A Wideband Sampling Wattmeter[1]

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Abstract - The design and operation of a wideband sampling wattmeter capable of measuring distorted power signals with fundamental frequencies from 1 Hz to 10 kHz and harmonics up to 100 kHz is described. The microprocessor controlled wattmeter uses asynchronous sampling of the voltage and current signals. The errors associated with this type of operation are described as are various methods of correcting some of these errors. The wattmeter uses both a hardware multiplier-accumulator and a direct-memory-access unit to capture the data. Programmable time delay circuits are used to compensate for differential time delays between the input channels. Performance checks show that measurement uncertainties of less than ±0.1 percent of full-scale range (FSR) are obtainable.

INTRODUCTION

With the development of systems which produce electric power with highly distorted waveforms, there is an increasing need for accurate measurement techniques and calibration services that cover these higher frequency power signals. For example, the dc/ac converter power systems used in electric vehicles operate with fundamental chopping frequencies ranging from 1 to 10 kHz [1] and produce significant harmonics above 50 kHz on both the ac output as well as on the input power. To determine the efficiency of these power converters to the desired accuracy requires measurement of these signals with an uncertainty of less than one percent. To support the calibration of instruments for such applications, the National Bureau of Standards (NBS) undertook the development of instrumentation capable of measuring power in distorted signals having fundamental frequencies up to 10 kHz and harmonics up to 100 kHz with an uncertainty of less than ±0.1 percent of full-scale range (FSR).

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Several techniques available for making power measurements were considered as candidates for the new wattmeter. Time-division-multipliers have good accuracy at power line frequencies, but the accuracy of these techniques drops rapidly at frequencies above 1 kHz. Thermal-converter-based techniques also promised very good accuracy but again only at lower frequencies. Analog multipliers, on the other hand, have the necessary frequency response but not sufficient accuracy. The approach selected was the development of a dual-channel sampling wattmeter using custom designed input amplifiers, commercial track-and-hold (T/H) amplifiers, and analog-to-digital (A/D) converters. An earlier sampling wattmeter that measured signals up to 2 kHz had already been successfully developed at NBS [2]. The available commercial devices are able to provide the necessary 12-bit accuracy and 3 μs conversion time. Thus, the NBS sampling wattmeter can operate at sampling rates up to 300 kHz. Since the speed and accuracy of the commercially available T/H amplifiers and A/D converters improves each year, these units are designed to fit into a plug-in module and the rest of the instrument is designed to accommodate 16-bit converters and 1 μs conversion times. Thus, the accuracy of this instrument can be increased by designing new input modules.

The NBS sampling wattmeter makes use of a high speed digital multiplier-accumulator to generate the power data in real time and uses a fixed sampling rate. The principles of this approach have been described previously [3]. The NBS sampling wattmeter has enhanced this approach with the addition of circuitry designed to minimize the errors associated with the use of a fixed (nonsynchronized) sampling rate. Other sampling wattmeters have avoided this problem by using synchronous sampling [2,4,5]. The design of the NBS sampling wattmeter combines the simple fixed frequency sampling with trigger and sample counter circuits to minimize the truncation error problem described below. This design makes operation over a wide frequency range possible. The NBS sampling wattmeter measures dc signals and ac signals from 1 Hz to 100 kHz. Measurements at higher and lower frequencies are possible; however, frequencies which are close to multiples of half the sampling rate must be avoided. The amount by which these frequencies must be avoided depends on many measurement parameters, as described in the Theory section. Because the truncation errors are not a problem, the NBS sampling wattmeter does not require long integration times at lower frequencies to get accurate readings.

A microcomputer is used to control the operation and perform the necessary calculations in the NBS sampling wattmeter. This approach has been combined with the use of direct-memory-access (DMA) channels which store data from both channels to allow examination of the voltage and current waveforms. Algorithms have been developed which improve the accuracy with which the average and rms values of the voltage and current can be determined from the DMA data. These values are used to determine the gain and offset of each channel. The algorithms are described in the Operation and Correction Formula section. The NBS sampling wattmeter also makes use of programmable time delay circuits to compensate for differential time delays (relative phase differences) between the voltage and current channels. These delays can seriously affect the accuracy of wattmeters used to measure high frequency ac signals.

