Physikalisch-Technische Bundesanstalt
Braunschweig und Berlin
Nationales Metrologieinstitut

Measurement of the Gravitational Constant by dropping three test masses – a proposal

projected and presented by

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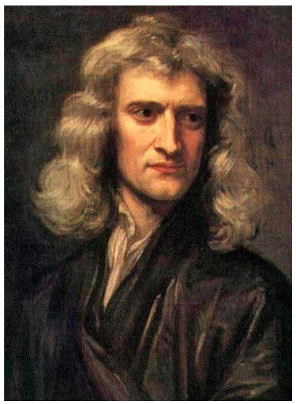
(currently working at PTB)

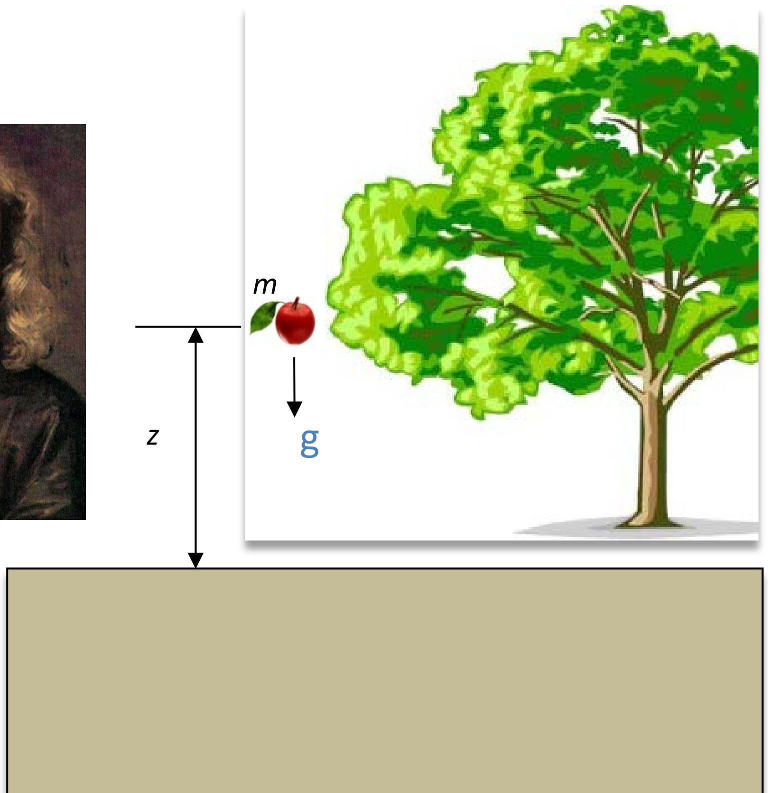
NIST, Gaithersburg, USA – 9/10 October, 2014

Outline

- The differential gravity gradiometer
  - Acceleration due to gravity, g – the gravimeter
  - Newtonian Constant of Gravitation, G – the gradiometer
- Similarities to atom gravimeters
- Proposed experiment

‚small‘ g vs. ‚big‘ G





Newtonian constant of gravitation:

