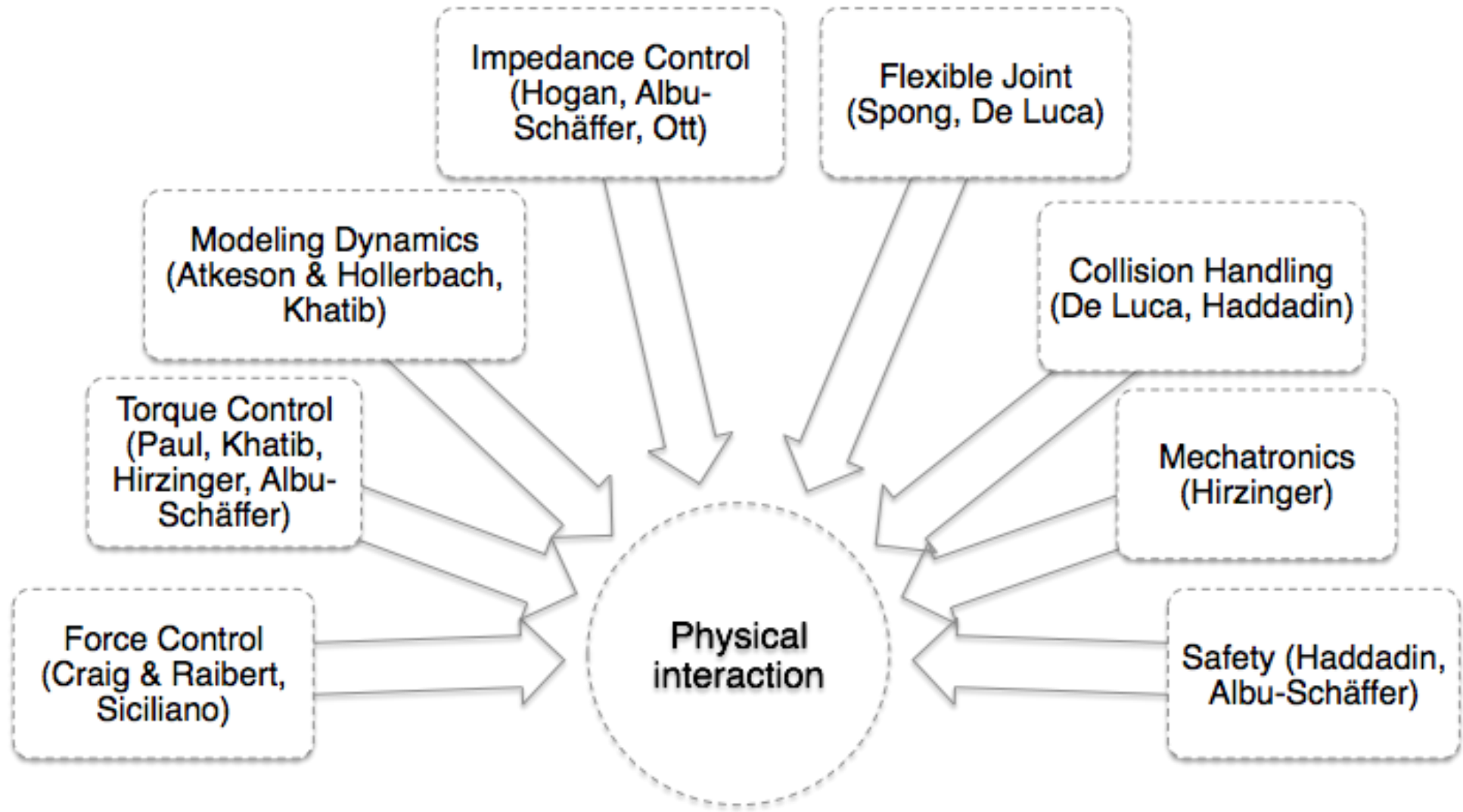


Robot Control

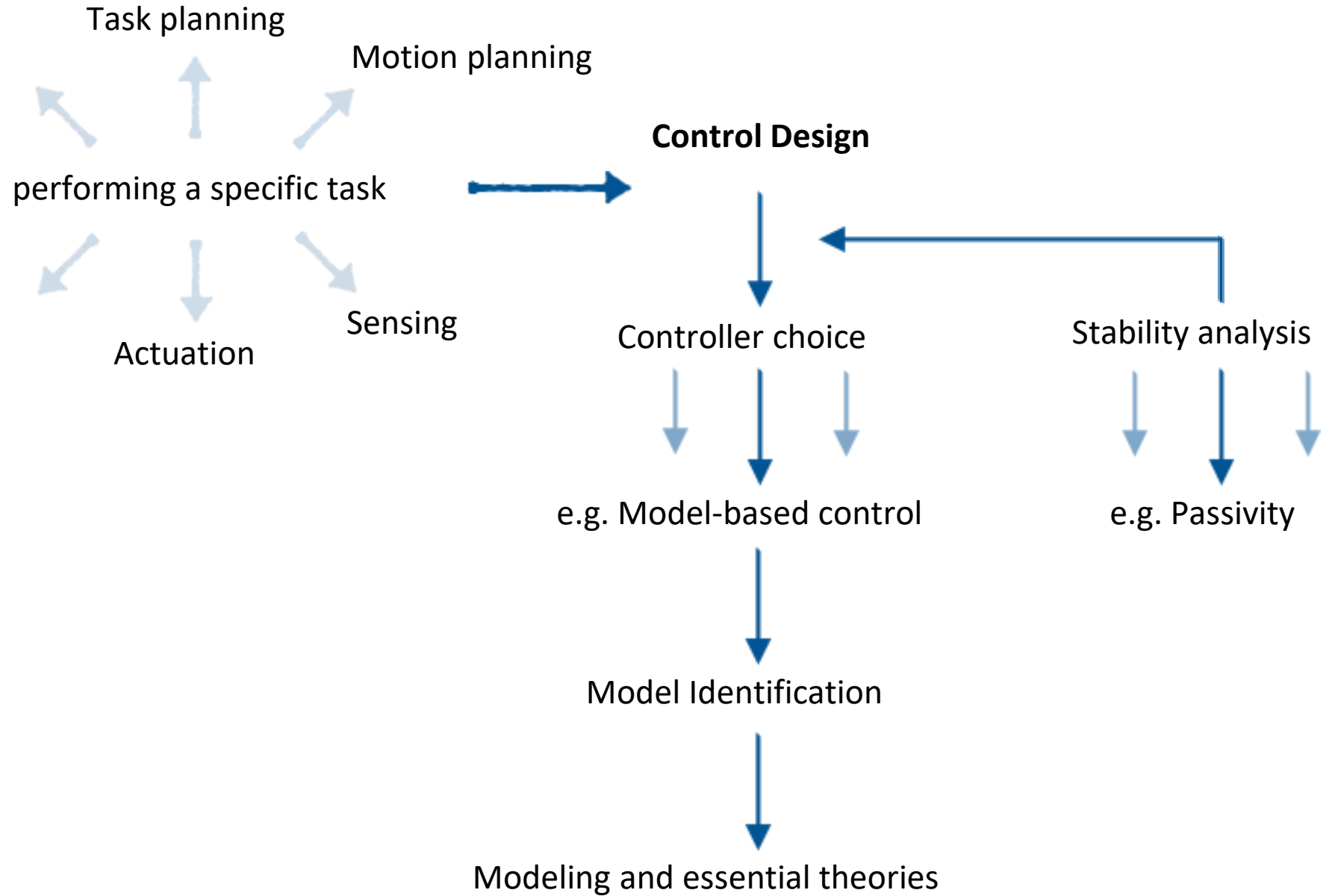
MSRM

Sami Haddadin, Lars Johansmeier

Physical Interaction

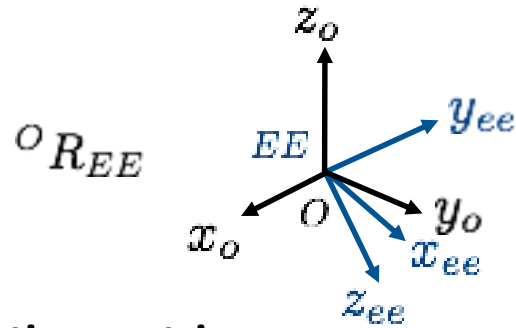


Overview



Modeling and Essential Theories

Kinematics



Rotation matrix $R_{3 \times 3}$
Orthonormal matrix

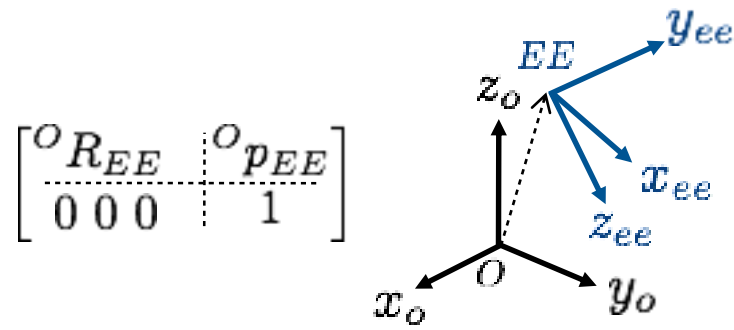
9 elements
-3 orthogonality relationships
-3 unitary relationships
= 3 independent elements

Euler angles $\{\theta, \phi, \psi\}$

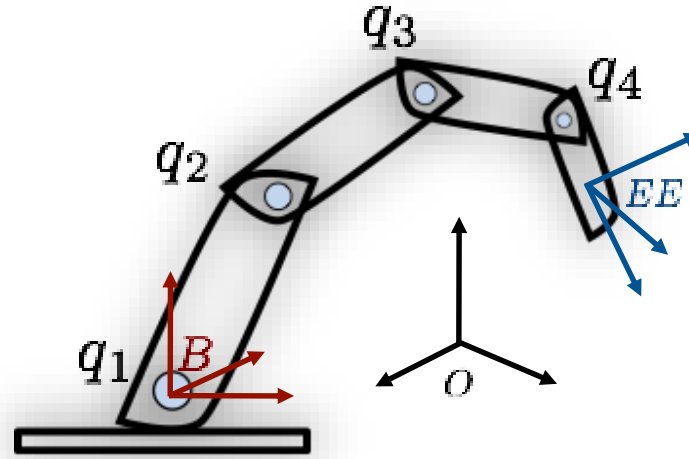
Non unique and singularities

Unit quaternions (k_0, \mathbf{k}_v)

