A Current Comparator System to Establish the Unit of Electrical Energy at 60 Hz

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Abstract—A compensated current comparator system to establish the United States legal unit of electrical energy at the National Bureau of Standards at energy levels of approximately 30 and 60 kJ is described. Analysis of the system uncertainties and experimental data indicates that the registrations of three standard type watthour meters were determined with total estimated uncertainties of about 30 ppm at unity power factor (PF) and 40 ppm at 0.5 PF. Of these uncertainties, 18 ppm represents the three standard deviation bound for the effects of random errors, and the remainder the root sum of squares of bounds to possible calibration and systematic effects. These results indicate that it should now be possible to disseminate the energy unit with uncertainties less than the presently quoted 500 ppm.

INTRODUCTION

THE unit of ac electrical energy established, maintained, and disseminated by the National Bureau of Standards (NBS) is used by watthour meter (WHM) manufacturers, electric utilities, and state utility commissions to insure the accuracy of the 75×10^6 WHM's which register the electrical energy sold in the United States. At the present time, the annual revenue from the sale of this energy exceeds 20×10^9.

The energy unit has previously been established and disseminated with an uncertainty that was within 500 ppm. This uncertainty was appropriate when considered with respect to the performance expected from both standard- and service-type WHM's. Recently, however, there have been improvements in both meter types, and a more accurate unit of energy is justified. Furthermore, statistical WHM testing techniques, adopted by many electric utilities as a substitute for periodic testing of all meters, have demonstrated the need for an improved energy standard.

Traditionally, a constant power-time interval method has been used at NBS to establish the unit of ac energy. This method makes use of an electrodynamic wattmeter to transfer a power measurement from dc to ac [1]. The wattmeter is then used to measure a constant ac power during an accurately measured time interval to calibrate a group of standard WHM's with an amount of energy which is known in terms of the units of voltage, resistance, and time interval as maintained at NBS. The established energy unit is maintained by the group of WHM's between calibrations.

This paper describes a current comparator (CC) system to establish the unit of electrical energy at 60 Hz at energy levels of approximately 60 kJ (PF = 1.0) and 30 kJ (PF = 0.5) with total system uncertainties less than 100 ppm. This new system also operates on a constant power-time interval principle, but differs from the standard wattmeter method in that the voltage and current of the phantom power load are separately established and maintained at fixed levels. In the CC method, a small current is measured by the voltage drop across a known resistance and scaled up to the desired level by a compensated current comparator. Both current and voltage are established and monitored by a technique which involves customary potentiometric measurements and dc/ac transfer by thermoelements.

Some advantages of the method are:

1) A wattmeter mechanical deflecting system is not required.

2) A compensated current comparator is used for current scaling; hence, the power dissipated in a current determining resistor is small.

3) The system balance is independent of phase angle.

4) An electrothermic instrument is used for dc/ac voltage transfer.

5) The system components are stable and frequent calibrations are not required.

6) The established energy unit is referred directly to the NBS maintained units of voltage, resistance, and time interval.

BASIS OF THE CC METHOD

Constant power supplied to a WHM at 120 V and 5 A for a time interval of 108 s establishes a nominal 64.8-kJ energy level at unity PF, and a nominal 32.4-kJ level at 0.5 PF, current lagging voltage.

The choice of the 108 s is largely a matter of convenience. The WHM's studied have light sensitive detectors whose light sources are interrupted by rotating disks which contain 1000 slots. At unity PF and rated excitation, the disks rotate at a nominal rate of $16\frac{2}{3}$ r/min, and, for a time interval of 108 s, will make about 30 revolutions. With 1000 slots in the disk, a total of about 30 000 pulses will be obtained during this time.