$G = 6.673\ 84 \times 10^{-11}\ m^3\ kg^{-1}\ s^{-2}$

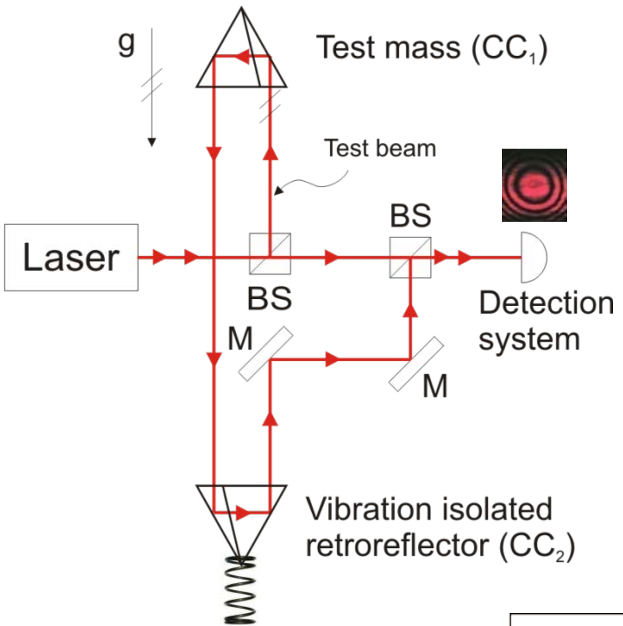
$$F_G = \frac{Gm_1 m_2}{r^2}$$

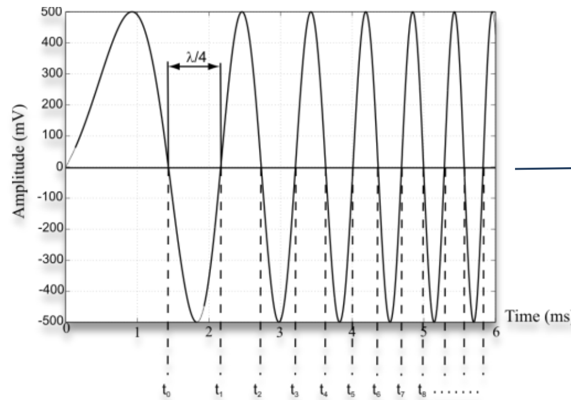
g

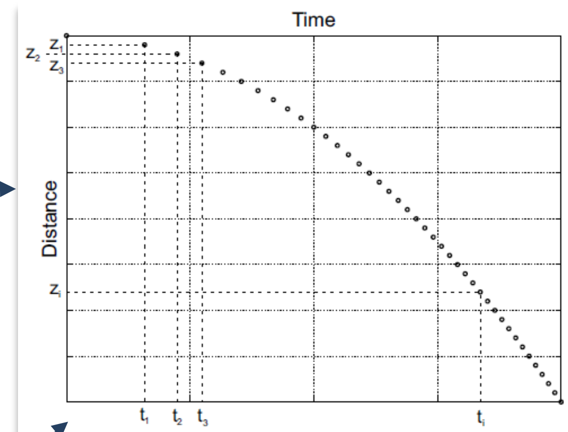
Acceleration due to gravity:

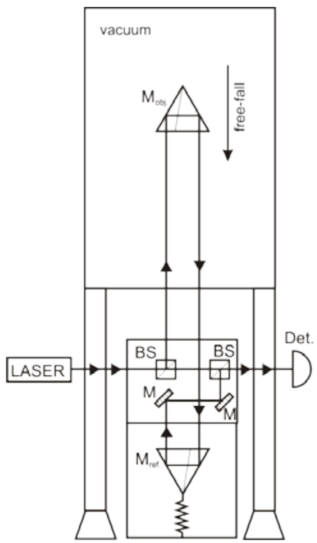
$g = 9.806\ 65\ m\ s^{-2}$

Principle of measurement









$$z(t) = z_0 + v_0 t + \frac{g}{2} t^2$$

acceleration due to gravity

Free-fall absolute gravimeter

FG5-X

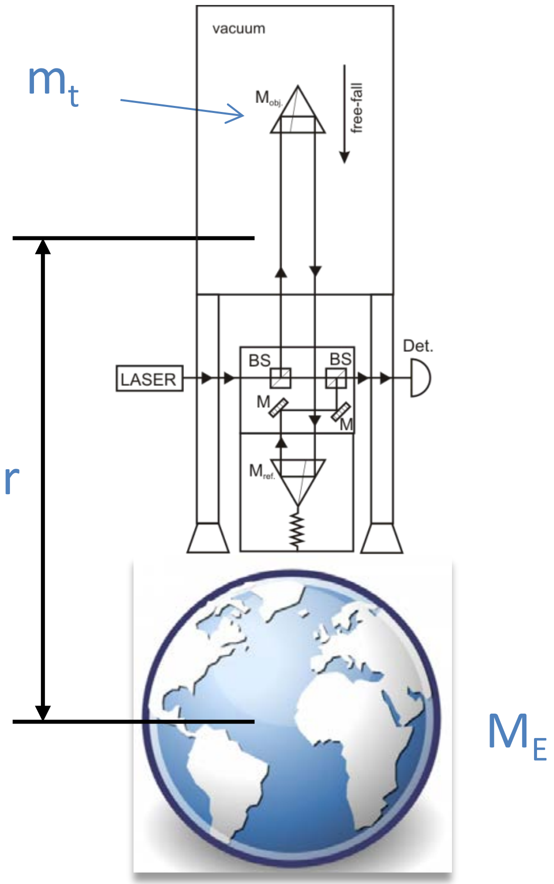
Microg-LaCoste
(Lafayette, CO, USA)