OPERATING PRINCIPLES

In the NBS sampling wattmeter the average power P for a periodic signal is calculated using the approximation [5] given by

$$P \approx W = \frac{1}{n}\sum_{k=0}^{n-1} V(t_k)\, I(t_k) \;, \qquad (1)$$

where $V(t_k)$ and $I(t_k)$ are uniformly spaced samples of the voltage and current, respectively, and W is the indicated power. The sampling times t_k are not synchronized with the input signal; thus, the summation interval will not, in general, coincide with an integral number of cycles of the input signal. This asynchronous sampling results in an error referred to as the truncation error. This error is minimized, however, by adjusting the number of samples n so that the difference between the summation interval and the time to complete an integral number of cycles of the input signal is less than one sample interval. The magnitude of the truncation error which results under these conditions is described in the Theory section below.

Figure 1 is a block diagram of the NBS sampling wattmeter which uses two identical amplifier/data converter modules optically isolated from the rest of the instrument. The optical isolation interface is indicated by the dashed lines in the figure. The optical isolation feature provides the sampling wattmeter with a high common mode rejection capability. It also ensures that, when used in a comparison mode with a wattmeter to be calibrated, the NBS sampling wattmeter will not introduce ground loops. Each input module has a gain selectable amplifier, T/H amplifier, and 12-bit A/D converter. An external shunt is used in the current channel. The sample strobe signals come from a crystal controlled clock and pass through delay circuits which allow for corrections of differential time delays between the voltage and current channels. The digital data from the A/D converters are routed to both a multiplier-accumulator, and to a DMA unit which records up to 4096 samples from both channels. The DMA unit is controlled by the microprocessor and (using the present software) records one set of data each time the measured quantities are displayed.

The wattmeter controls the number of samples in the summation interval by means of the trigger and cycle counter circuits which set the summation interval to a multiple of the input signal period. The sample counter determines the number of sample products that have been summed in that interval. This process represents an approximate synchronization of the summation interval with the input signal. The microcomputer can extend the summation interval beyond the limit imposed by the hardware (the 16-bit sample counter and 35-bit accumulator) to provide longer summation intervals which reduce the effects of truncation errors. Details of these operations are given in the section on Operation and Correction Formulas. The operator interfaces with the microcomputer via a keyboard and display and can control many of the system's measurement parameters.

