_2, \dots, x_k | z_1, z_2, \dots, z_k) = P(x_1 | z_1) P(x_2 | x_1, z_2) \dots P(x_k | x_{k-1}, z_k)$$

## Prognosis

- ✓ Predict the future state using the identified state propagation model (*a series of prior* PDF)

$$P(x_{k+1} | x_k) P(x_{k+2} | x_{k+1}) \dots$$

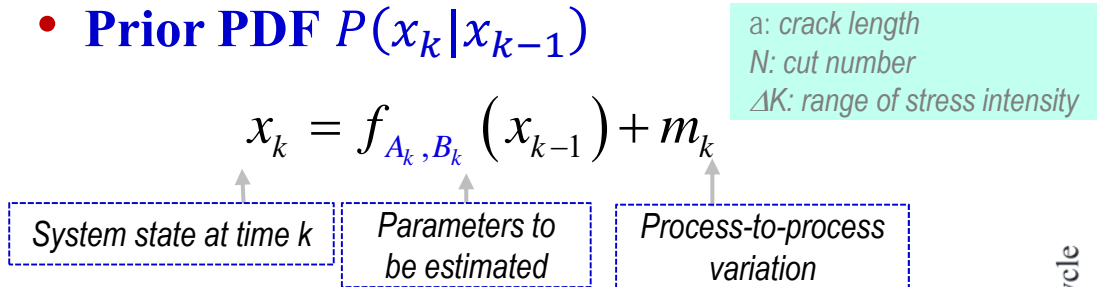
(1) PDF: Probability Density Function

(2) Bayes' Rule: posterior PDF can be estimated through prior pdf and likelihood

(3) Markov process: current state  $x_k$  is only dependent on preceding state  $x_{k-1}$

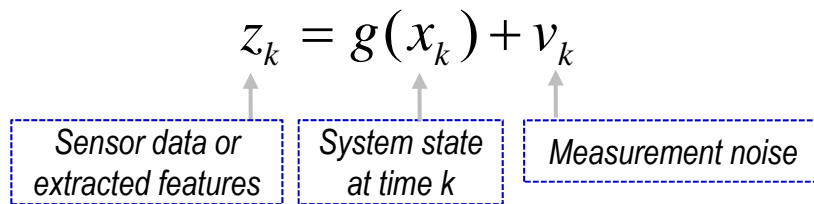
# Analytical Representation: (2)

- **Prior PDF**  $P(x_k | x_{k-1})$



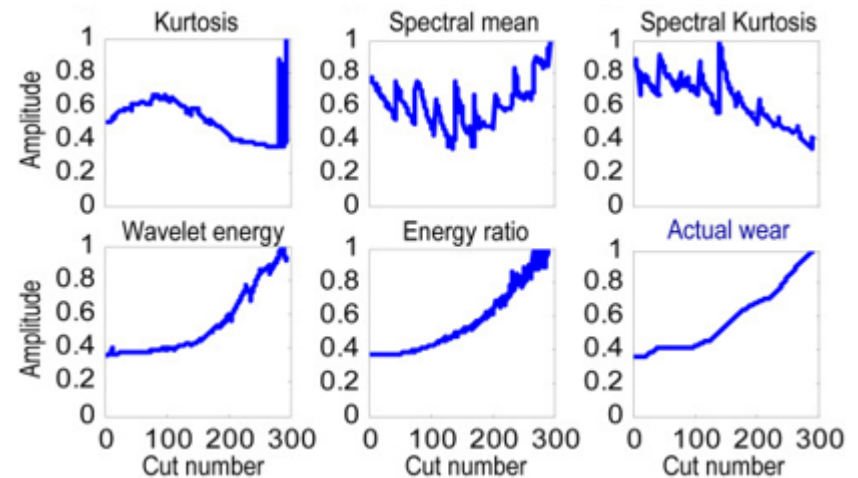
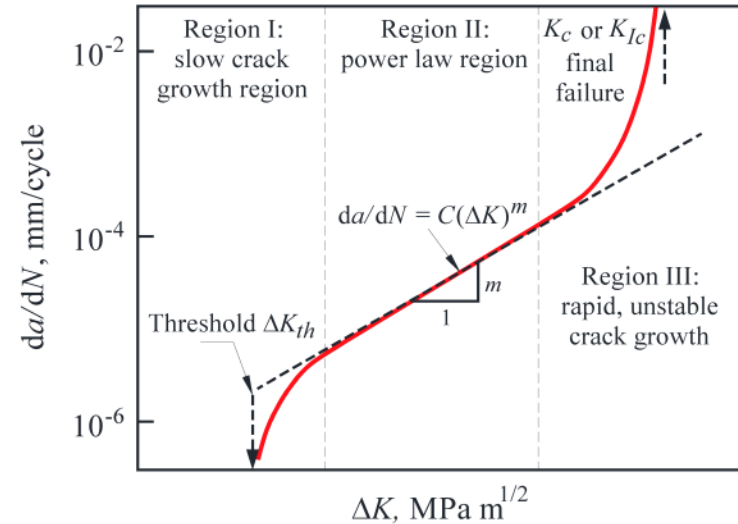
- ✓ Also called **State Evolution** model
- ✓ Obtained from physical or empirical knowledge
- ✓ Sometimes conditional on parameters,  $\{A_k, B_k\}$ , which determines the performance degradation rate and may be *time-varying*