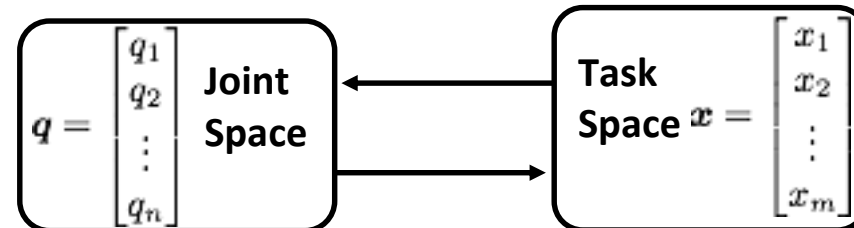
$$k_0 + k_{v,x}\vec{i} + k_{v,y}\vec{j} + k_{v,z}\vec{k}$$



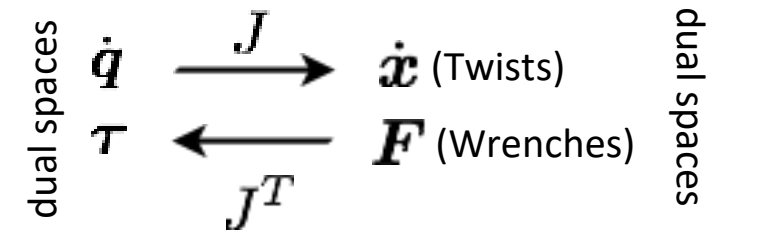
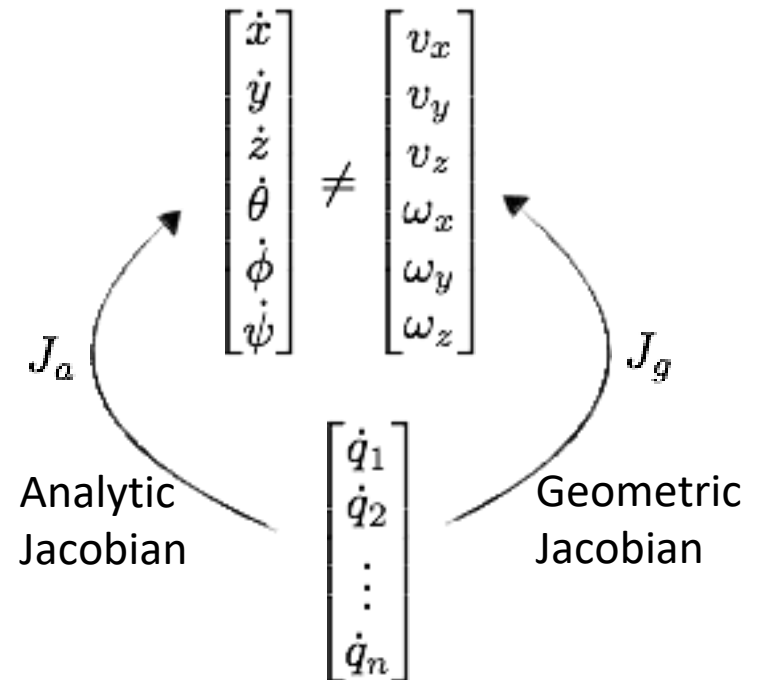
Transformation matrix $T_{4 \times 4}$
Always invertible
Composition: ${}^O T_{EE} = {}^O T_B {}^B T_{EE}$



Direct Kinematics



Inverse Kinematics



Space of linear operators on vector space V . $f: V \rightarrow \mathbb{R}$

When Jacobian loses rank.

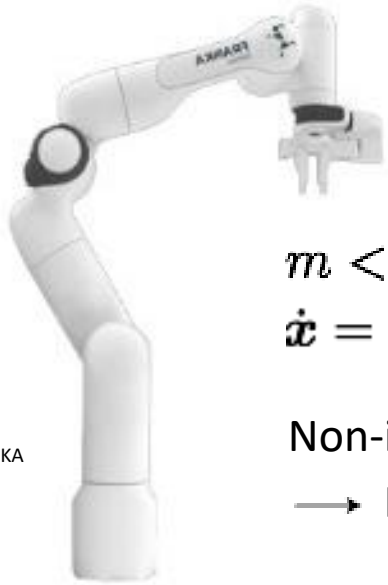
Manipulability vs. Singularity

How far from singularity?

$$\sqrt{\det(JJ^T)}$$

Redundancy

FRANKA EMIKA



$$m < n$$

$$\dot{\mathbf{x}} = \mathbf{J}_{m \times n} \dot{\mathbf{q}}$$

Non-invertible Jacobian

→ Inverse kinematics problems

Null-space control

No motion

$$\mathbf{0} = \mathbf{J} \dot{\mathbf{q}}_N$$

No Wrench

$$\boldsymbol{\tau}_N = \mathbf{J}^T \mathbf{0}$$

Moore-Penrose Pseudo-inverse

Unique and always exists

Such that (almost ident. for weighted):

$$\mathbf{J} \mathbf{J}^\# \mathbf{J} = \mathbf{J} \quad \mathbf{J}^\# \mathbf{J} \mathbf{J}^\# = \mathbf{J}^\#$$

$$(\mathbf{J} \mathbf{J}^\#)^T = \mathbf{J} \mathbf{J}^\# \quad (\mathbf{J}^\# \mathbf{J})^T = \mathbf{J}^\# \mathbf{J}$$

$$\dot{\mathbf{q}}_N = (\mathbf{I} - \mathbf{J}_W^\# \mathbf{J}) \dot{\mathbf{q}}'$$

$$\boldsymbol{\tau}_N = (\mathbf{I} - \mathbf{J}^T \mathbf{J}_W^{\#T}) \boldsymbol{\tau}'$$

Task prioritization

$$\dot{\mathbf{q}} = \mathbf{J}_W^\# \dot{\mathbf{x}} + (\mathbf{I} - \mathbf{J}_W^\# \mathbf{J}) \dot{\mathbf{q}}'$$

On velocity level

$$\boldsymbol{\tau} = \mathbf{J}^T \mathbf{F} + (\mathbf{I} - \mathbf{J}^T \mathbf{J}_W^{\#T}) \boldsymbol{\tau}'$$

On torque level

Primary task

Secondary task

$$\mathbf{J}_W^\# = \mathbf{W}^{-1} \mathbf{J}^T (\mathbf{J} \mathbf{W}^{-1} \mathbf{J}^T)^{-1}$$

$$\mathbf{J}^\# = \mathbf{J}^T (\mathbf{J} \mathbf{J}^T)^{-1}$$

Weighted pseudo-inverse

Differential inverse kinematics:

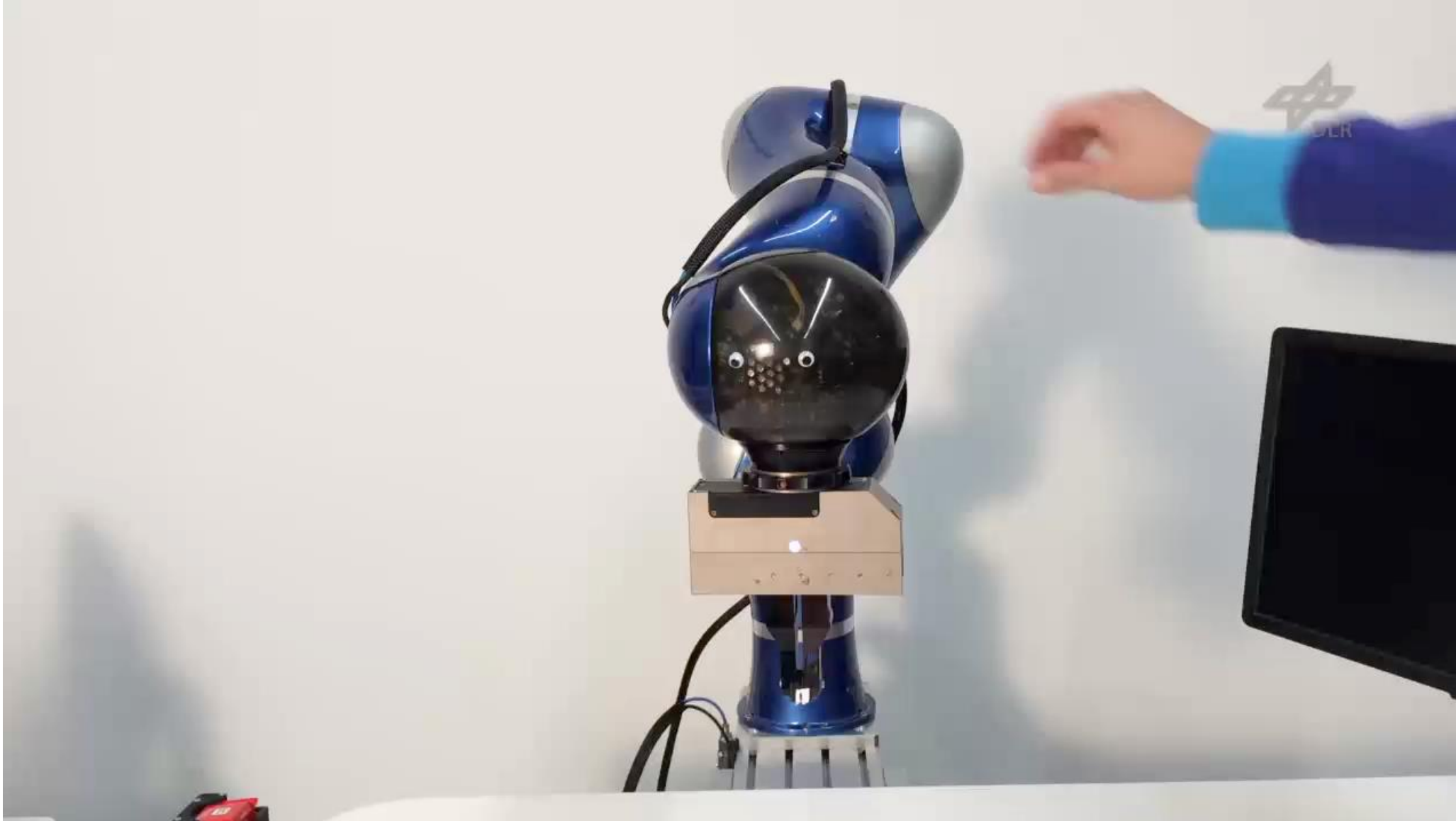
Minimization of:

$$\frac{1}{2} \dot{\mathbf{q}}^T \mathbf{W} \dot{\mathbf{q}}$$

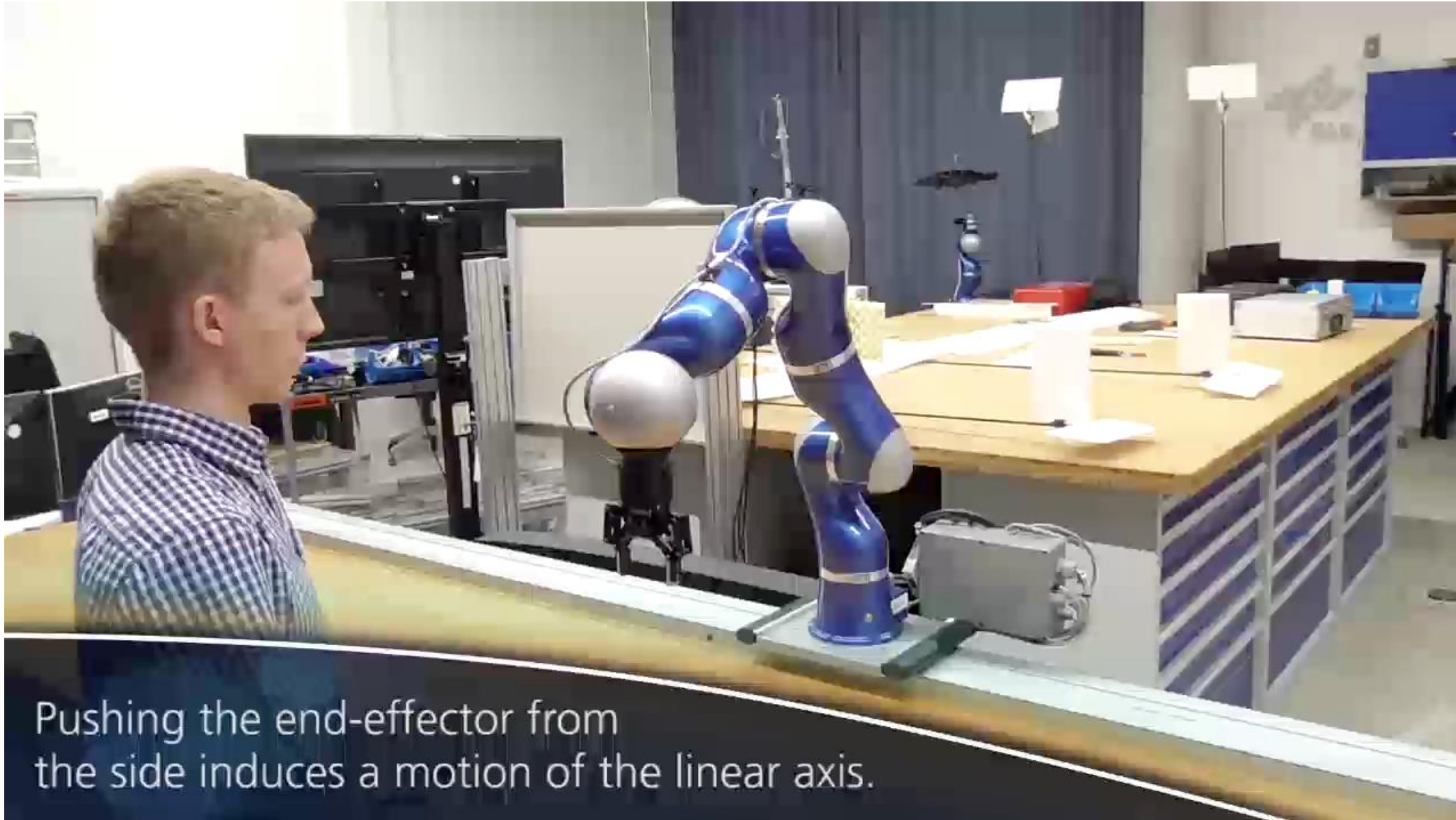
$\mathbf{J}^\#$

$\mathbf{J}_W^\#$

Redundancy

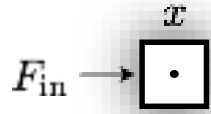


Redundancy



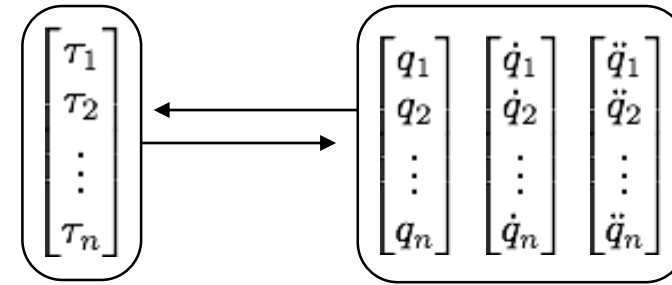
Pushing the end-effector from the side induces a motion of the linear axis.

Dynamics



$$F_{in} = M\ddot{x}$$

Direct Dynamics

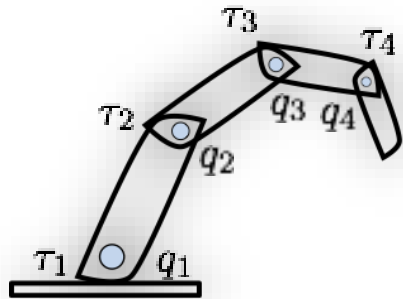


Inverse Dynamics

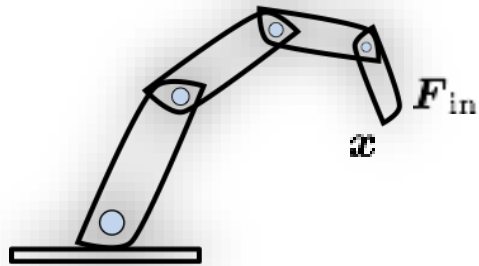
Euler-Lagrange method
(energy-based approach)
(symbolic)

vs.

Newton-Euler method
(Wrench balance approach)
(numeric)



$$\tau_{in} = M(q)\ddot{q} + \underbrace{C(q, \dot{q})\dot{q}}_{c(q, \dot{q})} + g(q)$$



$$F_{in} = J^{\#T}(q)\tau_{in} = M_x(q)\ddot{x} + C_x(q, \dot{q})\dot{x} + F_g$$

Operational Space Dynamics

$$M_x(q) = J^{\#T}(q)M(q)J^{\#}(q)$$

$$C_x(q, \dot{q}) = J^{\#T}(q)C(q, \dot{q})J^{\#}(q) - M_x(q)\dot{J}(q)J^{\#}(q)$$

Skew-symmetry of:

$$\dot{M}(q) - 2C(q, \dot{q})$$

$$\dot{M}_K(q) - 2C_K(q, \dot{q})$$

$$\Gamma \in \mathbb{R}^{n \times n} \longrightarrow \forall v \in \mathbb{R}^n \implies v^T \Gamma v = 0$$

& skew-symmetric

Flexible-joint robots

Fully coupled model

$$\begin{bmatrix} M(\mathbf{q}) & S(\mathbf{q}) \\ S^T(\mathbf{q}) & B \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{q}} \\ \ddot{\boldsymbol{\theta}} \end{bmatrix} + \begin{bmatrix} \mathbf{c}(\mathbf{q}, \dot{\mathbf{q}}) + \mathbf{c}_1(\mathbf{q}, \dot{\mathbf{q}}, \dot{\boldsymbol{\theta}}) \\ \mathbf{c}_2(\mathbf{q}, \dot{\mathbf{q}}) \end{bmatrix} + \begin{bmatrix} \mathbf{g}(\mathbf{q}) + K(\mathbf{q} - \boldsymbol{\theta}) + D(\dot{\mathbf{q}} - \dot{\boldsymbol{\theta}}) \\ K(\boldsymbol{\theta} - \mathbf{q}) + D(\dot{\boldsymbol{\theta}} - \dot{\mathbf{q}}) \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \boldsymbol{\tau}_m \end{bmatrix}$$

Reduced model (Large transmission ratio)