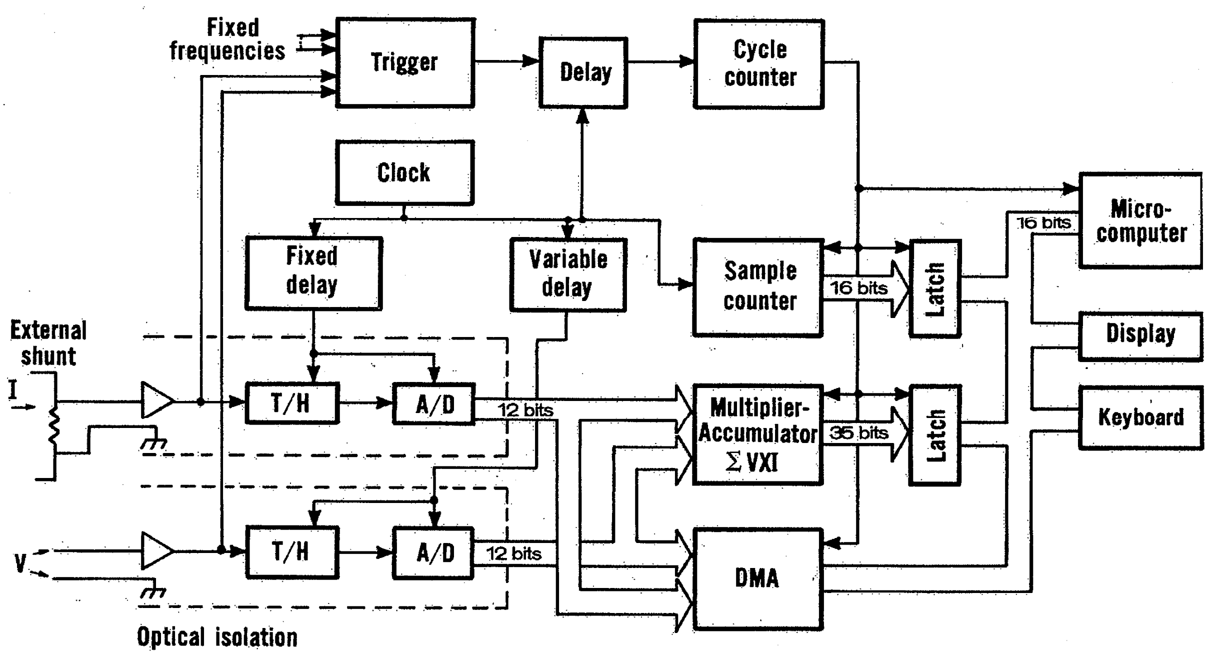

Figure 1. Block Diagram of Sampling Wattmeter

THEORY

This section presents the formulas for the truncation error for a sampling wattmeter and describes the theoretical basis for adjusting the summation interval to within one sample interval of an integral number of cycles of the input signals. The maximum error with such adjustment is compared to the case where no adjustment is made, for both sinusoidal input voltage and current signals and for periodic input signals with many harmonics present.

The truncation error formulas can be put into a compact form if several of the parameters are expressed in terms of the input signal phase angles. Figure 2 shows a simple example of sampling one cycle of sinusoidal voltage and current signals. If the sample interval γ is expressed in radians of the fundamental of the input signal, then the voltage and current samples are given by

$$V_k = V \sin(k\gamma + \alpha) , \tag{2}$$

and

$$I_k = I \sin(k\gamma + \alpha + \beta) , \tag{3}$$

where α is the starting voltage phase angle, β is the relative phase angle and V and I are peak values. If the power is measured using n samples, the indicated power can be shown to be

$$W = \frac{VI}{2} \left[\cos\beta - \frac{1}{n} \sum_{k=0}^{n-1} \cos(2k\gamma + 2\alpha + \beta)\right] . \tag{4}$$

Since the true power is just the first term in this expression, the summation term is the truncation error E. The difference between the summation interval, given by $n\gamma$, and the nearest integral number of cycles c of the input signal is defined as the truncation angle δ, that is $\delta \equiv 2\pi c - n\gamma$. Let c' be the number of cycles and partial cycles in the summation interval, or $2\pi c' = n\gamma$, then the truncation error can be expressed either as (see Appendix A for derivation)

$$E = \frac{VI}{2n} \frac{\sin\delta}{\sin\gamma} \cos(2\alpha + \beta - \delta - \gamma) , \tag{5}$$

or as

$$E = \frac{VI\gamma}{4\pi c'} \frac{\sin\delta}{\sin\gamma} \cos(2\alpha + \beta - \delta - \gamma) . \tag{6}$$

Consider this error for the case where the signal frequency is small compared to the sampling rate. When no restriction is placed on the summation interval, the maximum truncation error occurs when the $\sin\delta$ and cosine terms in (6) are ±1; thus,

$$\left|E_{max}\right| = \frac{VI}{4\pi c'} \left|\frac{\gamma}{\sin\gamma}\right| . \tag{7}$$

The angle γ can be expressed in terms of the input signal frequency f as $\gamma = 2\pi f/f_s$ where f_s is the sampling frequency. Thus, if f is much smaller than f_s, γ is small so $\sin\gamma \approx \gamma$ and (7) shows that the maximum error is inversely proportional to the number of signal cycles in the summation interval. However, if the summation interval $n\gamma$ is approximately synchronized with the input signal such that the truncation angle δ is less than or at most equal to one sample interval δ, the maximum truncation error occurs when the cosine term in (5)

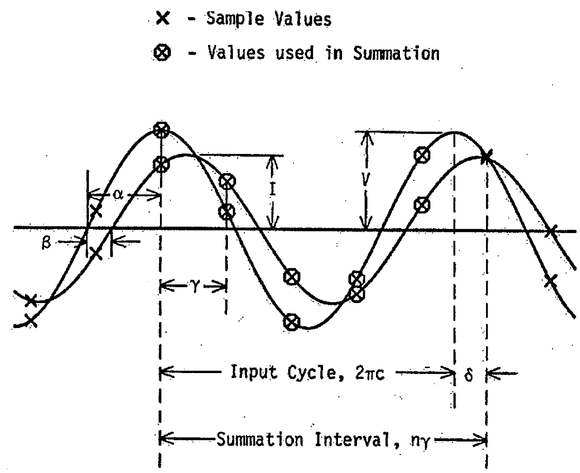

Figure 2. Summation example showing the relationship of various quantities. In this example c = 1, n = 5, $\alpha = 90°$, $\beta = -30°$, $\gamma = 80°$, and $\delta = -40°$.

is ±1 and $\sin\delta = \sin\gamma$, provided that $\gamma < \pi/2$; then

$$\left|E'_{max}\right| = \frac{VI}{2n} . \tag{8}$$

Thus, the maximum error is inversely proportional to the number of samples and independent of the signal frequency. As might be expected, the method of adjusting n reduces the maximum error only for lower frequencies and gives no improvement for signals sampled less than four times per cycle ($\gamma > \pi/2$).