- **Likelihood Function**  $P(z_k | x_k)$



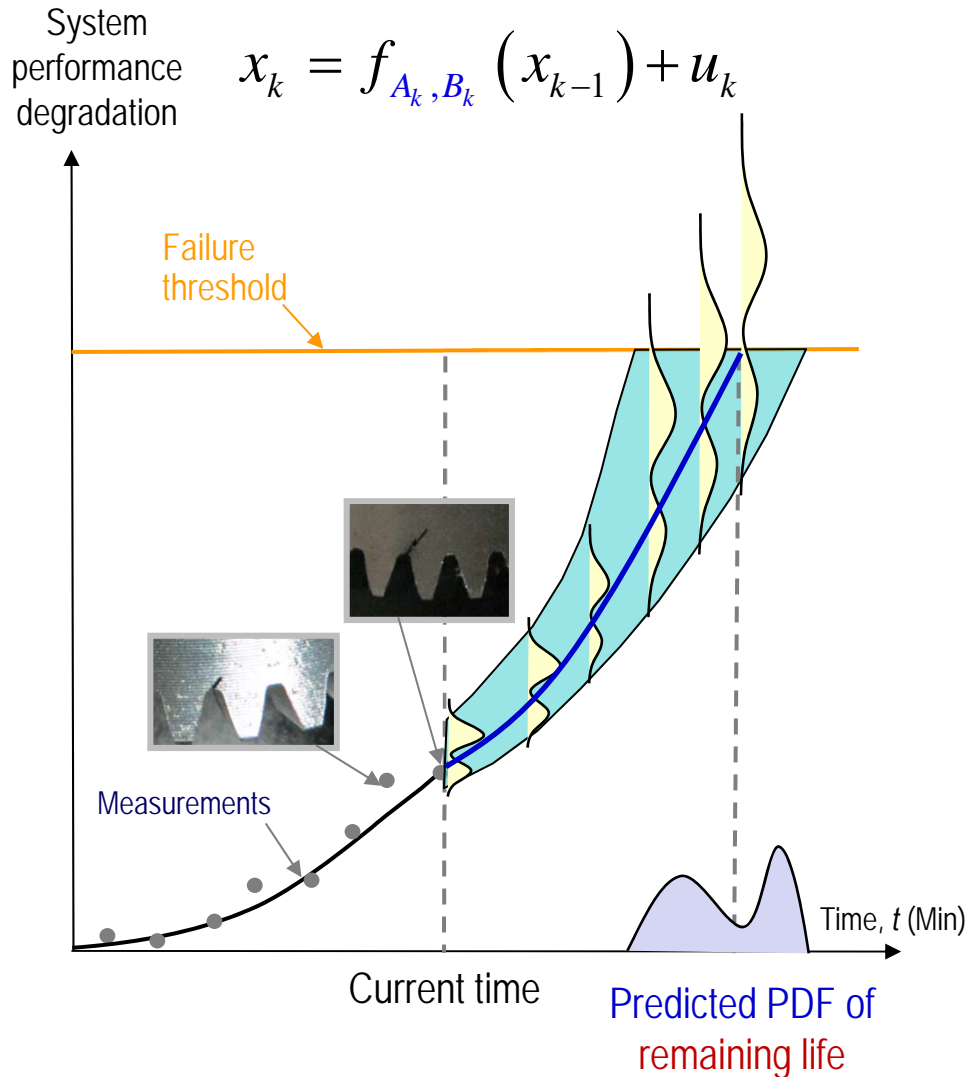
- ✓ Also called **Measurement** model
- ✓  $g$ : *nonlinear* mapping, from data-driven model

**Paris law**, describing crack propagation over time (i.e. from  $x_{k-1}$  to  $x_k$ )





# Particle Filter



- ✓ Each particle describes a specific **state** value and two **coefficients**  $\{x_k^i, A_k^i, B_k^i\}$  ( $k$ : time;  $i$ :  $i^{\text{th}}$  particle);
- ✓ System state (e.g. tool wear) at a certain time is the **statistical sum** of values from a chosen number  $N$  of particles, expressed as a **probability distribution**;
- ✓ Particles' evolution directly determined by **state evolution model**, which can be linear or **nonlinear**
- ✓ State **propagation** is presented as progressive updates of prior PDF

$$P(x_{k+1} | x_k) = \sum_{i=1}^N w_k^i P(x_{k+1}^i | x_k^i)$$

Recursively updated

Computed from state evolution model

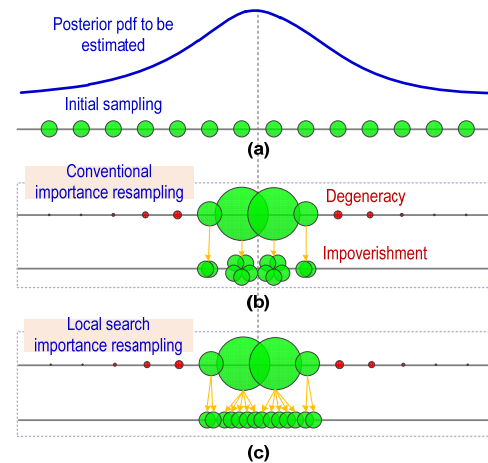
# Improvement on PF

- **Limitation of PF**

- ✓ *Particle degeneracy*: particles of weight 0 removed when estimating time-varying distribution
- ✓ Not able to track and predict *time-varying* performance degradation