The final quantity of interest is the WHM registration, $\mathcal{R}$, defined as

$$\mathcal{R} = \frac{\text{indicated energy}}{\text{applied energy}} = \frac{W_i}{W_a} \simeq \frac{W_i}{W_{an}} (1 - C_{wa}) \quad (1)$$
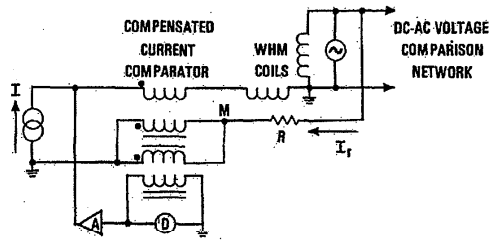
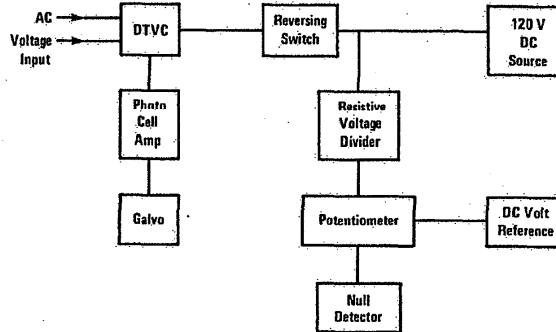Fig. 1. Simplified schematic of unity power factor system.

Fig. 2. DC–AC voltage comparison instrumentation.

where W_{an} is the nominal applied energy in joules, and C_{wa} is a small correction in proportional parts. In terms of the total pulse count, p, the indicated energy is $W_i = p \times 2.16$ J/pulse, and the registration at unity PF is

$$\Re \simeq \frac{p \times 2.16}{64.8 \times 10^3} (1 - C_{wa}) = \frac{p}{3 \times 10^4} (1 - C_{wa}). \qquad (2)$$

At 0.5 PF, the registration expressed in terms of the pulse count is

$$\Re \simeq \frac{2p}{3 \times 10^4} (1 - C_{wa}). \qquad (3)$$

The 1-s interval pulses from the in-house standard frequency are counted on a counter operated so that it will measure any selected time interval. At the beginning of the seconds count, a gating signal starts the totalizers which count pulses from the WHM's. At the end of the selected time interval, the gating signal is removed and the pulse count for each WHM is obtained. The sum of counts from the WHM's is also totalized by means of electronic summing circuits, and the average registration is calculated. This group mean is used to monitor the stability of the WHM's which maintain the energy unit.

A simplified schematic of the system, connected for unity PF operation, is shown in Fig. 1. Three WHM's are connected together for simultaneous calibration in the position indicated in the figure as "WHM Coils". The timing and pulse count circuits have been omitted.

A two-phase oscillator supplies the constant voltage and current amplifiers (not shown) which serve as the voltage and current sources. The phase angle between the oscillator reference and variable phase outputs is continuously variable throughout the range 0–π rad, in both lead and lag directions.

The compensated current comparator [2] is operated with one turn as the primary winding, and the secondary and compensation windings each have 100 turns. Hence, the current in the resistor, R, is scaled upward by a factor of 100 for application to the WHM current coil, and the power dissipated in the resistor is small.

The resistor has two nominally equal sections of 4.8 kΩ. One section is used for 0.5-PF operation, and the two sections are connected in parallel for unity PF. R is constructed of card-wound Evanohm wire [3], and the resistance–temperature characteristic is expressed as

$$R_T = R_{25}[1 + \alpha(T - 25) + \beta(T - 25)^2]. \qquad (4)$$

Values of the coefficients α and β were computed from low-power dc resistance measurements at three temperatures, and a table of resistance versus temperature was computed using these coefficients. Thermocouples were attached to the resistor sections so that wire temperature could be measured, and the dc values of the sections were measured in a bridge [4] with the equivalent of 120-V rms applied to the resistor. The measured values agreed with the computed table values within 5 ppm. The dc/ac resistance differences were also determined [5] because R is used for dc/ac current transfer.