For higher signal frequencies, (5) and (6) reflect the well known result that the error in using the sampling approach can become large if the sample interval γ approaches a multiple of π; in other words, the input signal frequency becomes a multiple of half the sampling rate. As γ approaches π, δ approaches 0 and, using l'Hospital's rule, one can show that the ratio of the sine terms approaches n. Thus, the error becomes

$$E\Big|_{\gamma=\pi} = -\frac{VI}{2} \cos(2\alpha + \beta) , \tag{9}$$

which is independent of the number of samples taken.

Equation (7) is useful for determining how close the input signal frequency can approach a multiple of half the sampling rate. The answer depends on the number of samples taken and the maximum relative error E_r to be tolerated, where E_r is the ratio of the maximum error to the input power at unity power factor, $E_r = \left|2E_{max}\right|/VI$. Considering (7) as an inequality and solving for γ gives

$$\left|\gamma - m\pi\right| > \arcsin\left(\frac{1}{nE_r}\right) , \tag{10}$$

where m is the multiple of half the sampling rate that γ is approaching. Expressing γ in terms of the signal frequency gives

$$\left|\frac{2f-mf_s}{f_s}\right| > \frac{1}{\pi}\ \arcsin\left(\frac{1}{nE_r}\right) . \quad (11)$$

As an example, consider $n = 150{,}000$, $m = 1$, and $f_s = 300{,}000$ samples per second. Then the relative truncation error is less than 0.05% if the signal frequency is more than 640 Hz away from half the sampling rate; i.e., $f < 149.36$ kHz or $f > 150.64$ kHz.

A result similar to the one above can be obtained for periodic signals with harmonics present. In this case one must consider the interaction of the voltage and current harmonics which create product harmonics [5] in the power signal. If the order of a voltage harmonic is designated by h and that of a current harmonic by i then these harmonics will result in product harmonics j given by $j = |h \pm i|$. Thus, the amplitude P_j and phase ϕ_j of the j^{th} product harmonic are due to all the voltage and current harmonics which have a sum and difference of j. In this case the truncation error can be written as [5]

$$E^* = W - P_0 = -\frac{1}{n}\sum_{j=0}^{\infty}[P_j \sum_{k=0}^{n-1} \cos(kj\gamma + \phi_j)], \quad (12)$$

where P_0 is the true power. The truncation error due to each product harmonic,

$$E_j^* = -\frac{P_j}{n}\sum_{k=0}^{n-1} \cos(kj\gamma + \phi_j) , \quad (13)$$

has the same form as the error considered for the sinusoidal case (c.f. eq. 4). Thus, the error will be a sum of terms similar to (6), that is

$$E_j^* = \frac{P_j\gamma}{2\pi c'}\ \frac{\sin(\frac{j\delta}{2})}{\sin(\frac{j\gamma}{2})}\ \cos(\phi_j - \frac{j\delta}{2} - \frac{j\gamma}{2}) , \quad (14)$$

where the terms c', δ, and γ are all relative to the number of cycles and phase angle of the fundamental. In a manner similar to the purely sinusoidal case, the truncation error becomes large if $j\gamma/2 = \pi$ or a multiple of π, which means that there are product harmonics present with the same frequency as the sampling rate.

As before, the maximum truncation error if n is not adjusted is

$$|E_{jmax}^*| = \frac{P_j}{2\pi c'}\ \frac{\gamma}{\sin(\frac{j\gamma}{2})} , \quad (15)$$

and the maximum error if n is adjusted is

$$|E_{jmax}^{*'}| = \frac{P_j}{2\pi c'}\ \gamma , \quad (16)$$

provided that $j\gamma < \pi$. Thus, the method of adjusting n gives a reduction in the maximum truncation error only for those product harmonics which are sampled more than twice per cycle of the product harmonic.