- **Improvement 1: Local Search PF (LSPF)**

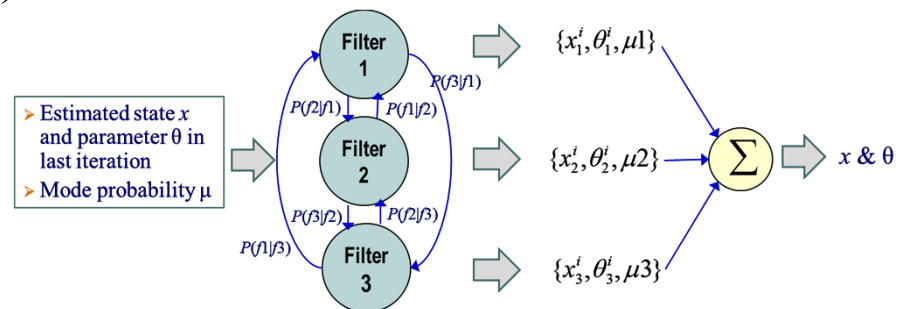
- ✓ For tracking and prognosis of *gradual* degradation with *time-varying* rates
- ✓ Improve particles' diversity through *adaptive* change of positions of resampled particles, by adding a *perturbation*



Wang & Gao, "Adaptive re-sampling-based PF", *JMS*, 2015

- **Improvement 2: Multi-Mode PF (MMPF)**

- ✓ For *transient* performance change
- ✓ Each filter mode corresponds to one deterioration scenario
- ✓ Mode transition *automatically* performed based on Bayesian inference



Wang & Gao, "Markov Nonlinear System", *ASME JGTP*, 2016

# Bearing Failure and Life Prognosis

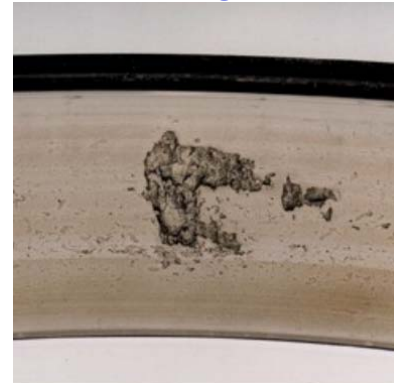
- Rolling bearing accounts for 30% rotating machine failure [1]
- Common causes for bearing failure [2]:
  - ✓ Fatigue
  - ✓ Excessive load
  - ✓ Overheating
  - ✓ Corrosion
  - ✓ Lubricant failure
  - ✓ Contamination
  - ✓ Misalignment
- **Statistical** bearing life,  $L_{10}$ :

$$L_{10} = \frac{\left(\frac{C}{P}\right)^e * 10^6}{60 * N}$$

**C**: dynamic load rating; **P**: equivalent bearing load; **N**: rotating speed; **e**: constant

- ✓ Not consider effect of operating conditions
- ✓ May deviate significantly from actual life, in some cases by nearly a *factor of 5* [3]

Fatigue



Corrosion



Contamination



Misalignment



- [1] Tandon, A review of vibration, 1999
- [2] SKF, Bearing damage chart
- [3] Zaretsky, Bearing life prediction, 2000

# Multi-Stage Life Modeling

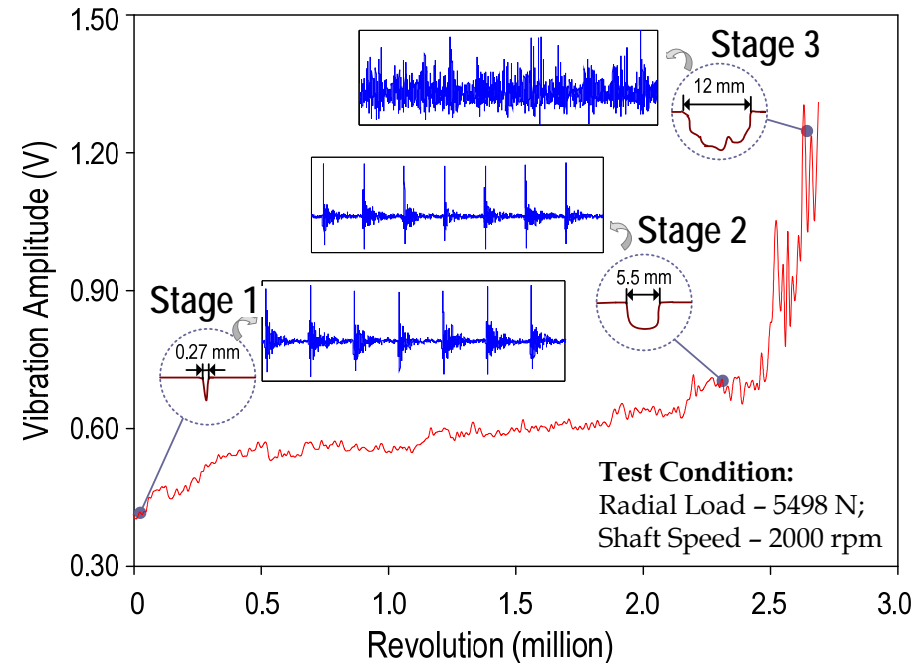
- **Bearing life stages**
  - ✓ Stage 1: Normal operation
  - ✓ Stage 2: Defect initiation
  - ✓ Stage 3: Accelerated performance degradation