Fig. 2 shows the instrumentation used to establish the ac voltage applied to the WHM's. DTVC is a differential thermocouple voltage comparator [6] containing two
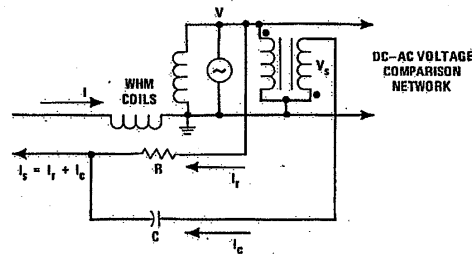
Fig. 3. Simplified schematic of 0.5-PF system.

nominally identical thermal voltage converters (TE's). The 120-V dc input to the DTVC is established by means of the voltage divider, potentiometer, and dc reference voltage. The two TE's are first connected in parallel to the dc voltage source so that their voltage outputs may be adjusted to equality. One TE is then switched to measure the ac voltage (V of Fig. 1) which is adjusted until a DTVC null is again obtained. Hence, $V = V_{dc}(1 + \delta)$, where V_{dc} is the corrected dc input, and δ is the dc/ac difference of the DTVC in proportional parts. To eliminate dc reversal errors in the TE's, WHM tests are taken in pairs with the dc input to the DTVC reversed between tests. The average count from the "direct" and "reversed" dc tests is taken as the calibration result.

With the chosen value of R (2.4 kΩ) for unity PF, $I_r = 120/(2.4 \times 10^3) = 50$ mA at 120 V. Hence, the current in the primary of the CC and in the WHM current coils is established at $I \simeq nI_r = 100 \times 50 \times 10^{-3} = 5$ A. At a current level of 50 mA, the resistor dissipates only 6 W, and is sufficiently cooled with a small fan, since operation is in a temperature controlled room. At unity PF the measured resistor operating temperature was 27.5°C. The CC null balance is obtained by simultaneously adjusting the current source and the phase angle between the current and voltage sources.

The CC feedback network in Fig. 1 is used to compensate for small instabilities in V and I. This network amplifies the CC unbalance voltage and injects a small correcting current into the current circuit. This technique also insures that the M point is kept near ground potential. Hence, the voltage across the WHM coils, V to ground, and the voltage across the resistor, V to M, are almost equal. The voltage M to ground was measured, and a correction due to this voltage was included in the expression for C_{wa}. The measured magnitude of the M point voltage, for both unity and 0.5-PF operation, was $M = 150 \cdot \mu$V. This amounts to a correction of $\psi = M/V = (150 \times 10^{-6})/120 \simeq 1.3$ ppm.

The expression for the energy applied to the WHM's at unity PF is given by

$$W_a \simeq W_{an}(1 + C_{wa})$$

$$\simeq (ntV_{dc}^2/R_n)[1 + 2\delta - (\psi + \Delta + \zeta + \eta)], \qquad (5)$$

where W_a equals the applied energy in joules, n equals the current comparator turns ratio, t equals the time interval in seconds, V_{dc} equals the corrected dc voltage at DTVC dc input terminals, R_n equals the nominal dc resistor value, δ equals the DTVC dc/ac difference, ψ equals the measured correction for M point potential, Δ equals the resistor dc/ac difference, ζ equals the correction to R_n, and η equals the current comparator in-phase correction to nominal ratio. The magnitude of the correction at unity PF is

$$C_{wa} \simeq 2(0) - [1.3 + (-5)$$

$$+ (-13.4) + .1] \times 10^{-6} = 17 \text{ ppm.} \qquad (6)$$

The manner in which the CC system is operated at 0.5 PF is shown in Fig. 3. For simplicity, the CC network and current source to the left of the WHM current coil and M point (see Fig. 1) are not shown. The dc/ac voltage comparison instrumentation is the same as in Fig. 2. The 0.5 PF circuit incorporates a phase inverting voltage transformer whose secondary is connected in series with a capacitor, C. This modification essentially produces a negative capacitance; hence, the WHM current I lags the WHM voltage V. The negative C technique avoids the instability problems that would arise if the lagging current had been obtained by means of an inductor. Now, $I_s = I_r + I_c$ and, since $R = 4.8$ kΩ and $I_r = 25$ mA at 120 V, C is chosen so that I_s is 50 mA and the phase angle between V and I is $\pi/3$ rad.