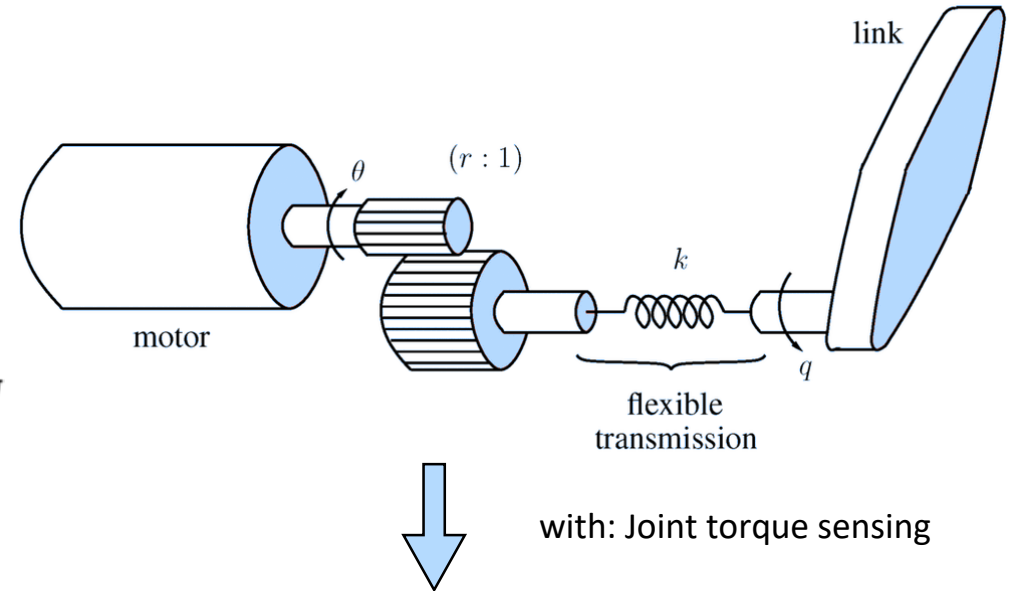
$$M(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{c}(\mathbf{q}, \dot{\mathbf{q}}) + \mathbf{g}(\mathbf{q}) - \boldsymbol{\tau}_J = \mathbf{0}$$

$$B_\theta \ddot{\boldsymbol{\theta}} + \boldsymbol{\tau}_J = \boldsymbol{\tau}_m$$

$$K(\boldsymbol{\theta} - \mathbf{q}) + D_s(\dot{\boldsymbol{\theta}} - \dot{\mathbf{q}}) = \boldsymbol{\tau}_J$$

$$B \ddot{\boldsymbol{\theta}} + \boldsymbol{\tau}_J = \boldsymbol{\tau}_m$$

$$K(\boldsymbol{\theta} - \mathbf{q}) + D(\dot{\boldsymbol{\theta}} - \dot{\mathbf{q}}) = \boldsymbol{\tau}_J$$



$$\boldsymbol{\tau}_m = BB_\theta^{-1} \boldsymbol{\tau}_{in} + \boldsymbol{\tau}_J - BB_\theta^{-1} (K(\boldsymbol{\theta} - \mathbf{q}) + D_s(\dot{\boldsymbol{\theta}} - \dot{\mathbf{q}}))$$

Inertia shaping

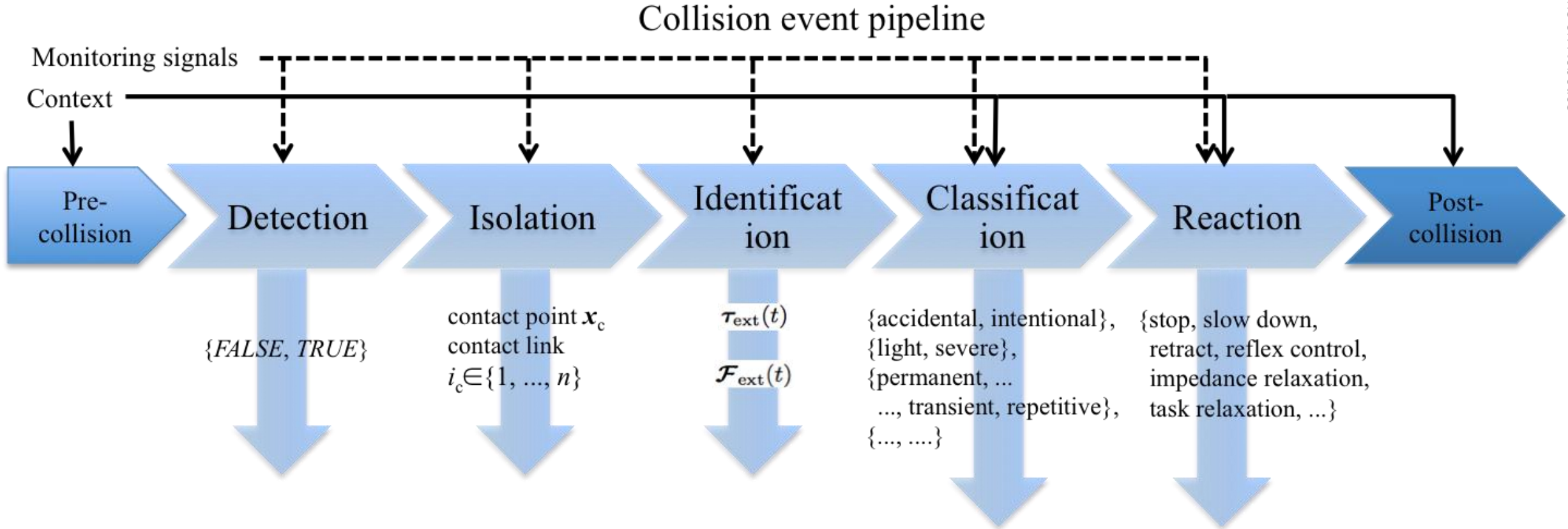


Damping shaping

Take Me by the Hand!



Collision Handling



External Joint Torque Observer

Generalized momentum: $\mathbf{p} = M(\mathbf{q})\dot{\mathbf{q}}$

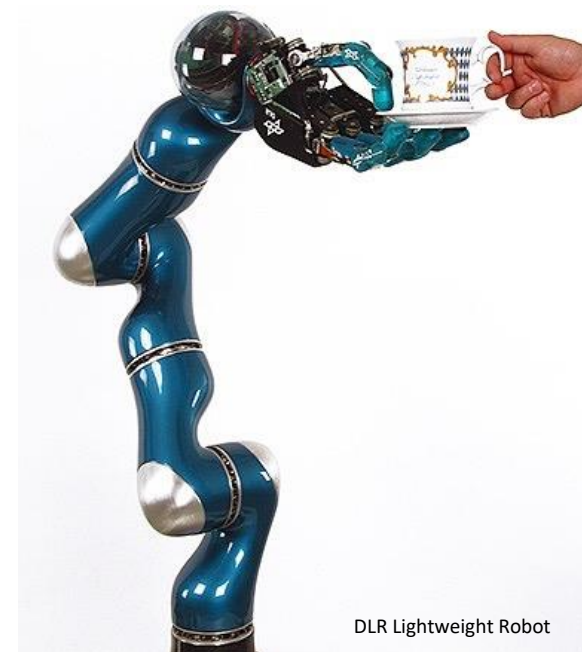
$$\mathbf{r}(t) = K_O \left[\mathbf{p}(t) - \int_0^t (\boldsymbol{\tau}_J + C^T(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} - \mathbf{g}(\mathbf{q}) + \mathbf{r}) ds - \mathbf{p}(0) \right]$$

$$\dot{\mathbf{r}} = -K_O \mathbf{r} + K_O \boldsymbol{\tau}_{\text{ext}}$$

Component-wise: $\frac{r_j(s)}{\tau_{\text{ext},j}(s)} = \frac{K_{O,j}}{s + K_{O,j}}$

$$K_O \rightarrow \infty \longrightarrow \mathbf{r} \approx \boldsymbol{\tau}_{\text{ext}}$$

Collision on the i -th link: $\mathbf{r} = [* \dots * \underset{\substack{\uparrow \\ i+1}}{0} \dots \underset{\substack{\uparrow \\ n}}{0}]$