FG5-X : Precision : 15 µGal/√(Hz)*
Accuracy: 2 µGal*

(*from http://www.microglacoste.com/absolutemeters.php)

Measurement of G with a gravimeter

Take Earth as source mass M_E



g

$$F_G = \frac{Gm_1 m_2}{r^2}$$

$$G = 6.7x10^{-11} \frac{Nm^2}{kg^2}$$

$M_E = m_1$

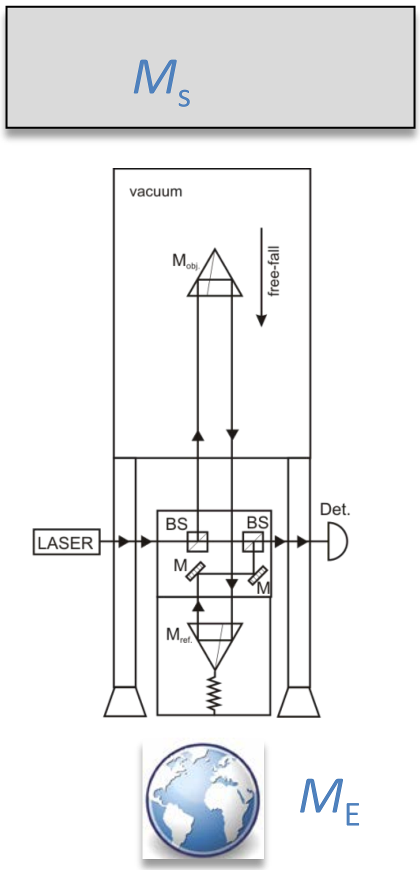
$m_t = m_2$

- Calculate g with theoretical model
- Measure g
- Determine G

Problem: Mass, geometry and density distribution of Earth are not well known!

use of well defined source mass, M_S

configuration 1



M_s produces perturbing acceleration *P(z,G)*

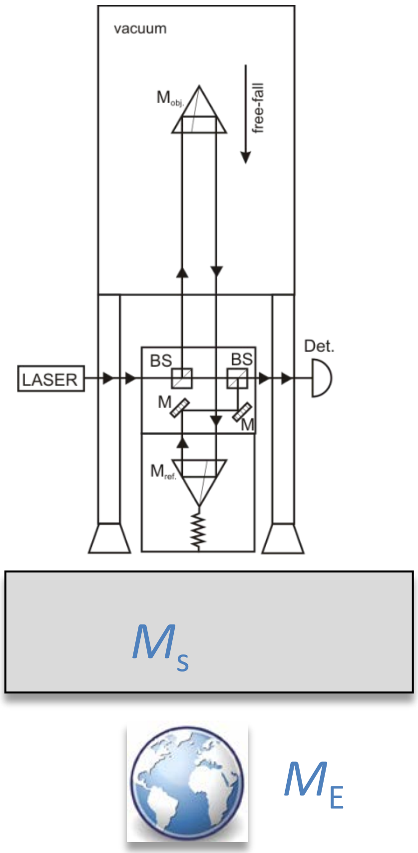
Total signal is

gravity gradient

$$g_1 = \underbrace{g_0 + \gamma z}_{Earth} - \underbrace{P(z,G)}_{source\ mass}$$