The test procedure is the same as for unity PF. V is established in terms of V_{dc}, then I and the phase angle between V and I are adjusted until the CC is brought to null. The feedback network again maintains the CC null during a 108-s interval.

The expression for the energy applied to the WHM's at 0.5 PF is similar to that for unity PF. However, corrections for the capacitor conductance and transformer phase angle must be included. These additional terms enter from the expression $I_c = V_sY_c = -V(1 + \alpha + j\beta)(G_c + j\omega C)$, and, since the WHM's respond only to active and not reactive power, $\text{Re}\{I_c\} = -VG_c(1 + \alpha - \beta/D)$, where G_c is the capacitor conductance in siemens, D is the capacitor dissipation factor in radians, and α and β are as previously defined. The product $G_c\alpha$ is

TABLE I
UNITY POWER FACTOR UNCERTAINTIES IN PPM

Item	Magni-tude	Sub-total	Total	Uncertainty Source
DC Voltage				
standard cell	1			calibration
potentiometer	3			calibration
voltage divider	1			calibration
reversing switch	0.5			see text
dc power supply regulation	0.5			see text
root sum of squares		3		
AC Voltage				
dc/ac difference of DTVC		3		calibration
root sum of squares (dc + ac)		4		
Total Voltage 2 × sum (dc + ac)			8	
Current Ratio				
CC unbalance	0.1			see text
CC corrections	1			calibration
M point voltage	0.2			see text
detector current	0.6			see text
root sum of squares			1	
Resistor				
dc/ac difference	.3			calibration
dc value	5			see text
root sum of squares			6	
Total Systematic Uncertainty Bound				
Square Root of Sum of Squares			10	

negligibly small; hence,

$$\text{Re}\{I_c\} \simeq -VG_c(1 - \beta/D). \quad (7)$$

The expression for the energy applied to the WHM's at 0.5 PF is then

$$W_a \simeq W_{an}(1 + C_{wa}) = (ntV_{dc}^2/R_n)[1 + 2\delta$$
$$- (\psi + \Delta + \zeta + \eta) - G_cR_n(1 - \beta/D)] \quad (8)$$

where all terms are as previously defined. Using this expression, the correction to the applied energy is

$$C_{wa} \simeq 2\delta - (\psi + \Delta + \zeta + \eta) - G_cR_n(1 - \beta/D)$$
$$= [9.8 - 196 - 525] \times 10^{-6} = -711 \text{ ppm.} \quad (9)$$

UNCERTAINTIES

The uncertainties in the process of calibrating watthour meters with the CC system involve the random variations in the process combined with the offset from the correct values due to systematic effects. The latter include inaccuracies in the reference standards used, plus biases arising from individual properties of instruments which, for a given measurement, combine to give a unique value for the displacement of the center of the probability distribution of the random errors of the measurement process. A bound for the possible effect of these systematic biases [an interval $(-E,E)$ for symmetrical bounds] is needed. To this bound, a limit (e.g., three times the standard deviation of the distribution of random errors) must be added to achieve a total uncertainty to be attached to the process.

The estimated bounds to possible uncertainties from systematic effects in the CC system at unity PF are listed in Table I. Contributions from voltage, current ratio, and resistor items are evaluated from information supplied with calibrated items or from direct measurement as indicated in the table. Component drifts or ageing effects between calibrations are not included.

In the dc voltage section, the value for the "reversing switch" is an allowance for the voltage drop through the switch, since (see Fig. 2) the voltage is measured at the switch input and not at the DTVC input. With a current of 20 mA and a switch resistance of 3 mΩ, this contribution is 0.5 ppm. The "dc power supply regulation" item arises because one TE is switched to the ac voltage; the measured voltage change at the switch input is ≤ 0.5 ppm. These possible systematic uncertainties are combined in a root sum of squares manner. Since the applied energy is proportional to V^2, twice the sum of the dc and ac voltage uncertainties is used.