- **Degradation modes**
  - ✓ Mode 1: *gradual degradation* (small vibration variation denoted by noise )

$$x_k = x_{k-1} + N(\mu_k, \sigma_k)$$



Life stages	Degradation modes
Stage 1	All Mode 1
Stage 2	Onset denoted by Mode 2
Stage 3	Most are Mode 2



- ✓ Mode 2: *exponential defect growth* (derived from spall propagation model)

$$\frac{dx}{dt} = \left( \frac{\Delta\tau}{C(1-x)} \right)^m = C' x^m$$

$C$  and  $m$  material dependent parameters  
 $\Delta\tau$ : shear stress range, constant

$$x_k = \left[ x_{k-1}^{(1-m_k)} + C_k (1-m_k) \right]^{1/(1-m_k)}$$

# Bearing Time to Failure Prognosis

- **Remaining Useful Life (RUL)**

- ✓ Calculated as the *first passage time*  $T$ , at which the predicted vibration first passes the defined failure threshold

$$\left\{ \begin{array}{l} RUL_p = \inf \left\{ T : x_k + \sum_{i=k+1}^T N(\mu_k, \sigma_k) \geq Threshold \right\} \quad \text{for Mode 1} \\ RUL_p = \inf \left\{ T : [x_{T-1}^{(1-m_r)} + C_T(1-m_r)]^{1/(1-m_r)} \geq Threshold \right\} \quad \text{for Mode 2} \end{array} \right.$$

$\mu$  and  $\sigma$ : to be estimated by PF

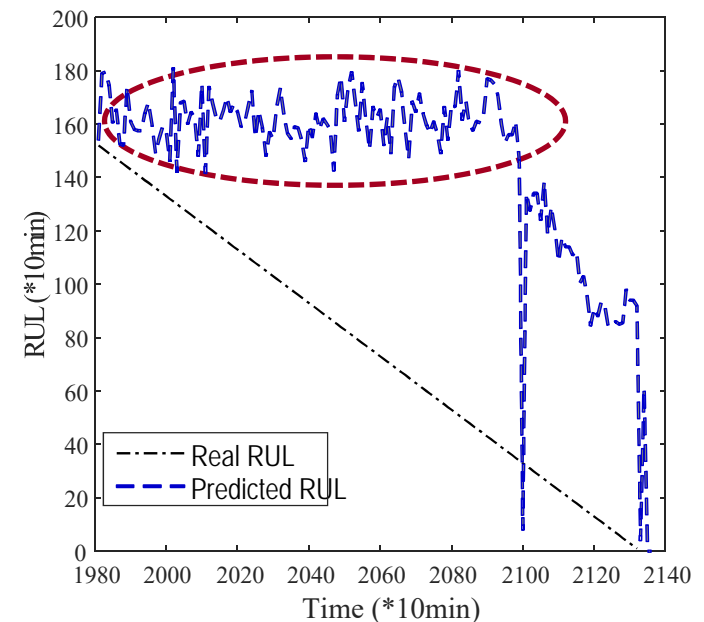
$C$  and  $m$ : to be estimated by PF

$\inf(\bullet)$  denotes the first passage time

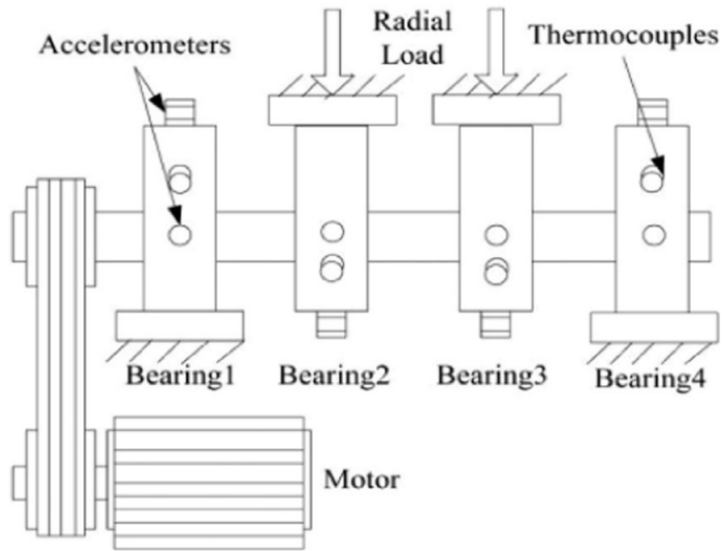
- **RUL Model Compensation**

- ✓ Vibration maintain at a level before defect initiation  
→ predicted RULs by Mode 1 almost *constant*
- ✓ A *compensation* needed for Mode 1

$$RUL_{k,comp} = RUL_{k,predicted} - k \left( 1 - \frac{x_k - x_1}{x_1} \right)$$