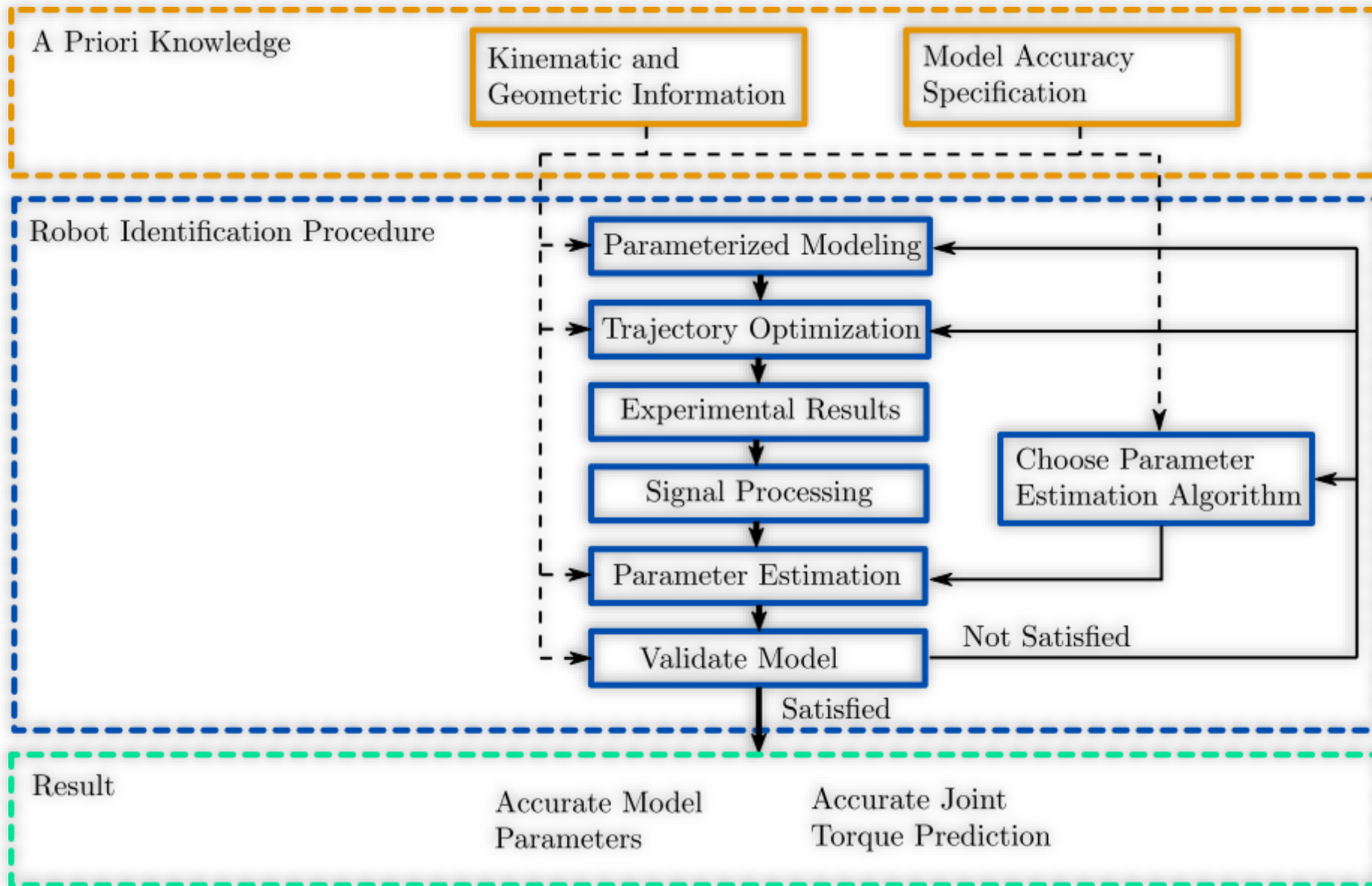


Collision Handling



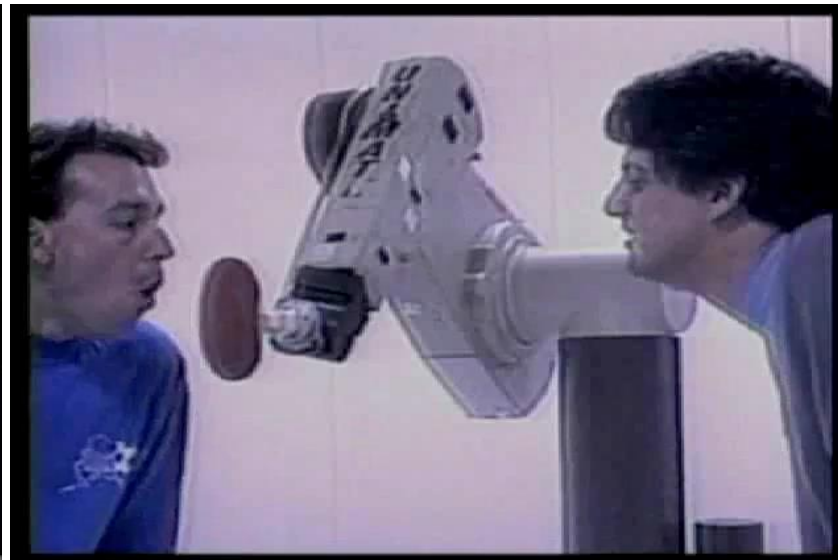
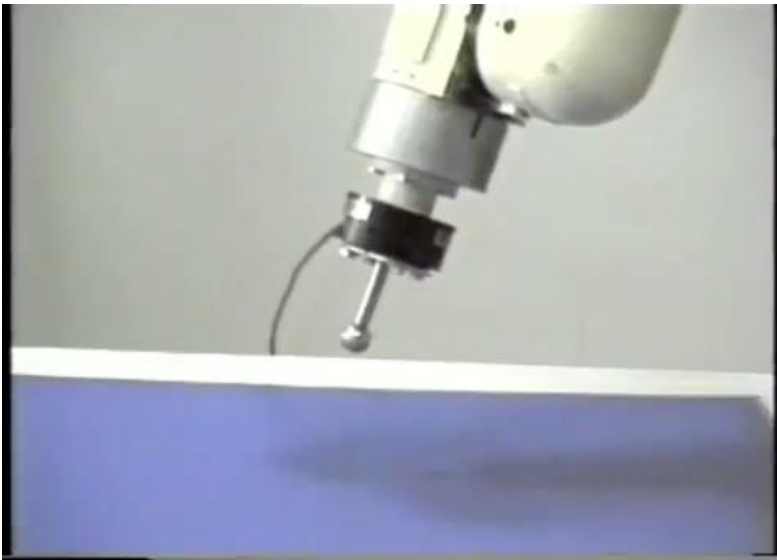
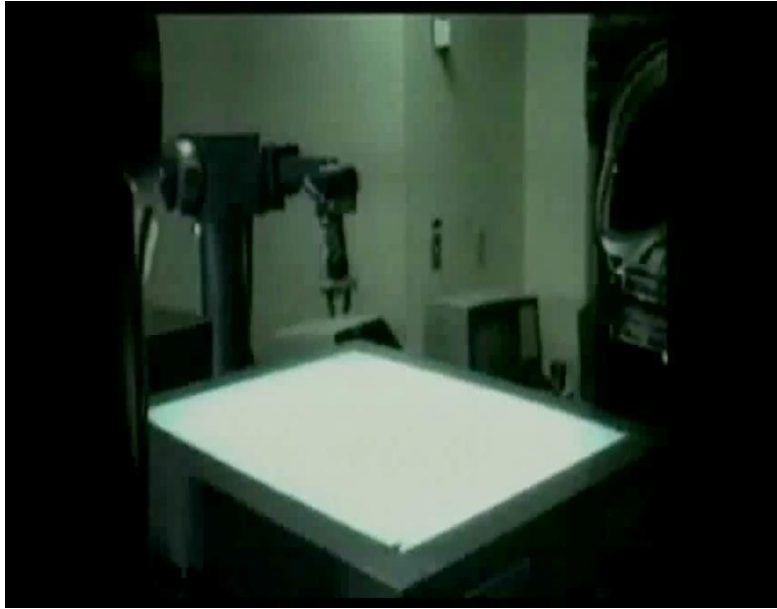
Model Identification

Identification Procedure



Control Design

Early pHRI



Joint Impedance Control

Desired impedance behavior:

$$\tilde{\mathbf{q}} = \mathbf{q} - \mathbf{q}_d$$

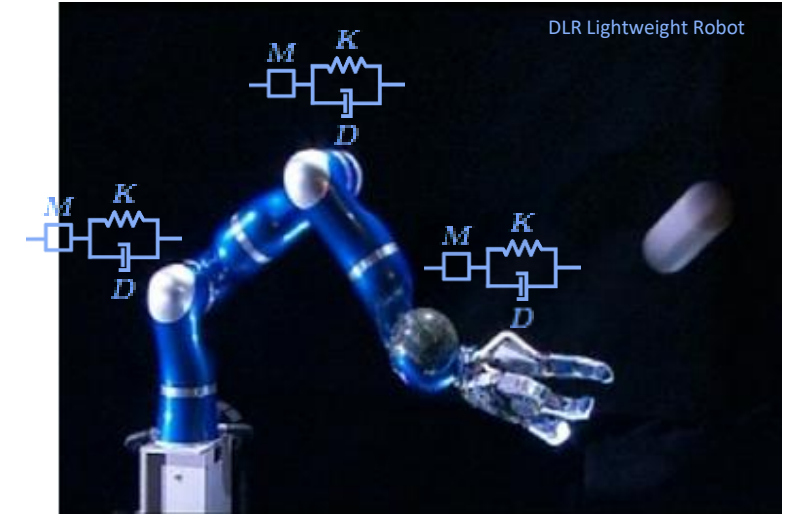
$$M' \ddot{\tilde{\mathbf{q}}} + D \dot{\tilde{\mathbf{q}}} + K \tilde{\mathbf{q}} = \boldsymbol{\tau}_{\text{ext}}$$

Robot dynamics:

$$M(\mathbf{q}) \ddot{\mathbf{q}} + C(\mathbf{q}, \dot{\mathbf{q}}) \dot{\mathbf{q}} + \mathbf{g}(\mathbf{q}) = \boldsymbol{\tau}_{\text{in}} + \boldsymbol{\tau}_{\text{ext}}$$

Required control input:

$$\boldsymbol{\tau}_{\text{in}} = M(\mathbf{q}) \ddot{\mathbf{q}}_d + C(\mathbf{q}, \dot{\mathbf{q}}) \dot{\mathbf{q}} - M(\mathbf{q}) M'^{-1} (D \dot{\tilde{\mathbf{q}}} + K \tilde{\mathbf{q}}) + (M(\mathbf{q}) M'^{-1} - I) \boldsymbol{\tau}_{\text{ext}} + \mathbf{g}(\mathbf{q})$$



Avoidance of inertia shaping:

$$\begin{aligned} M' &\longrightarrow M(\mathbf{q}) \\ D &\longrightarrow D + C(\mathbf{q}, \dot{\mathbf{q}}) \end{aligned}$$

$$\boldsymbol{\tau}_{\text{in}} = M(\mathbf{q}) \ddot{\mathbf{q}}_d + C(\mathbf{q}, \dot{\mathbf{q}}) \dot{\mathbf{q}}_d - D \dot{\tilde{\mathbf{q}}} - K \tilde{\mathbf{q}} + \mathbf{g}(\mathbf{q})$$

No need for external joint torque sensing

PD+ controller

$$\begin{aligned} \dot{\mathbf{q}}_d = 0 &\longrightarrow \text{Compliance control} \\ \boldsymbol{\tau}_{\text{in}} &= -D \dot{\tilde{\mathbf{q}}} - K \tilde{\mathbf{q}} + \mathbf{g}(\mathbf{q}) \end{aligned}$$

Closed-loop dynamics:

$$M(\mathbf{q}) \ddot{\tilde{\mathbf{q}}} + (D + C(\mathbf{q}, \dot{\mathbf{q}})) \dot{\tilde{\mathbf{q}}} + K \tilde{\mathbf{q}} = \boldsymbol{\tau}_{\text{ext}}$$

Cartesian Impedance Control & Damping Design

Desired impedance behavior (without inertia shaping):

$$M_x(\mathbf{q})\ddot{\tilde{\mathbf{x}}} + (D_x + C_x(\mathbf{q}, \dot{\mathbf{q}}))\dot{\tilde{\mathbf{x}}} + K_x\tilde{\mathbf{x}} = \mathbf{F}_{\text{ext}}$$

Required control input:

$$\boldsymbol{\tau}_{\text{in}} = J^T(\mathbf{q}) [M_x(\mathbf{q})\ddot{\tilde{\mathbf{x}}}_d + C_x(\mathbf{q}, \dot{\mathbf{q}})\dot{\tilde{\mathbf{x}}}_d - D_x\dot{\tilde{\mathbf{x}}} - K_x\tilde{\mathbf{x}}] + \mathbf{g}(\mathbf{q})$$

Design of stiffness: Constant & defined by the application (Normally symmetric & positive).