use of well defined source mass MS

configuration 2



M_s produces perturbing acceleration *P(z,G)*

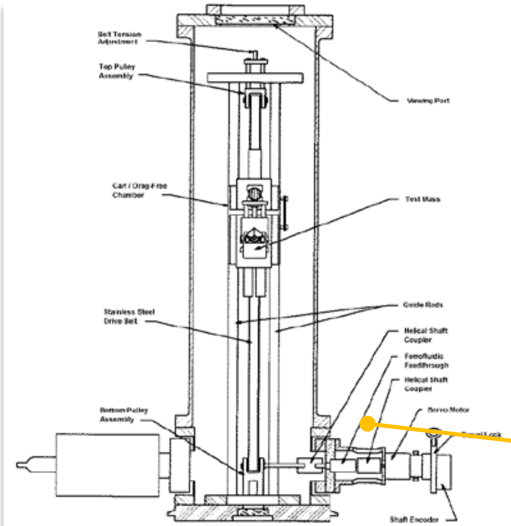
Total signal is

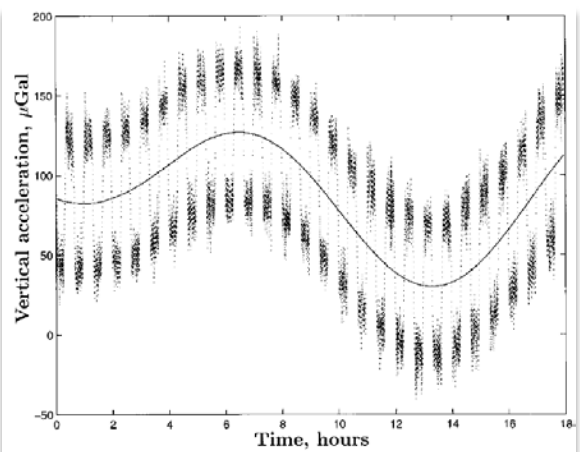
$$g_2 = \underbrace{g_0 + \gamma z}_{Earth} + \underbrace{P(z,G)}_{source\ mass}$$

Differential signal

$$g_2 - g_1 = 2\underbrace{P(z,G)}_{source\ mass}$$

Schwarz et al., 1998, University of Colorado – Free-Fall Gravimeter

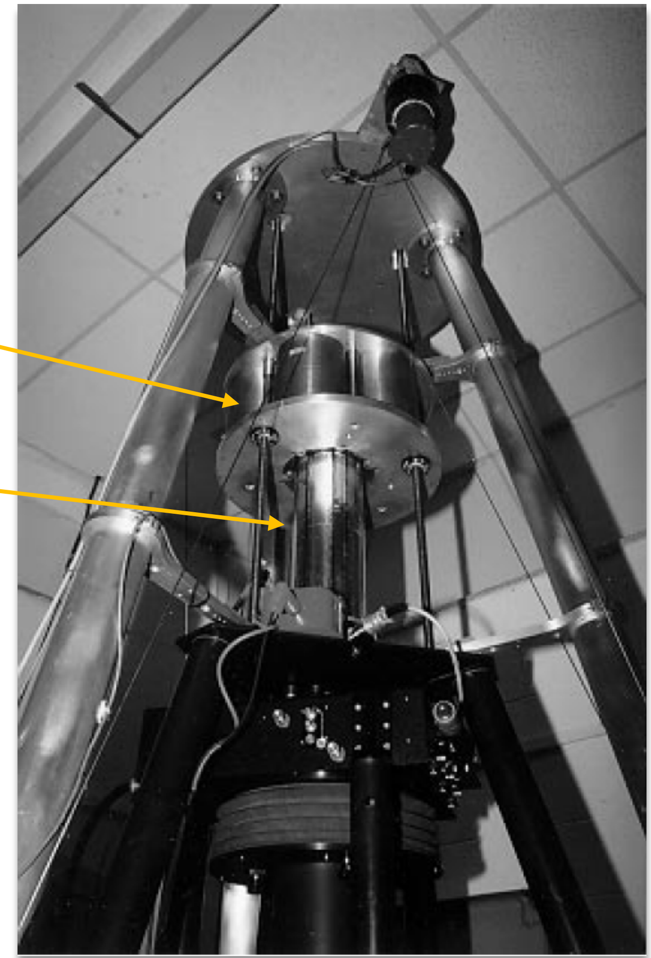




source mass:

~500 kg

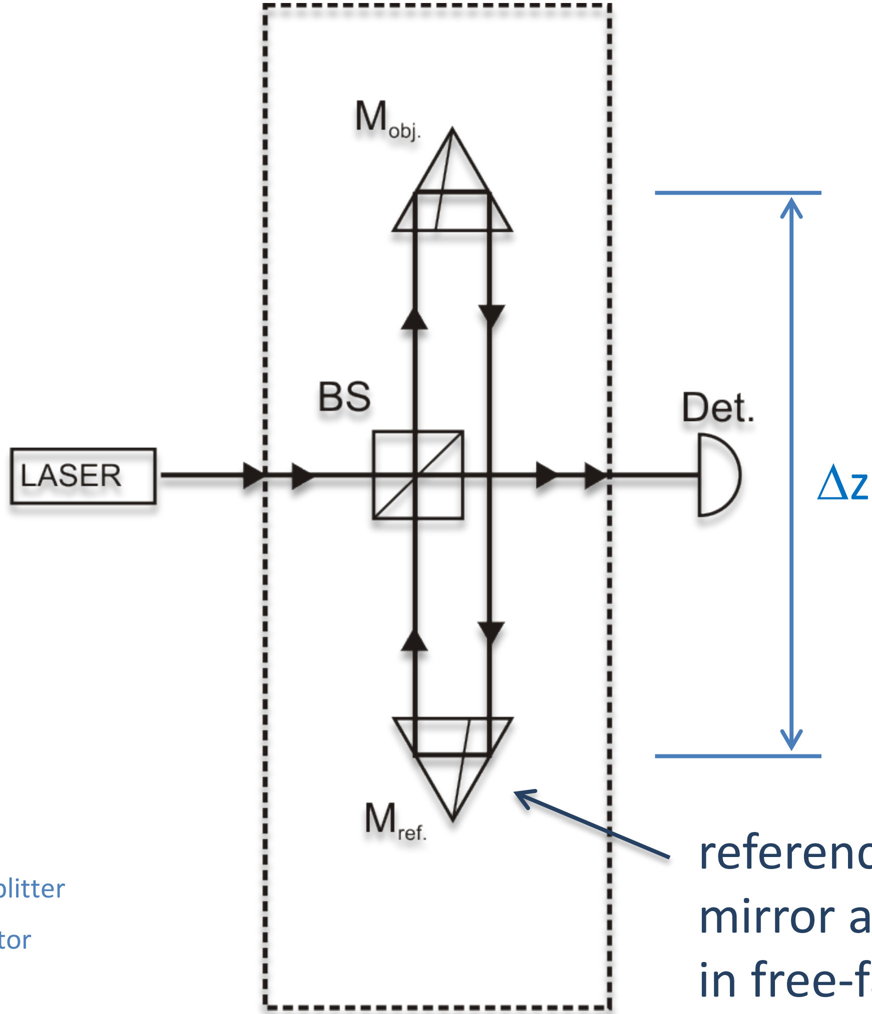
FG5 gravimeter



Result:
$\Delta G/G = 1.4 \cdot 10^{-3}$

JP Schwarz, DS Robertson, TM Niebauer & JE Faller (1998). *A free-fall determination of the universal constant of gravity*. Science, **18**, 2230-2234