# Experimental Setup



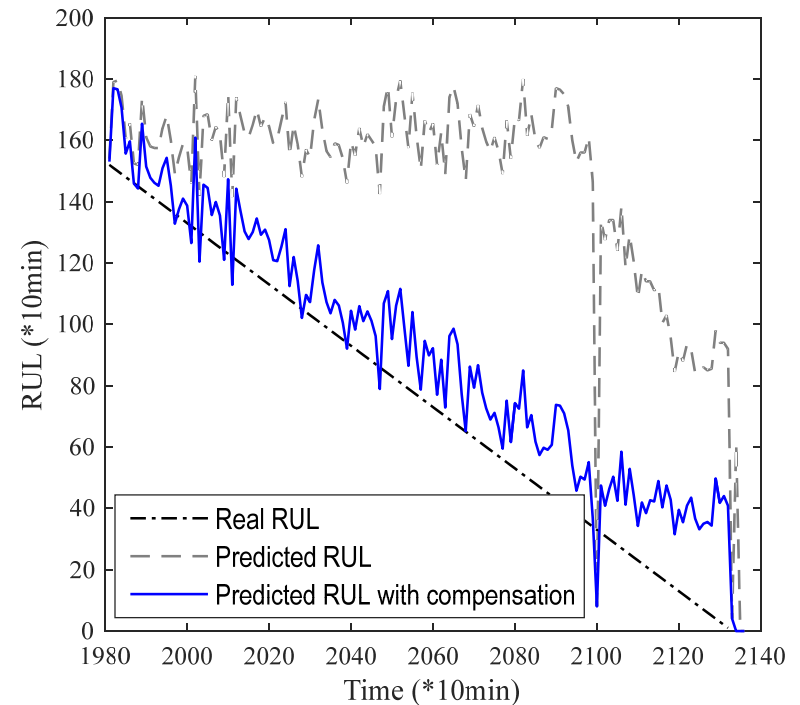
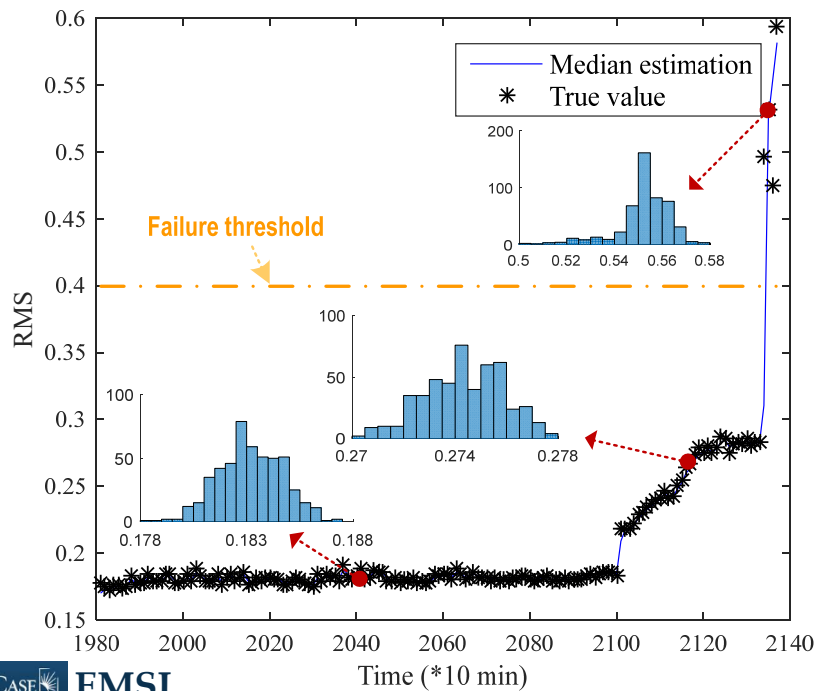
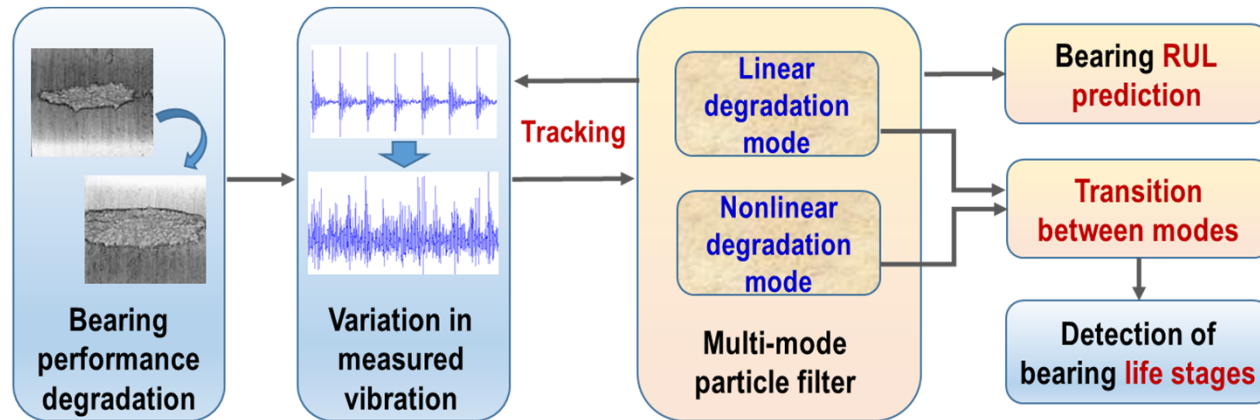
Inner-race defect