OPERATION AND CORRECTION FORMULAS

The 16-bit microcomputer (see fig. 1) is used to control the NBS sampling wattmeter and to perform the various calculations. Table I lists some of the parameters that can be set and their respective ranges. The syncnronization source can be set to one of the internal (crystal-based) fixed frequencies when pure dc signals are being measured or to either channel for ac signals. As mentioned above, the system does only an approximate synchronization of $n\gamma$ to an integral number of signal periods. If a fixed frequency is selected while measuring an ac signal, the successive power readings will vary in value around the true power reading because of the truncation error.

TABLE I

Range and Description of Measurement Parameters

Sample Frequency	2.34 to 300 kHz by powers of 2. Rate at which the input signals are sampled.
Track Time	0.33 to 2.67 μs by 0.33 μs steps. Track time for T/H amplifier.
Differential Sample Delay	-60 to 195 ns in 1 ns steps. Differential time delay between sample pulses for the two channels.
Synchronization Source	One of eight synch. sources. Either input channel with two filters each, or a fixed frequency of 2.34, 4.69, 18.75 or 75 kHz.
Trigger Level	-5 to 5 V in 0.04 V steps. Level the trigger circuit compares to the selected synch. source.
Trigger Delay	1 to 65,535 sample intervals. Sets start of summation interval relative to trigger event.
Summation Interval	2 to 65,535 synch. cycles. Summation interval in number of cycles of the selected synch. source.

The data from the multiplier-accumulator and sample counter are used to calculate the power, fundamental frequency, and period of the input signal. In general, these values are calculated using 150,000 or more samples which represent a summation interval of one half second or more. With this many samples, and with the summation interval not adjusted, the truncation error can be as large as ±0.3 percent of FSR for a 100 Hz signal. With the summation interval appropriately adjusted, the truncation error will be less than ±0.0007 percent of FSR. This error is much less than the expected uncertainty of the sampling wattmeter so it will not contribute significantly to the overall error.

The data stored via the DMA module are used to calculate the mean and rms value of both input channels. The accuracy of these measurements is important because the values are used to adjust the offset and gain for each channel. However, the number of samples is limited by the size of the DMA memory to 4096 for these calculations, so the uncorrected truncation error can be large. If the maximum number of samples is used, the error can be as large as 1.1 percent of FSR when measuring a 1 kHz signal and over 11 percent for a 100 Hz signal.

Three methods are available to minimize the truncation error. First, the summation can be performed over only complete cycles of the input signal to within one sample interval. The number of samples n to use is determined using the frequency value obtained from the sample counter data as described above. Using the voltage channel as an example, the average and rms values are calculated using the usual formulas as

$$V_{ave} = \frac{1}{n}\sum_{k=0}^{n-1} V_k \quad , \tag{17}$$

and

$$V_{rms} = \left(\frac{1}{n}\sum_{k=0}^{n-1} V_k^2\right)^{1/2} . \tag{18}$$

With this method the maximum truncation error for both a 100 Hz and 1 kHz signal will be about ±0.03 percent of FSR.

The second correction method estimates and corrects for the truncation error. If the first correction method is used and if the truncation angle δ is expressed in terms of the sample interval γ as $\delta = a\gamma$, then the parameter a, referred to as the truncation factor, will have a value between 0 and 1. This factor can be estimated either by calculation of an auto-correlation-like function from samples taken at the beginning and near the end of the data, or it can be estimated from the sample counter data. This latter method takes advantage of the large number of samples, typically over 150,000, and thus the large number of cycles N1 which the sample counter circuit and software extension can handle. These data can be used to determine the largest number of complete cycles N2 of the incoming signal which will fit into 4096 samples, as well as how many samples n and fractions of samples (the desired truncation factor a) will fit into that number of cycles. Since N1 is typically 40 to 50 times N2, the factor a can be determined with a resolution of about 2 percent. The estimated value for a and the nth sample data are used to make a truncation error correction using the following formulas:

$$V'_{ave} = \frac{1}{n+a}\sum_{k=0}^{n-1} V_k + \frac{aV_n}{n+a} \quad , \tag{19}$$

and

$$V'_{rms} = \left(\frac{1}{n+a}\sum_{k=0}^{n-1} V_k^2 + \frac{aV_n^2}{n+a}\right)^{1/2} . \tag{20}$$

Note that these equations make no correction for a = 0 (when there is no error) and an exact correction for a = 1.

