where $K_c$ is the thermal conductance from the heater to the TC base region through the insulating bead and thermocouple, and $K_2$ is the thermal conductance of the heater.

The temperature at the midpoint of the heater is given approximately by $t_a \sim t_d \approx e_o/S_c \approx \lambda I^2/S_c$, and hence,

$$\delta_c = (S_c/2\lambda)\,[4/(4 + K_c/K_2)]\,(\partial\pi_{12}/\partial T)\,(\pi_{12}/K_1^2). \tag{18}$$

From the Kelvin relations, $\partial\pi_{12}/\partial T$ may be rewritten as

$$\partial\pi_{12}/\partial T = \pi_{12}/T + \tau_1 - \tau_2 \tag{19}$$

and substituting in (18),

$$\delta_c = (S_c/2\lambda)\,[4/(4 + K_c/K_2)]\,(\pi_{12}/T + \tau_1 - \tau_2)\,(\pi_{12}/K_1^2). \tag{20}$$

$\delta_d$ Error due to second-order Thomson heat in the support leads arising from Peltier heat in the heater/support-lead junctions.

The temperature gradient in each support lead will give rise to a second-order Thomson heat component of $-\tau_1 I\Delta T_j$, which in turn leads to a second-order change of $-\tau_1\pi_{12}I^2/2K_1^2$ in the temperature at each junction. Following a similar procedure to that given above for $\delta_c$, it follows that

$$\delta_d = -(S_c/\lambda)\,[4/(4 + K_c/K_2)]\,(\tau_1\pi_{12})/4K_1^2. \tag{21}$$

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REFERENCES

[1] F. L. Hermach, "Thermal converters as ac-dc transfer standards for current and voltage measurements at audio frequencies," *J. Res. NBS*, vol. 48, pp. 121-138, 1952.
[2] F. C. Widdis, "The theory of Peltier and Thomson effects in thermal ac-dc transfer devices," *Proc. Inst. Elect. Eng.*, Monograph 497M, pp. 328-334, 1962.
[3] B. D. Inglis, "A method for the determination of ac-dc transfer errors in thermoelements," *IEEE Trans. Instrum. Meas.*, vol. IM-27, pp. 440-444, 1978.
[4] F. L. Hermach and E. S. Williams, "Thermal converters for audio frequency voltage measurements of high accuracy," *IEEE Trans. Instrum. Meas.*, vol. IM-15, pp. 260-268, 1966.
[5] B. D. Inglis, "Errors in ac-dc transfer arising from a dc reversal difference," *Metrologia*, vol. 17, pp. 111-117, 1981.
[6] —, "AC-DC transfer standards—present status and future directions," *IEEE Trans. Instrum. Meas.*, pp. 285-290, this issue.
[7] —, "Ac-dc transfer measurements with a precision of <0.1 ppm," *Metrologia*, vol. 18, pp. 133-138, 1982.
[8] F. J. Wilkins, T. A. Deacon, and R. S. Becker, "Multijunction thermal converter," *Proc. Inst. Elect. Eng.*, vol. 112, pp. 794-806, 1965.
[9] A. L. Bond, F. M. Gay, and R. G. Borley, "Multijunction thermocouples," British Patent Spec. 1 357 217, Patent Office, London, 1974.
[10] D. Zhang and Z. Zhang, "Method for reduction of ac-dc transfer error caused by the Thomson effect for the multijunction thermal converter," *IEEE Trans. Instrum. Meas.*, vol. IM-29, pp. 412-415, 1980.

# A Test of the Quantum Hall Effect as a Resistance Standard

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*Abstract*—This paper demonstrates that the quantum Hall effect can be used to monitor a laboratory unit of resistance. A 6453.2-Ω room temperature reference resistor was calibrated relative to two quantum Hall effect devices with a 0.017-ppm (1 σ) uncertainty for each 1 h measurement period. This accuracy was achieved by correcting for a measurement system offset error and for the temperature dependences of each quantum Hall device. Hamon series-parallel resistor networks were then used to calibrate the 6453.2-Ω resistor in terms of the five one ohm resistors which comprise the NBS ohm. The total 1σ accuracy for the transfer between the quantum Hall devices and the 1-Ω resistors was 0.047 ppm.