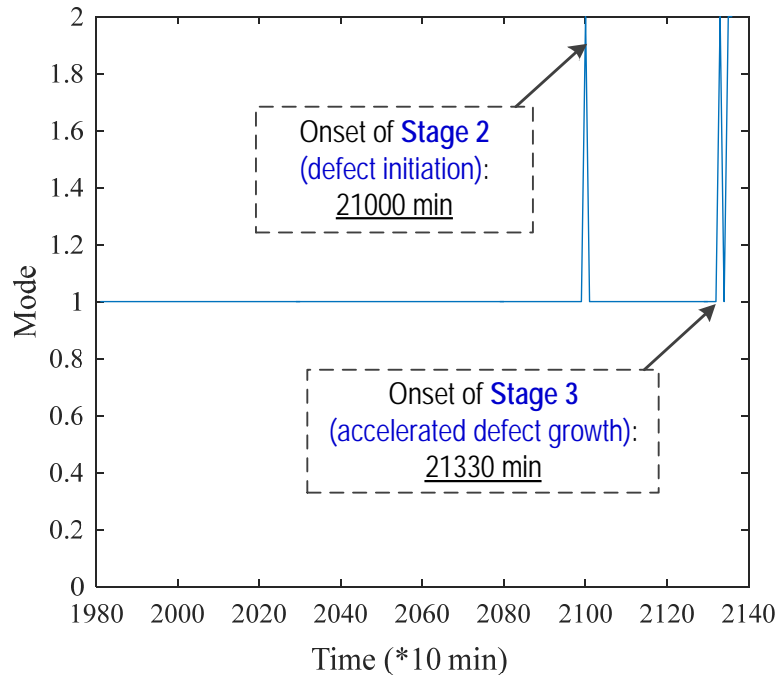
Outer-race defect

- ✓ *Constant rotational speed* at 2,000 rpm, a *radial load* of 6,000 lb
- ✓ Four ZA-2115 double row bearings, with force *lubricated*
- ✓ vibration data were collected every 10 minutes, with 20 kHz sampling rate
- ✓ A magnetic plug placed in the oil feedback pipe to collect debris; experiments stopped when the *accumulated debris* exceeded a certain level

# Multi-Mode Prognosis



# Performance Evaluation



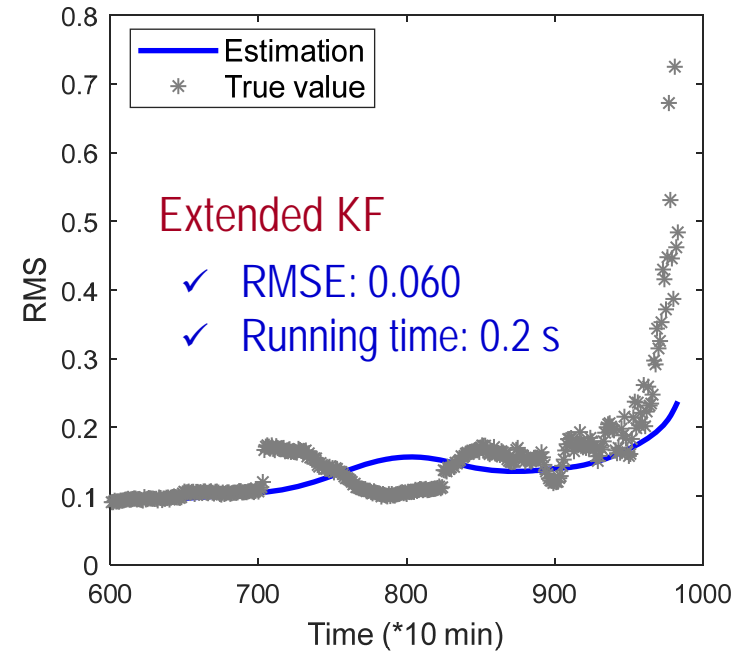
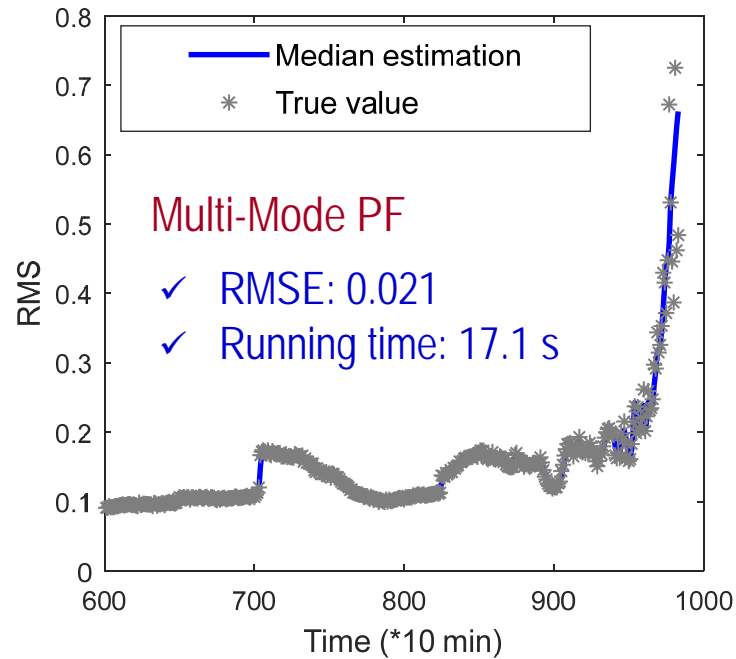
- **Automatic detection of transition among different bearing life stages**
  - ✓ Onset of stage 2: the *first transition* point in time from Mode 1 to Mode 2  
→ a *jump* occurs in the measured vibration
  - ✓ Onset of stage 3: the point in time when Mode 2 is switched on  
→ large vibration variation caused by *accelerated* spall propagation, accelerated material fracture