The final method available to minimize the truncation error is to use the trigger delay parameter. This parameter determines the phase α of the start of the summation interval. Thus, α can be adjusted to minimize the cosine term in (6).

Another error that can be compensated is the differential time delay between the two channels. This delay has the effect of changing the power factor of the input signals. As shown in Table I the resolution of this adjustment is 1 ns. To appreciate the importance of this compensation, consider a 10 kHz power signal at half power-factor, i.e., $\beta = 60°$. The differential time delay which causes an error of ±0.1 percent of FSR is only 18 ns. Measurements on the amplifier/data converter modules have shown that without compensation and with the two channels on different gain settings the differential time delay can be as large as 44 ns. Thus, even though these amplifiers have a 3 dB bandwidth of several megahertz, they could cause significant errors at 10 kHz for low power-factor signals if the differential time delay was not compensated.

PERFORMANCE CHECKS

The accuracy of the NBS sampling wattmeter has been checked by several methods. Its frequency response has been measured using a simulated unity power-factor signal; its differential time delay has been measured using a simulated zero power-factor signal; its performance at lower frequencies has been compared with an accurate thermal converter-type wattmeter; the temperature sensitivity of its input modules has been determined. The results of these tests are described below.

The frequency response of the NBS sampling wattmeter was determined by comparing its measurements with those of an ac voltmeter whose frequency response up to 100 kHz had been checked with a calibrated ac/dc thermal converter against an accurate dc source. The same ac signal was connected to both input channels to simulate a unity power-factor signal. Figure 3 shows the errors measured for both channels in the rms mode and the simulated power when both channels were on the 1 V range. These results are typical of the errors measured for the 1, 2, and 5 V ranges. Results for the 0.1, 0.2, and 0.5 V ranges were always better than these and the results for the 10 to 500 V ranges using the 100-to-1 attenuator are generally poorer by a factor of about 2.

The differential time delays for the various gain settings of the input modules were determined using the NBS Phase Angle Standard [6] to produce a simulated zero power-factor signal. The differential time delays measured remained constant with frequencies from 1 to 50 kHz to within ±5 ns. The differential time delays obtained when using the 1 V range as reference were 0 ns for the 0.1 V range, 20 ns for the 0.2 and 2 V ranges, and 44 ns for the 0.5 and 5 V ranges.

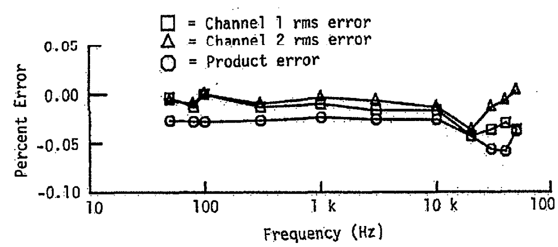

Figure 3. Percent error for each channel and the product vs frequency for 0.75 V rms into both channels simulating a unity power-factor signal.

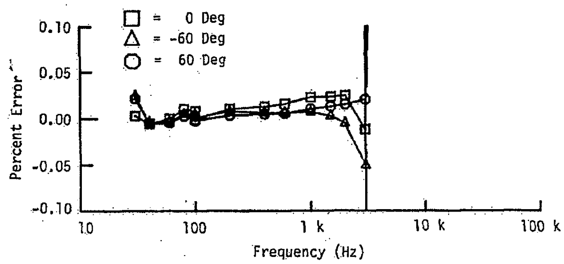

Figure 4. Difference between NBS sampling wattmeter and thermal wattmeter in percent of full-scale while measuring power in signals with 120 V and 5 A and with power-factors of 1 and ±0.5 (0 and ±60 degree phase angles, respectively).

The NBS sampling wattmeter was compared against the standard thermal wattmeter [7] used in the NBS power and energy calibration facility. The uncertainty of this wattmeter is less than ±50 ppm (±0.005 percent) for 40 to 100 Hz ac signals. According to its designer [7], however, it should operate satisfactorily with ac signals up to 1 kHz. Figure 4 shows the difference between these two wattmeters with input signals of 120 V and 5 A rms and with power-factors of 1 and ±0.5 (phase angles of 0 and ±60 degrees). These two instruments agreed to within ±0.03 percent over the frequency range from 40 Hz to 2 kHz.