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## I. INTRODUCTION

THE HALL resistance $R_H$ of a two-dimensional electron gas is, under certain conditions, quantized in units of $h/e^2$ [1]:

$$R_H(i) = \frac{V_H}{I} = \frac{h}{e^2 i} = \frac{25812.80}{i} \tag{1}$$

where $V_H$ is the Hall voltage across the sample, $I$ is the current through the sample, $h$ is the Planck constant, $e$ the elementary charge, and $i$ is an integer quantum number. Ultimately one would like to determine the value of $R_H$ in order to verify that it is indeed equal to submultiples of $h/e^2$ [2] and thus depends only on fundamental constants of nature. If this were proved

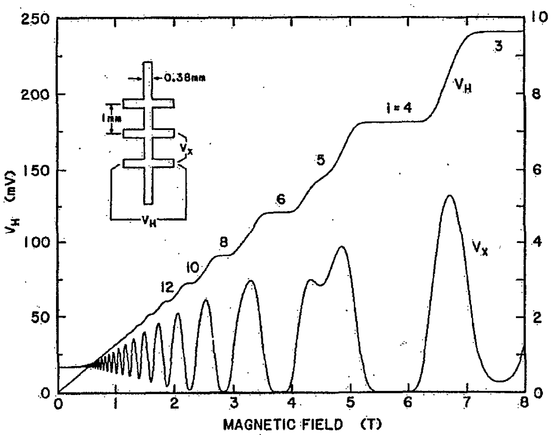


Fig. 1. Recording of $V_H$ and $V_x$ versus magnetic field for a GaAs device cooled to 1.2 K. The current is 25.5 μA.

to be correct, then the quantum Hall effect could be used as an absolute resistance standard.

An an intermediate step in achieving this goal it is necessary to demonstrate that the quantum Hall effect can be used as a relative standard to maintain a laboratory unit of resistance based on wire-wound resistors analogous to the way in which the ac Josephson effect is used to maintain a laboratory unit of voltage. To be of practical use this standard would need to be capable of calibrating resistors to a relative accuracy of a few parts in $10^8$. We demonstrate that this requirement is indeed achievable.

## II. Quantum Hall Effect Measurements

Two high-quality, quantum Hall effect devices were used; both were GaAs-$Al_xGa_{1-x}As$ heterostructures grown by molecular beam epitaxy by A. C. Gossard at AT&T Bell Laboratories, and then prepared into Hall bars and screened by D. C. Tsui at Princeton University, Princeton, NJ. They are 4.6 mm long and 0.38 mm wide, and have three sets of potential probes, with two sets symmetrically displaced ±1.0 mm along the channel from the center set. Their zero magnetic field mobilities were $\sim 10^5$ $cm^2/(V \cdot s)$ at 4.2 K. Both devices were optimized for the $i = 4$ Hall step, where $R_H(4) \approx 6453.2$ Ω. Fig. 1 shows a low-sensitivity recording of $V_H$ and $V_x$ versus magnetic field for one of the devices for a current of 25.5 μA at 1.2 K. The $V_H$ and $V_x$ plots were equally as good for the second device.

The first part of the experiment consisted of making potentiometric comparisons of the quantum Hall voltage $V_H$ with the voltage drop $V_R$ across a series-connected room temperature reference resistor using the measurement circuit indicated in Fig. 2. The wire-wound reference resistors were adjusted to have a resistance within a few parts-per-million (ppm) of $R_H$. They were then hermetically sealed in silicone-fluid-filled containers and placed in temperature regulated enclosures controlled to within ±0.002°C of a nominal temperature of approximately 28°C.

To measure $R_H$ in terms of a reference resistor, the potentiometer voltage is made almost equal to the voltage drop across $V_H$ or $V_R$. An electronic detector, $D$, with an input

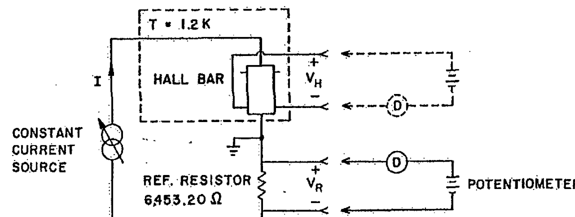


Fig. 2. A simplified schematic of the measurement circuit.