## Comparison between standard PF and multi-mode PF

	Standard PF with nonlinear degradation function	Multi-mode LSPF
Prediction error RMSE [hour]	7.9	2.9



# Comparison: PF vs. KF



Particle Filter		Extended Kalman Filter	
Comp. Steps	Computational Complexity	Comp. Steps	Computational Complexity
Prediction	$N * (n^2 + n)$	Prediction	$n^3 + 2n^2 + n$
Update	$N * (2p^2n + pn + p)$	Kalman gain	$2p^3 + 2p^2n + 2pn + p$
Resampling	$N * (2anN + n^2 + 2n + qn)$	Update	$pn^2 + 2n^2 + 2pn + p + n$

Note: **N**: number of particles in PF; **n**: state dimension; **p**: measurement dimension  
**a**: coefficient related to resampling strategy, between 0 and 1; **q**: constant



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Wang & Gao, "Adaptive re-sampling-based PF", JMS, 2015

# Broad Applications of PF

(Wang and Gao, JMS, 2015)



Machine tool



Motor



Gearbox

(Wang *et al.* JMS, 2017)

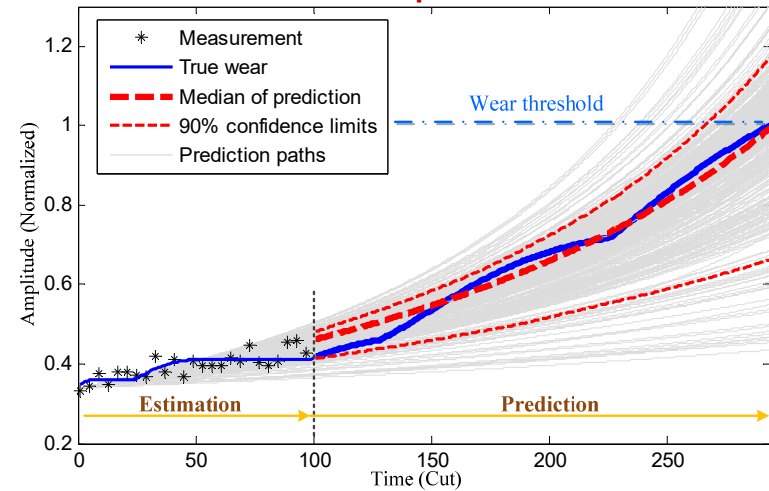


HVAC system

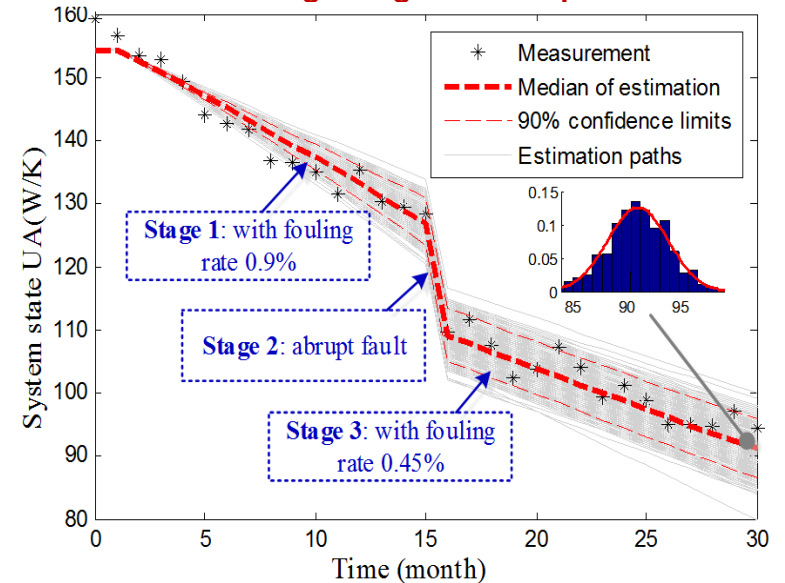
(Wang and Gao, IEEE TASE, 2017)



## Tool wear prediction



## Heat exchange degradation prediction



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# Moving Forward

## Machines

## Sensor Measurement

## Data Analytics



Vibration



Temp



Speed



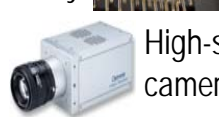
AE



Micro wireless sensing networks



Acoustic array



High-speed cameras



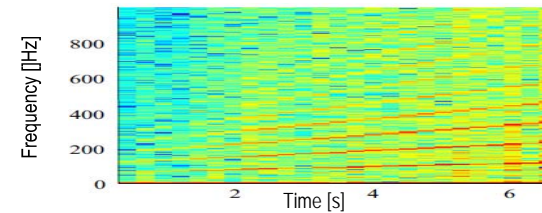
Current  
React and Repair

### Techniques

- Deterministic feature extraction
- Feature fusion through PCA
- Time-freq. analysis (e.g. STFT, wavelet)