Finally, temperature tests were performed on the input modules over the range of 10° to 40° C. These tests showed a temperature coefficient of gain of about 0.008 percent per degree C averaged over all ranges. The worst case sensitivity was 0.019 percent per degree C.

CONCLUSIONS

The desired objective of the NBS sampling wattmeter described herein is to enable the measurement of the power of highly distorted waveforms with fundamental frequencies up to 10 kHz and significant harmonics up to 100 kHz with an uncertainty of less than ±0.1 percent of FSR. Most of the performance checks implemented so far have used sine wave input signals. The wattmeter has performed within the desired accuracy level for these signals. Theoretical considerations suggest that similar accuracy should be available for distorted waveforms within the specified frequency range. Verification of performance under distorted waveforms is planned. Such a verification requires a calibration circuit with a physical standard for electric power that has equal or higher accuracy with distorted signals than the wattmeter to be investigated. A more detailed description of the NBS sampling wattmeter will be available in the form of an NBS Technical Note which is being prepared.

Future work on the NBS sampling wattmeter includes designing more accurate input modules using 14- or 16-bit A/D converters. Also, a phase-locked-loop circuit will be incorporated which will allow for a direct comparison in the same instrument between a phase-locked sampling approach and the one used at present.

ACKNOWLEDGEMENTS

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APPENDIX A

Derivation of truncation error formulas

The steps required to show that the truncation error calculated according to (5) or (6) is equivalent to the summation terms in (4) are given in this appendix. The summation terms in (4) represent the error E in the indicated power obtained by averaging the products of the voltage and current samples given by (2) and (3), respectively; that is

$$E = -\frac{VI}{2n}\sum_{k=0}^{n-1}\cos(2k\gamma + 2\alpha + \beta) , \qquad (A1)$$

where, as in the above text, V and I are the peak voltage and current values, n is the number of samples, γ is the sample interval expressed in radians of the input signal, α is the starting voltage phase angle, and β is the relative phase angle (see fig. 2). By writing the cosine term as the real part of a complex exponential, (A1) becomes

$$E = -\frac{VI}{2n}\,\mathrm{Re}\Big[\sum_{k=0}^{n-1} e^{2k\gamma i}\; e^{(2\alpha+\beta)i}\Big] , \qquad (A2)$$

or

$$E = -\frac{VI}{2n}\,\mathrm{Re}\Big[e^{(2\alpha+\beta)i}\sum_{k=0}^{n-1} e^{2k\gamma i}\Big] . \qquad (A3)$$

Now the terms in the sum are a geometric progression, so their sum can be written as

$$\sum_{k=0}^{n-1} e^{2k\gamma i} = \frac{1-e^{2n\gamma i}}{1-e^{2\gamma i}} , \qquad (A4)$$

or as

$$\sum_{k=0}^{n-1} e^{2k\gamma i} = e^{(n\gamma-\gamma)i}\,\frac{\sin n\gamma}{\sin \gamma} . \qquad (A5)$$

Thus, the error becomes

$$E = -\frac{VI}{2n}\,\frac{\sin n\gamma}{\sin\gamma}\,\mathrm{Re}\big[e^{(2\alpha+\beta+n\gamma-\gamma)i}\big] , \qquad (A6)$$

or

$$E = -\frac{VI}{2n}\,\frac{\sin n\gamma}{\sin\gamma}\,\cos(2\alpha + \beta + n\gamma - \gamma) , \qquad (A7)$$

where $n\gamma$ is the summation interval. If $n\gamma$ is expressed in terms of the nearest integral number of cycles c and a difference angle δ, called the truncation angle, i.e.,

$$n\gamma = 2\pi c - \delta , \qquad (A8)$$

then the error becomes

$$E = -\frac{VI}{2n}\,\frac{\sin(2\pi c-\delta)}{\sin\gamma}\,\cos(2\alpha+\beta+2\pi c-\delta-\gamma)\ , \quad (A9)$$

or

$$E = \frac{VI}{2n}\,\frac{\sin\delta}{\sin\gamma}\,\cos(2\alpha+\beta-\delta-\gamma)\ . \quad (A10)$$

If c' is defined as the number of cycles and partial cycles of the input signal in the summation interval, i.e. $2\pi c' \equiv n\gamma$, then using this to substitute for n in (A10) gives the alternate form of E as

$$E = \frac{VI\gamma}{4\pi c'}\,\frac{\sin\delta}{\sin\gamma}\,\cos(2\alpha+\beta-\delta-\gamma)\ . \quad (A11)$$

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From 1963 to 1969 he was with Vitro Laboratories in Silver Spring, Maryland. He joined the National Bureau of Standards in 1969 where his experiences include work in semiconductor device processing and instrument reliability analyses. He is presently developing wideband power measurement standards.

