

Panel on Confidence Intervals for Likelihood Ratios

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Question 1 – A confidence interval for what?

- Bayes Theorem and the Likelihood Ratio
- E = evidence (E_x = crime scene; E_y = suspect)
- H_p = proposition that two samples have the same source
H_d = proposition that two samples have a different source

$$\frac{\Pr(H_p | E)}{\Pr(H_d | E)} = \frac{\Pr(E | H_p)}{\Pr(E | H_d)} \frac{\Pr(H_p)}{\Pr(H_d)}$$

- Three terms:
 - Far right: “a priori” (before evidence) odds in favor of the common source proposition
 - Middle term: the likelihood ratio / Bayes factor
 - Far left term: “a posteriori” (after evidence) odds

Question 1 – A confidence interval for what?

$$BF = \Pr(E | H_p) / \Pr(E | H_d)$$

- Key issue is that probabilities often depend on parameters (e.g., copper example)

$$\Pr(E | H, \theta)$$

- Bayes Factor – Average over uncertainty in these parameters

$$\Pr(E | H) = \int \Pr(E | \theta, H) \Pr(\theta | H) d\theta$$

- The fully subjective Bayesian approach thus does not admit interval estimates (e.g., Taroni et al., LPR, 2016)

Question 1 – A confidence interval for what?

$$LR = \Pr(E | H_p) / \Pr(E | H_d)$$

- Key issue is that probabilities often depend on parameters (e.g., copper example)

$$\Pr(E | H, \theta)$$

- Likelihood ratio $LR = LR(\theta_p, \theta_d)$
(the two parameter vectors may have elements in common)
- One **can** build a CI for this function – this would address sampling variability for parameter estimates

Question 2 – How would we use a CI?

- Evidence that decision makers have a very hard time using LRs
 - They often do not appear to understand the definition or interpretation
 - They do not appear to consistently update prior probabilities in a manner consistent with the LR interpretation
- If they could understand and interpret LRs, then it seems reasonable to assume they could probably handle CIs for LRs.
 - Indeed they might even desire it (conveys info on uncertainty)
- But that's a very big “if”!!

Question 3 – Can a CI approach do harm?

- The debate between dogmatic Bayesian statisticians and non-Bayesian statisticians can definitely do harm
 - To whom should the forensic community listen?
 - Will they tune the statistical community out?
 - Some indications of this in OSAC discussions
- A relevant anecdote:
the ASA's recent p-value statement
- Note: OSAC Statistics Task Group has worked very well together across this divide

A role for “multiple” LR_s

- Confidence intervals address sampling variability in parameter estimates
- More important to consider a range of LR_s that address other factors (i.e., a sensitivity analysis)
 - Different parametric assumptions
 - Parametric vs non-parametric models (see, e.g., Lucy and Aitken, 2004)
 - Different estimation approaches for unknown parameters (diff’t estimators, diff’t databases)