Design of damping: Constant & diagonal not good.

Non-constant & non-diagonal inertia

e.g. based on general eigenvalue decomposition of symmetric matrices

For any positive-definite matrix A and symmetric matrix B , there is a non-singular matrix Q and a diagonal matrix B_0 such that:

$$Q^T Q = A$$

$$Q^T B_0 Q = B$$

In quasi-static case, at each position $\mathbf{x}_0 = \mathbf{f}(\mathbf{q}_0)$

$$M_x(\mathbf{q}_0)\ddot{\tilde{\mathbf{x}}} + D_x(\mathbf{x}_0)\dot{\tilde{\mathbf{x}}} + K_x\tilde{\mathbf{x}} = \mathbf{F}_{\text{ext}}$$

$$Q(\mathbf{x}_0)^T Q(\mathbf{x}_0)\ddot{\tilde{\mathbf{x}}} + D_x(\mathbf{x}_0)\dot{\tilde{\mathbf{x}}} + Q(\mathbf{x}_0)^T B_0(\mathbf{x}_0) Q(\mathbf{x}_0)\tilde{\mathbf{x}} = \mathbf{F}_{\text{ext}}$$

$$D_x(\mathbf{x}_0) = 2Q(\mathbf{x}_0)^T \text{diag}(\zeta_i \sqrt{b_{ii}}) Q(\mathbf{x}_0)$$

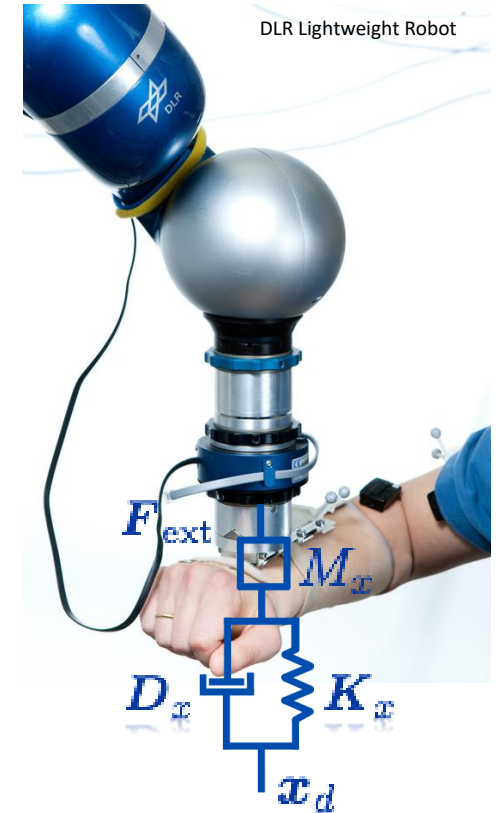
i-th diagonal element of B_0 $i = 1, \dots, 6$

Damping factor:
 $0 \leq \zeta_i \leq 1$

$$\tilde{\mathbf{x}} = \mathbf{x} - \mathbf{x}_d$$

$\dot{\tilde{\mathbf{x}}}_d = \mathbf{0} \longrightarrow$ Compliance control

$$\boldsymbol{\tau}_{\text{in}} = J^T(\mathbf{q}) (-D_x\dot{\tilde{\mathbf{x}}} - K_x\tilde{\mathbf{x}}) + \mathbf{g}(\mathbf{q})$$



Stiffen up!



Adaptive Impedance Control

Cartesian impedance control with the feedforward wrench

$$\tilde{\mathbf{x}} = \mathbf{x} - \mathbf{x}_d$$

$$\boldsymbol{\tau}_{\text{in}} = \mathbf{J}^T(\mathbf{q}) [M_x(\mathbf{q})\ddot{\mathbf{x}}_d + C_x(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{x}}_d - D_x\dot{\tilde{\mathbf{x}}} - K_x\tilde{\mathbf{x}} - \mathbf{F}_{ff}] + \mathbf{g}(\mathbf{q})$$

$$K_x(t) = K_x(t - T) + \delta K_x$$

$$\mathbf{F}_{ff}(t) = \mathbf{F}_{ff}(t - T) + \delta \mathbf{F}_{ff}$$

Similar to the principles of motor adaptation:

$$\delta K_x = \alpha_K (\text{diag}(\boldsymbol{\epsilon} \circ \tilde{\mathbf{x}}) - \gamma_K(t) K_x(t))$$

$$\delta \mathbf{F}_{ff} = \alpha_F (\boldsymbol{\epsilon} - \gamma_F(t) \mathbf{F}_{ff}(t))$$

$$\boldsymbol{\epsilon} = \tilde{\mathbf{x}} + \kappa \dot{\tilde{\mathbf{x}}}$$

α_K & α_F \longrightarrow Learning rate (positive definite)

γ_F & γ_K \longrightarrow Forgetting factor (positive definite)

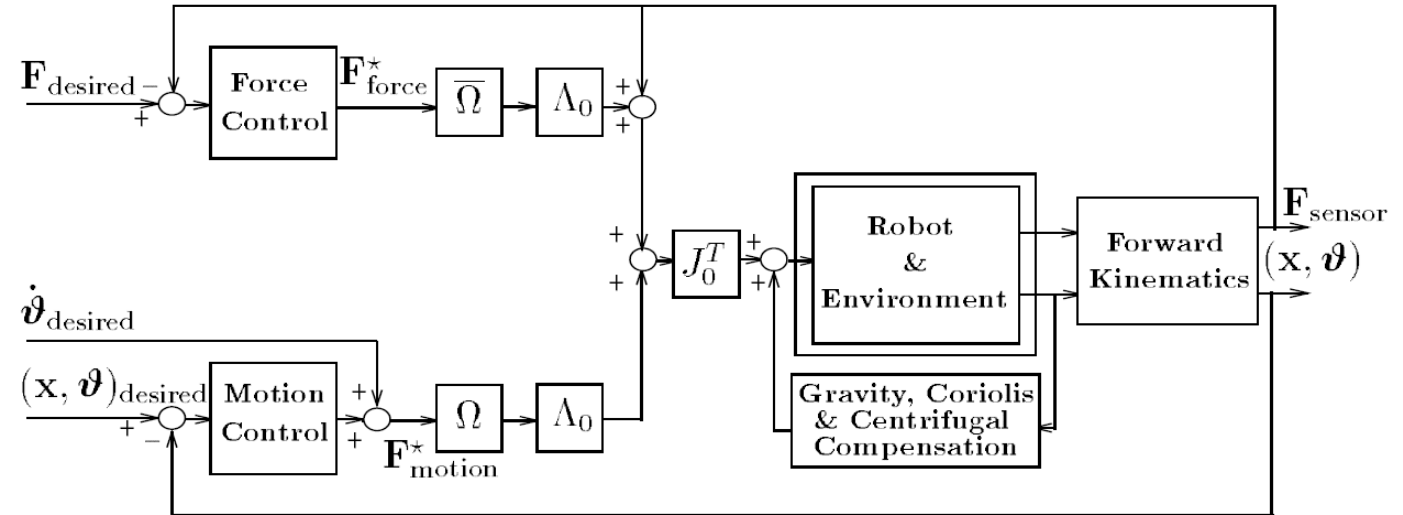
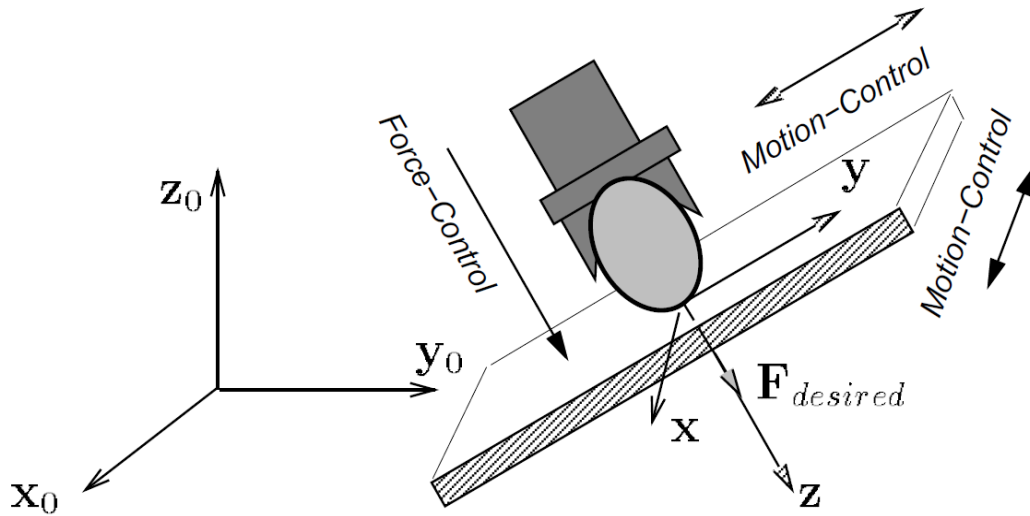
e.g. $\frac{a}{1 + b\|\boldsymbol{\epsilon}\|^2}$