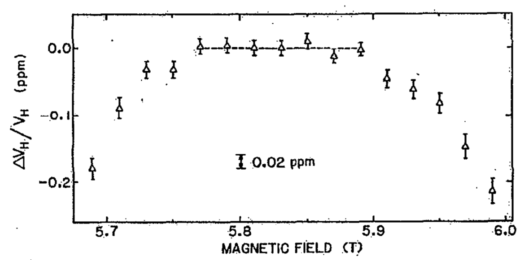


Fig. 3. A 6453.2-Ω GaAs quantum Hall step measured to high precision at 1.2 K, and $I = 25.5$ μA.

current less than $10^{-15}$ $A$, amplifies the difference-voltage signal. (Note that the potentiometer does not require calibration in this arrangement.) The current source, potentiometer, and electronic detector are all battery operated. Thermally induced voltages and linear drifts in the current source and the potentiometer are cancelled by reversing the current through the device and the reference resistor. A series of reversals in the order + - - + is made for each of two measurements of $V_H$ which bracket in time one measurement of $V_R$ in order to obtain a single data point.

Fig. 3 shows a high resolution mapping of the $i = 4$ Hall step for one of the GaAs devices cooled to 1.2 K with $I = 25.5$ μA. Each data point was obtained in one hour using the procedure described above, with a ±0.011 ppm random, or type $A$, measurement uncertainty. This Hall step is flat to within ±0.01 ppm over a range in magnetic field that is 2 percent of the central field value. The $i = 4$ step of the second GaAs sample was equally as flat, had a nearly identical shape, and occurred at 6.02 $T$ central field. Both devices clearly have Hall step shapes that make them suitable for use as resistance standards.

## III. Quantum Hall Effect Results

One of the room temperature, 6453.2 Ω, reference resistors was compared with two different Hall probe sets of each of the two GaAs devices for both magnetic field directions over a 12-month time period starting in May 1983. The results are shown in Fig. 4, where $\Delta R/R = (V_H - V_R)/V_R$. For a measurement time of 1 h the data typically had a ±0.011-ppm random uncertainty, and was corrected for a measurement system offset error [3] which was sometimes as large as (0.025 ± 0.013) ppm. This offset error was determined by replacing the Hall device by a second room temperature 6453.2 Ω resistor and then intercomparing the two resistors with the measurement system. Also, a correction for the temperature depen-

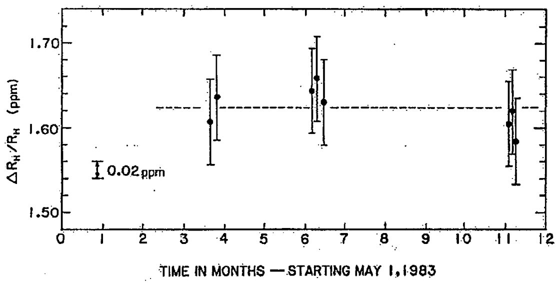


Fig. 4. Relative comparisons as a function of time of a 6453.2-Ω room temperature reference resistor with two different quantum Hall devices.

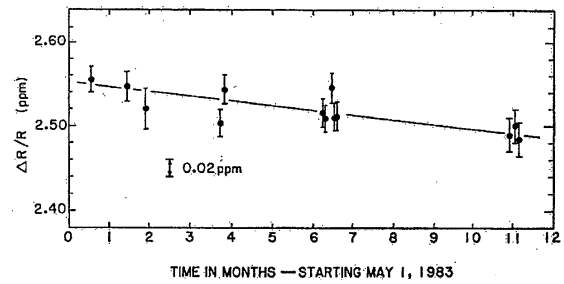


Fig. 5. Monitoring as a function of time the value of $R_H$ expressed as a difference in ppm from a nominal value of 6453.2 $\Omega_{NBS}$.

dence [4] of $R_H$ for each sample was applied to every measurement, the largest correction being (0.026 ± 0.002) ppm. No current dependence [5] was observed for $I \leq 25.5$ μA, so no correction for finite I was required. Our one standard deviation uncertainty, the root-sum-square of the above three uncertainties, is ±0.017 ppm.

The corrected data were independent of the Hall device, the Hall probe set, and the magnetic field direction, so they are not distinguished in Fig. 4. From a least squares fit, these data indicate that the value of the reference resistor is decreasing at the rate of (1.81 ± 0.46) parts in $10^{10}$ per day or (0.066 ± 0.017) ppm/year.