Discussion

Mario Savino (University of Bari, Bari, Italy): The author presents a very valuable sampling watt-meter for measuring distorted power signals. The increase of nonlinear loads causes high levels of electric waveform distortion and requires special care in analysis and measurement of the degree of disturbance caused by rectifiers, inverters, cycloconverters, pulse-burst-modulation systems, etc. The author is to be congratulated on introducing an interesting perspective to the problem of measuring power harmonics and for high accuracy measurements. I agree with the considerations that techniques employing time-division-multipliers, or thermal-convertors, or analog multipliers give rise to instruments which do not have a wideband or a sufficient accuracy.

I would like to suggest a FET-based possible approach. Recently, the use of Fast Fourier Transform allows for instrumenting high accuracy digital meters, which are very suitable for measurements in nonsinusoidal systems. The main limitations in the use of FET-based instruments in the power frequency range come from leakage effects, due to fundamental frequency fluctuations on distribution systems. These limitations can be overcome by using special windows and interpolation algorithms. The windows are useful for removing the long-range leakage and the resulting harmonic interference, while the interpolation algorithms are suitable for deleting the effects of short-range leakage [1], [2].

In our laboratory we have set up a digital instrument, which can measure frequency, voltage, current, active and reactive powers in electric nonsinusoidal systems [3], [4]. The simplified functional block diagram of the digital instrument is shown in Fig. 1. DMA logic is implemented in order to carry out the elaboration during the acquisition and to continue to obtain an acquisition as much as possible. The FET algorithm can be implemented in hardware so that data elaboration would be fast enough for a real-time signal processing. We intend to develop the real-time implementation in the near future. The instrument can be used for measuring grounding impedances and on-line power of inverterfed induction machines. The capabilities of the instrument have been satisfactorily demonstrated. It gives a high resolution and accuracy which varies between 0.05 percent and 0.5 percent for typical applications.

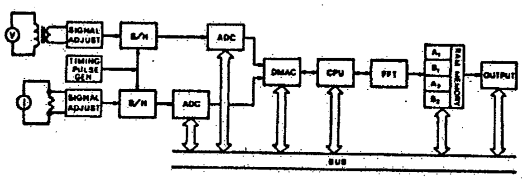

Fig. 1.

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G. N. Stenbakken: I thank Dr. Savino for his discussion and interest in this paper and for his sharing of the work he is doing in this area.

The problem he describes in the FFT approach is closely related to the error I analyzed in the paper. In both cases the error arises because the sampling is not synchronized with the signal being sampled. Thus, in general, the sampling interval will not cover an integral number of signal cycles. The errors that appear in the FFT results are referred to as leakage, whereas in the time domain this same error is referred to as a truncation error. The approach he references for eliminating the leakage error [2 in his reference] appears to be very effective. The advantage of the FFT approach is the ability to calculate a number of parameters in addition to total power, such as fundamental power, fundamental phase angle, harmonic powers and harmonic phase angles. The disadvantage is that, because of the large amount of computation involved, only a small portion of the data available from the converter is used or special hardware is needed to speed up the computation process as Dr. Savino proposes.

The particular measurement problem Dr. Savino mentions, that of measuring the power of signals with a fluctuating frequency is interesting in that it precludes synchronizing the sampling process. The approach used in the NBS Wideband Sampling Wattmeter is able to measure this kind of signal. First, the approximate synchronization method used tracks any frequency changes and second, since the computations are performed in real-time, the data summation period can be made very large. A special case of this problem that often occurs on power systems is the one in which the frequency of fluctuation is a subharmonic of the fundamental. For this case the total power is best obtained by summing the instantaneous power samples over complete cycles of the subharmonic. The NBS Wideband Sampling Wattmeter approach allows this mode of operation since the summation interval is an operator specified number of cycles of the fundamental.

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