DLR Lightweight Robot with Adaptive Impedance control in peg-in-hole experiment

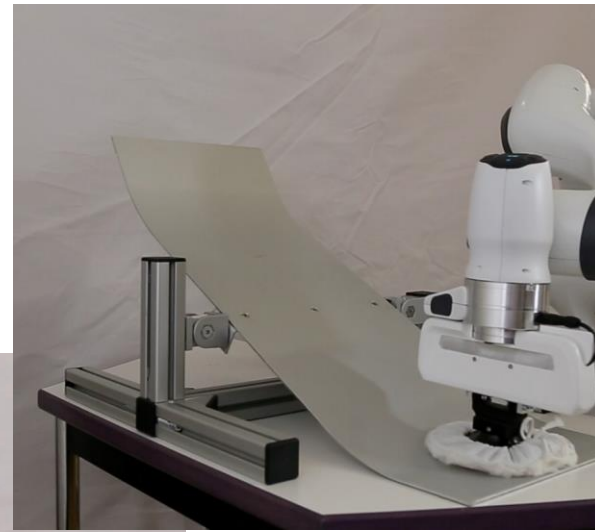
Hybrid Force Position Control

Classical approach: partition motion and force space via selection matrix Ω

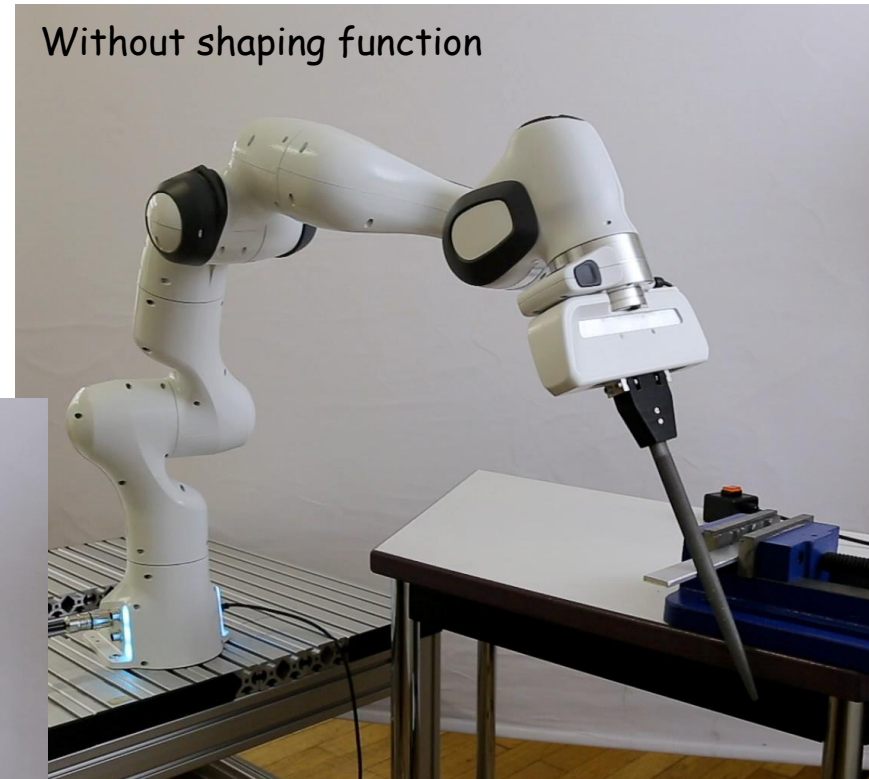
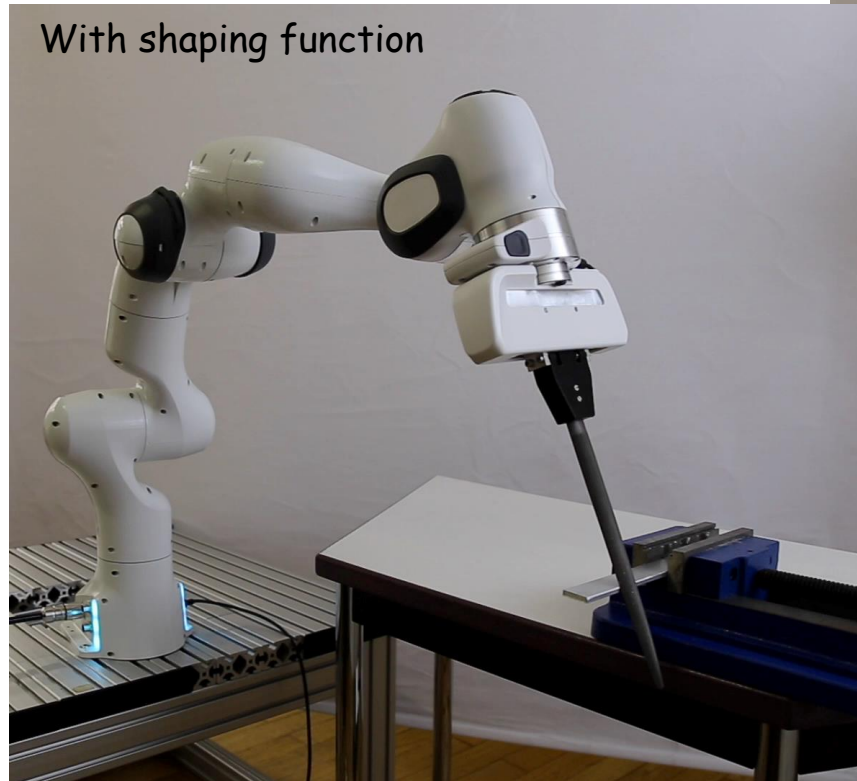


Disadvantage: Exact environment model has to be known!

Unified Force/Impedance Controller



Unified Force/Impedance Controller

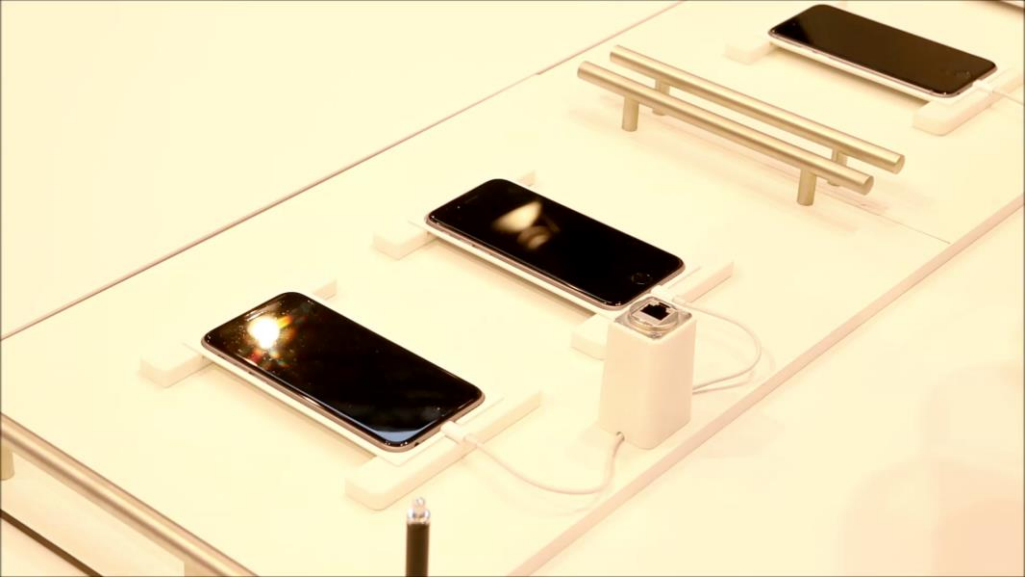
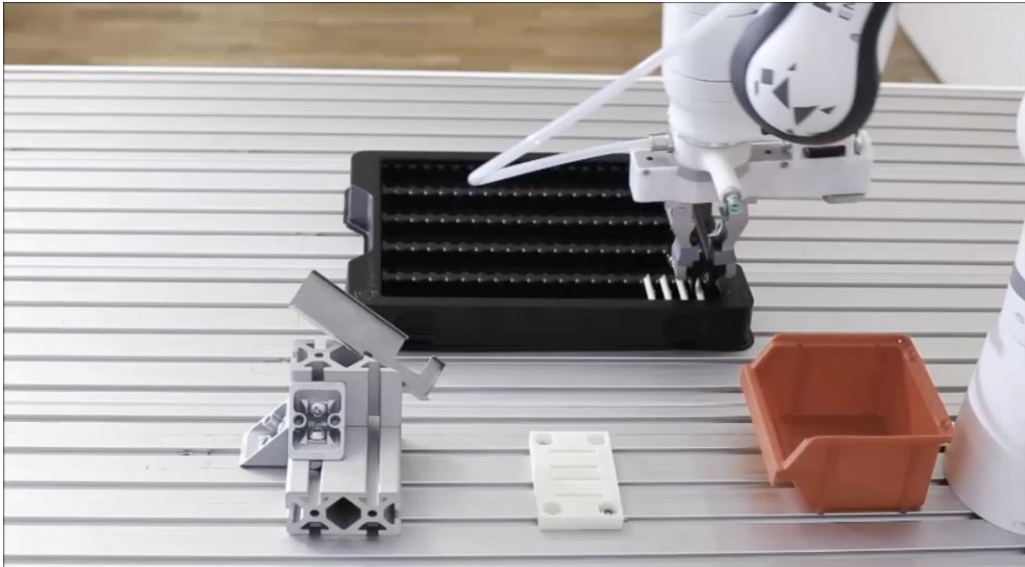
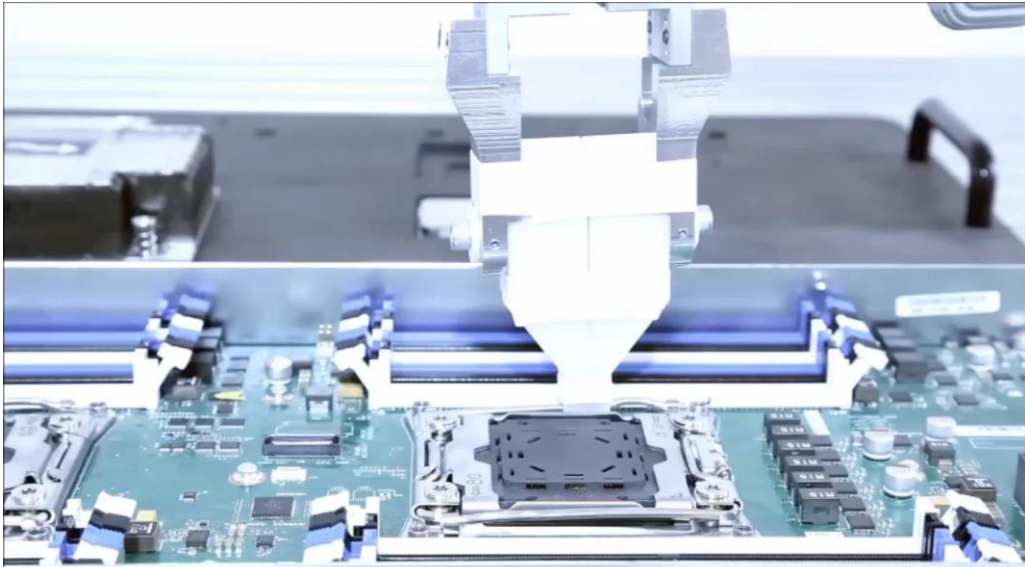