## IV. Step-Down to the NBS Ohm

The second part of the experiment consisted of calibrating the 6453.2-Ω resistors in terms of the set of five one ohm resistors which comprise the NBS ohm. To carry out this calibration we constructed two 6453.2 to 100 Ω series-parallel Hamon resistor networks [6] consisting of eight 800-Ω resistors plus a series-connected 53.2-Ω resistor. The eight 800-Ω resistors in the parallel (100 Ω) configuration, as well as the 53.2-Ω resistor, were each compared with the series (100 Ω) configuration of an existing 100 to 1 Ω Hamon resistor network using a dc current comparator resistance bridge. The 53.2-Ω resistor need only be measured within 1.2 ppm to achieve a 0.01 ppm accuracy for the 6453.2-Ω Hamon network. The transfer from 1 $\Omega_{NBS}$ to 6453.2 Ω is estimated to be accurate to within 0.044 ppm (one standard deviation) by this method.

Measurements involving the entire sequence (quantum Hall resistance compared with the 6453.2-Ω resistors and then stepped-down to $\Omega_{NBS}$) have been made since August 1983. Fig. 5 shows the results of these measurements. The total one standard deviation root-sum-squared uncertainty of this sequence is typically ±0.047 ppm, and the mean value of $R_H$ is (1.627 ± 0.047) ppm larger than a nominal value of 6453.2 $\Omega_{NBS}$. (An uncertainty of 0.047 ppm was assigned to the mean value because the resistance step-down errors are mostly systematic, or type B.)

## V. Conclusions

Although the precision and reproducibility of these measurements is 0.047 ppm, the inaccuracy with respect to the SI ohm is at least 0.5 ppm due to possible drifts of the NBS ohm since it was determined in SI units in 1974 via a calculable capacitor experiment [7]. Also, many factors such as the temperature [4] and current [5] dependencies must be closely examined and understood before we can confidently use the quantum Hall effect as an absolute resistance standard. Clearly, however, quantum Hall resistance devices can be used to monitor or maintain a laboratory unit of resistance to the level of accuracy required at national standards laboratories.


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## References


[1] K. v. Klitzing, G. Dorda, and M. Pepper, "New method for high-accuracy determination of the fine-structure constant based on quantized Hall resistance," *Phys. Rev. Lett.*, vol. 45, pp. 494–497, Aug. 1980.

[2] D. C. Tsui, A. C. Gossard, B. F. Field, M. E. Cage, and R. F. Dziuba, "Determination of the fine-structure constant using GaAs-$Al_x$-$Ga_{1-x}$As heterostructures," *Phys. Rev. Lett.*, vol. 48, pp. 3–6, Jan. 1982.

[3] M. E. Cage, R. F. Dziuba, B. F. Field, R. J. Wagner, D. C. Tsui, A. C. Gossard, and N. N. Tadros, "Progress report on investigations of the quantum Hall effect as a possible resistance standard," *Com. Consult. d'Electricité, Com. Int. Poids Mes., Trav. 16ᵉ Sess.*, Doc. CCE/83-32, Mar. 1983.

[4] M. E. Cage, B. F. Field, R. F. Dziuba, S. M. Girvin, D. C. Tsui, and A. C. Gossard, "Temperature dependence of the quantum Hall resistance," *Phys. Rev. B.*, vol. 30, pp. 2286–2288, Aug. 1984.

[5] M. E. Cage, R. F. Dziuba, B. F. Field, E. R. Williams, S. M. Girvin, A. C. Gossard, D. C. Tsui, and R. J. Wagner, "Dissipation and dynamic nonlinear behavior in the quantum Hall regime," *Phys. Rev. Lett.*, vol. 51, pp. 1374–1377, Oct. 1983.

[6] B. V. Hamon, "A 1-100 ohm build-up resistor for the calibration of standard resistors," *J. Sci. Instrum.*, vol. 31, pp. 450–453, Dec. 1954.

[7] R. D. Cutkosky, "New NBS measurements of the absolute farad and ohm," *IEEE Trans. Instrum. Meas.*, vol. IM-23, pp. 305–309, Dec. 1974.