In the current ratio section, the "CC unbalance" item arises from the fact that null is not precisely achieved. At unity PF, the feedback network limits the maximum null voltage to 20 μV, resulting in an unbalance uncertainty < 0.1 ppm. The "M point voltage" item arises from the measured 1.3-ppm correction term for the voltage difference between M and ground. Assuming a 10-percent measurement uncertainty in this correction (based on manufacturer's stated full scale accuracy of the voltmeter used), a 0.2-ppm allowance is tabulated. The "detector current" item arises from the contribution to the primary current resulting from the unbalance current in the detection circuit, and is approximated by $N_DV_d/N_pI_pZ_d$, where N_D/N_p is the ratio of detector to primary turns, V_d is the detection winding voltage, and Z_d is the input impedance of the CC null detector. This uncertainty amounts to $1.2 \times 10^{-2}V_d$, and for the worst case at unity PF (20 μV), the uncertainty is $<$ the allowed 0.6 ppm.

TABLE II
0.5-PF UNCERTAINTIES IN PPM

Item	Magnitude	Subtotal	Total	Uncertainty Source
DC Voltage				
same as Table I		3		see Table I
AC Voltage				
dc/ac difference of DTVC		3		calibration
root sum of squares (dc + ac)		4		
Total Voltage 2 × sum (dc + ac)			8	
Current Ratio				
CC unbalance	1.0			see text
CC corrections	1.0			calibration
M point voltage	0.2			see text
detector current	3.8			see text
root sum of squares			4	
Resistor				
same as Table I			6	see Table I
Voltage Transformer				
Quadrature Correction	5.3		5.3	calibration
Capacitor Conductance	19.6		19.6	calibration
Total Systematic Uncertainty Bound				
Square Root of Sum of Squares			23	

As an overall bound to the possible uncertainty of systematic effects from each of these known sources, a root sum of squares value is computed to be 10 ppm. In the sequence of measurements made by the process, each of the items listed represents an offset from the intended value. These offsets are persistent and cannot be reduced by measurements internal to the process.

Table II lists the estimated uncertainties for the CC system at 0.5 PF. The only additional terms are those for the "voltage transformer quadrature correction" and the "capacitor conductance" calibrations. The effect of the transformer's quadrature correction, β, is to introduce a correction of 525 ppm into the expression for the applied energy. Since β has a calibration uncertainty of 1 percent, an uncertainty of 5.3 ppm is listed. The corresponding conductance uncertainty is $\leq$ 10 percent, and the energy correction due to conductance is 196 ppm; hence, 19.6-ppm uncertainty is listed.

RESULTS

The average registrations of three standard type WHM's were determined with the CC energy system at unity and 0.5 PF. For each PF, 50 observations were made, and the mean result of the observations has a computed standard deviation of 6 ppm for both PF conditions. By using 3 standard deviations as the bound for the effect of random errors, the limit becomes 3 × 6 = 18 ppm. For the total uncertainty, allowance is made for a possible shift in the center of the random error probability distribution. Hence, the unity PF values obtained with the CC system are believed to have a total uncertainty of 10 + 18 = 28 ppm. Similarly, the total uncertainty of the 0.5 PF values is believed to be 23 + 18 = 41 ppm.

The WHM's were also calibrated with the standard wattmeter system, and the average registrations of 28 observations obtained at each PF condition yielded a standard deviation of 16 ppm.

CONCLUSIONS

The results of the work reported in this paper indicate that the application of a compensated current comparator system to establish the national unit of ac electrical energy appears to be a significant improvement over the NBS standard wattmeter method. From the analysis of estimated uncertainties, it appears that the unit of energy can now be established with uncertainties less than 100 ppm.

Since the standard deviation of the mean of a group of runs, using the standard wattmeter method amounted to as much as 16 ppm, in contrast to the 6-ppm value for the CC method, it appears that it should now be possible to better evaluate the instability of standard type watthour meters.

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