Assembly and Soft Manipulation

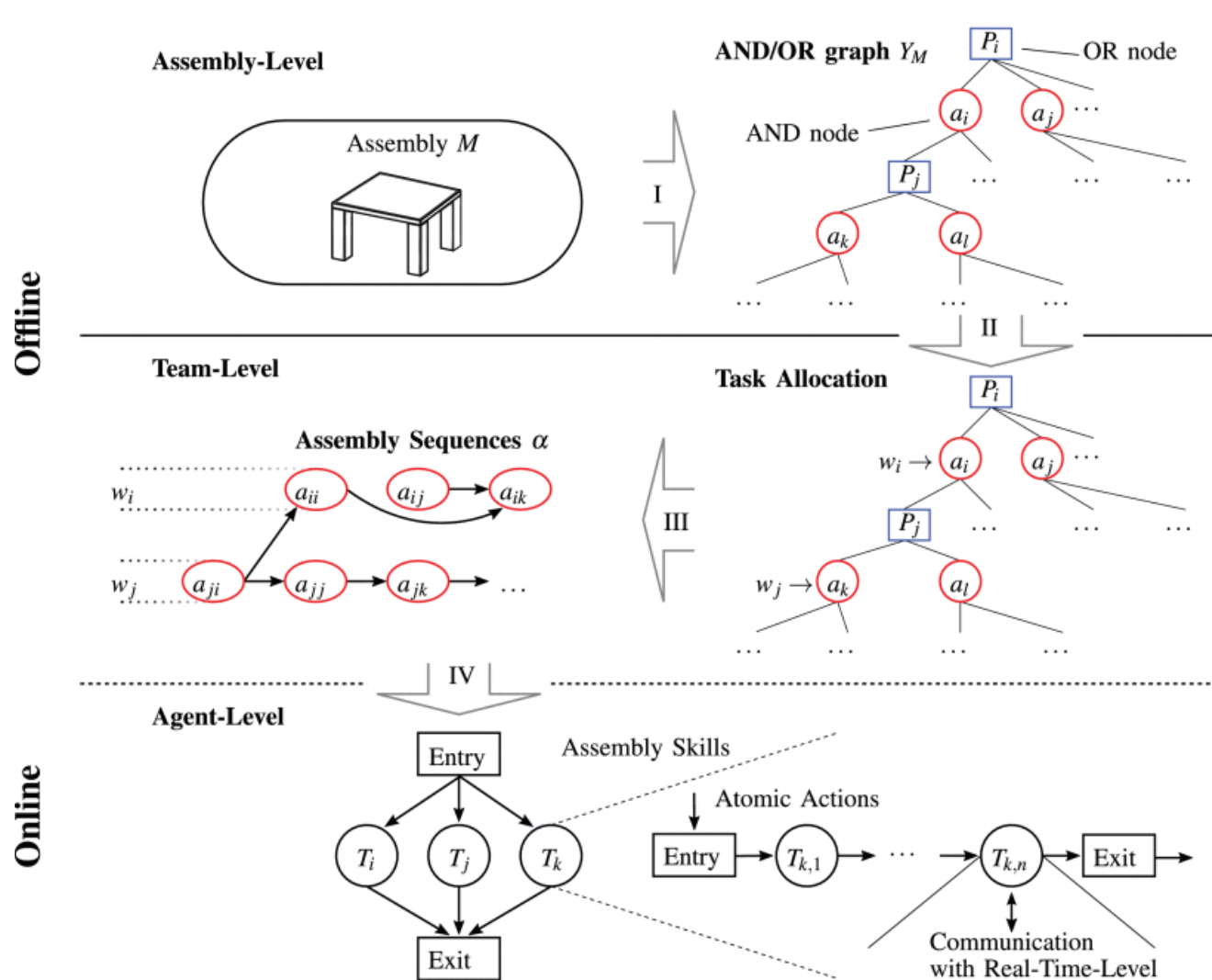
Assembly and Soft Manipulation



Assembly and Soft Manipulation

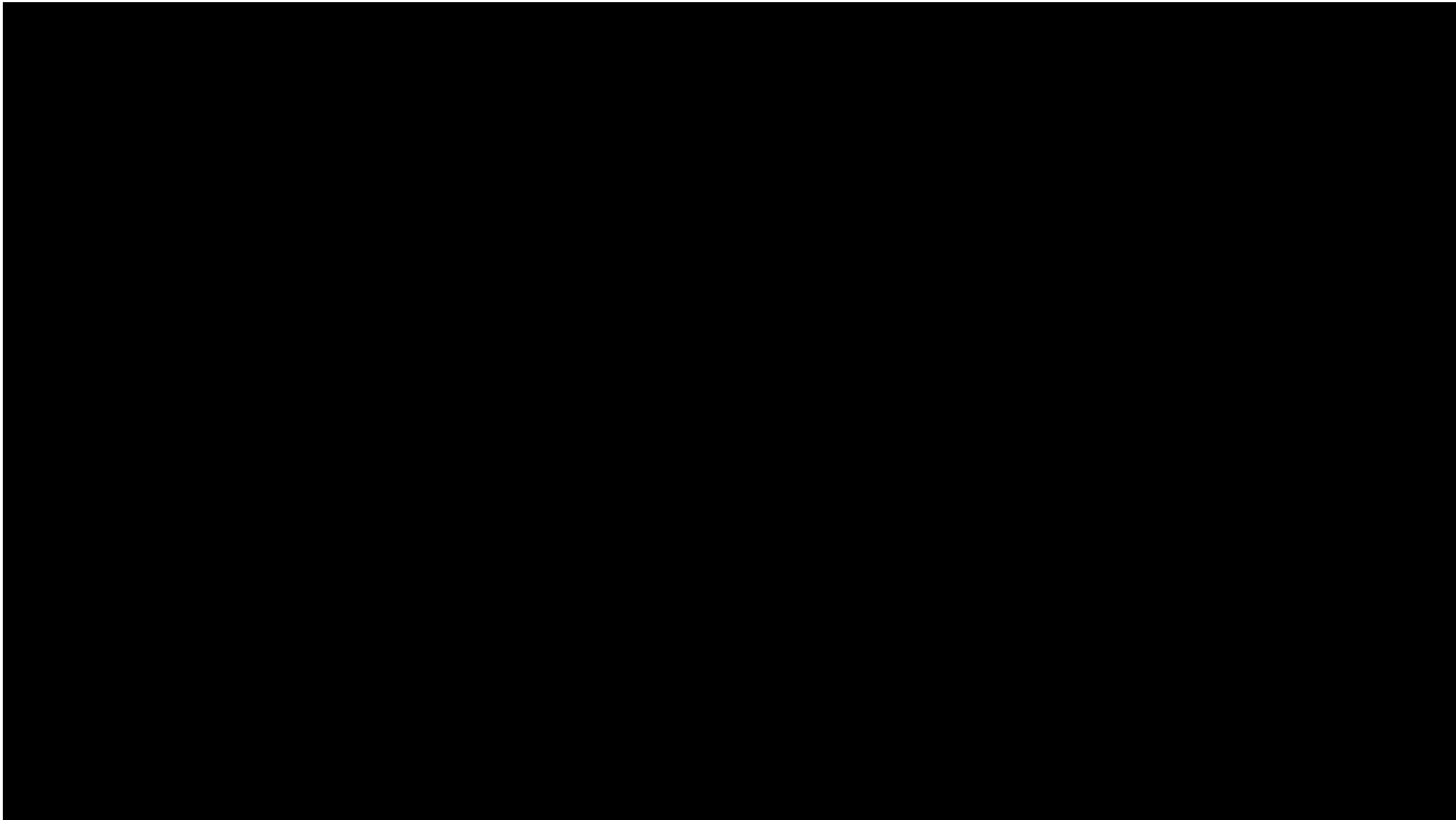


Assembly Planning



- Framework for multi-agent assembly
- Optimal assignment of agents to tasks is planned
- Local motion and manipulation planning

Assembly Planning



Assembly Planning

A Framework for Robot Manipulation:
Skill Formalism, Meta Learning and Adaptive Control

Lars Johannsmeier, Malkin Gerchow and Sami Haddadin

Institute of Automatic Control
Gottfried Wilhelm Leibniz Universität Hannover

irt



- Robot knows a general strategy
- Controller and skill parameters are learned
- Parameter space limits are derived from known system limits and task context