### Results

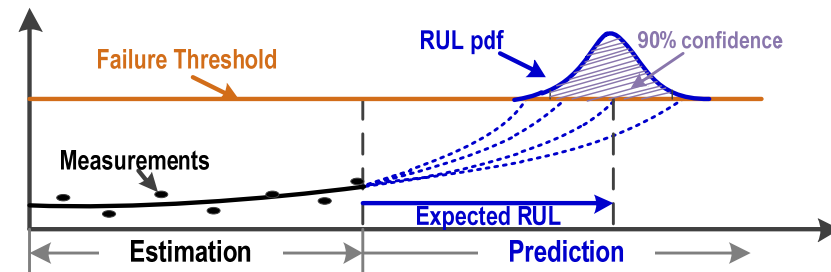
- Fault diagnosis: type and degree
- Condition-based maintenance



Future  
Predict and Prevent

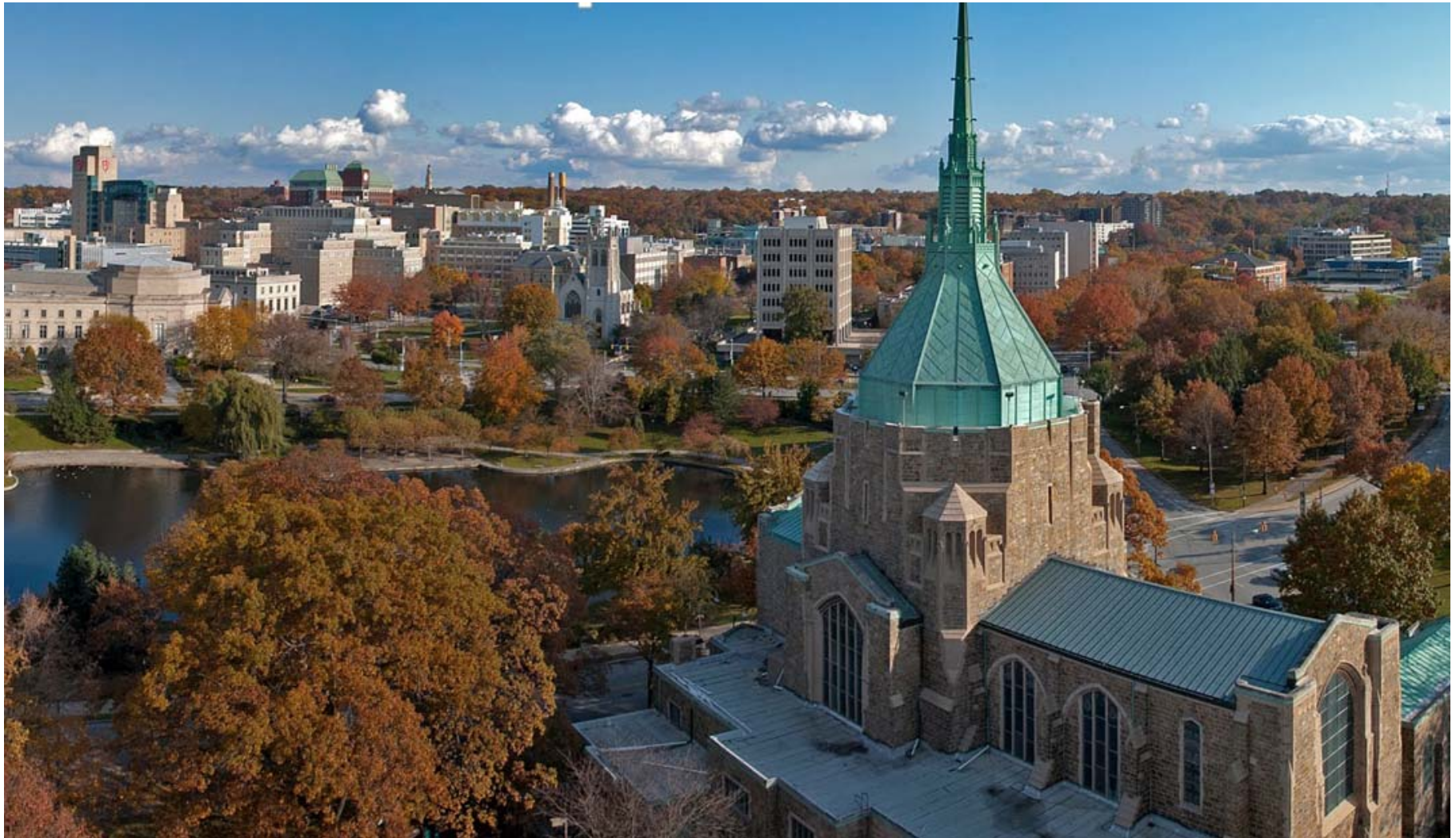
- Probabilistic inference (e.g. PF, Bayesian network)
- Deep Learning (e.g. DBN, DCNN)
- Multi-physics data fusion

- Fault Prognosis: degradation tracking and prediction
- Intelligent predictive maintenance



# Thank You!

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