Free-fall Gradiometer



reference mirror also in free-fall

height 1: g_T

$$\gamma := \frac{\Delta g}{\Delta z} = \frac{g_B - g_T}{\Delta z}$$

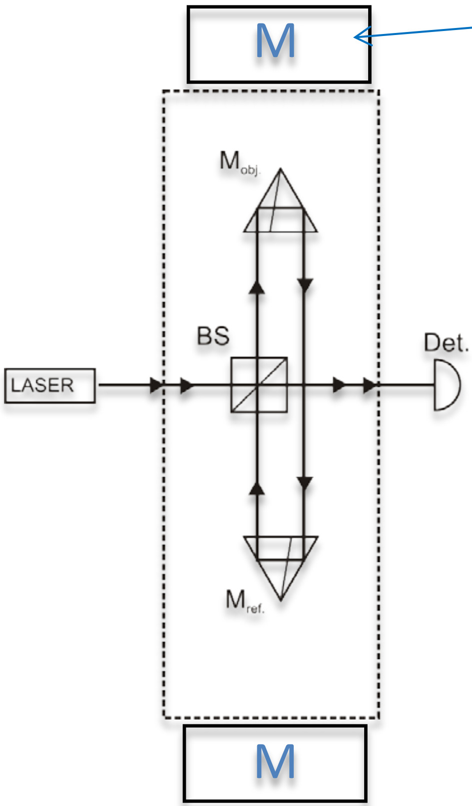
height 2: g_B

BS: Beam splitter

Det.: Detector

Measurement of G with a gravity gradiometer

Configuration 1



second source mass
to increase signal

acceleration due to external mass

$$g_T = {}^0g_T - {}^M\Delta g$$

top

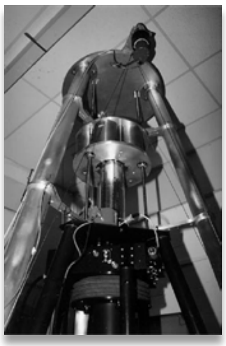
acceleration due to Earth

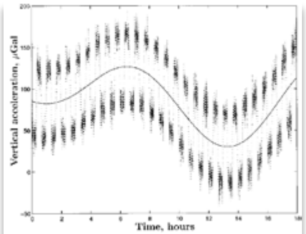
$$\Rightarrow g_B - g_T = \Delta z \cdot \gamma + 2\,{}^M\Delta g$$

$$g_B = {}^0g_B + {}^M\Delta g$$

bottom

equivalent to

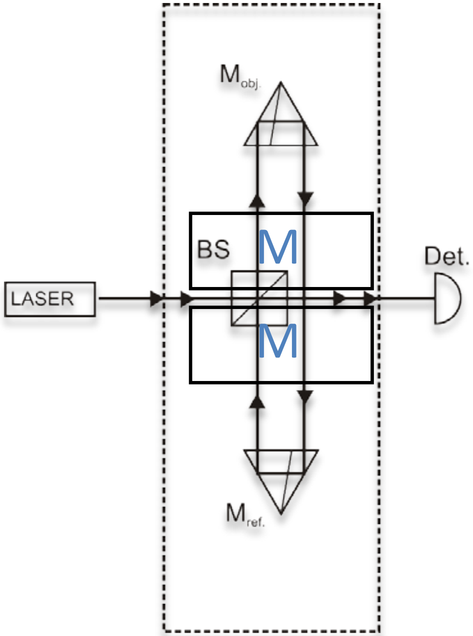




however, cancels tides, ocean loading, etc.

Measurement of G with a gravity gradiometer

Configuration 2



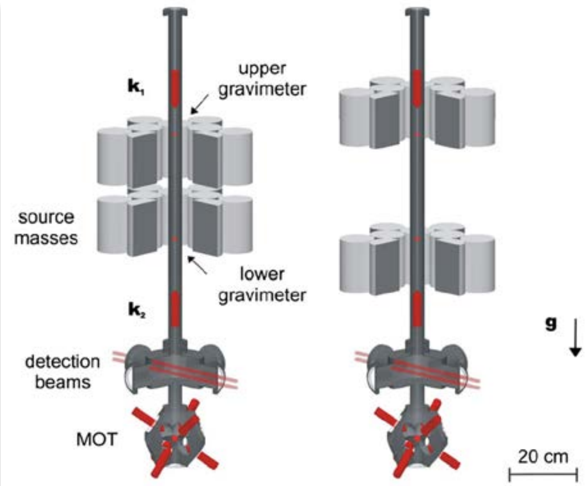
$$g_T = {}^0g_T + {}^M\Delta g$$ top

$$\Rightarrow g_B - g_T = \Delta z \cdot \gamma - 2\ {}^M\Delta g$$

$$g_B = {}^0g_B - {}^M\Delta g$$ bottom

Config. 2 – Config. 1: $$(g_B - g_T)_{,2} - (g_B - g_T)_{,1} = -4\ {}^M\Delta g$$

University of Florence – Atom interferometer (Raman interferometry)



Test mass:

Rubidium atoms

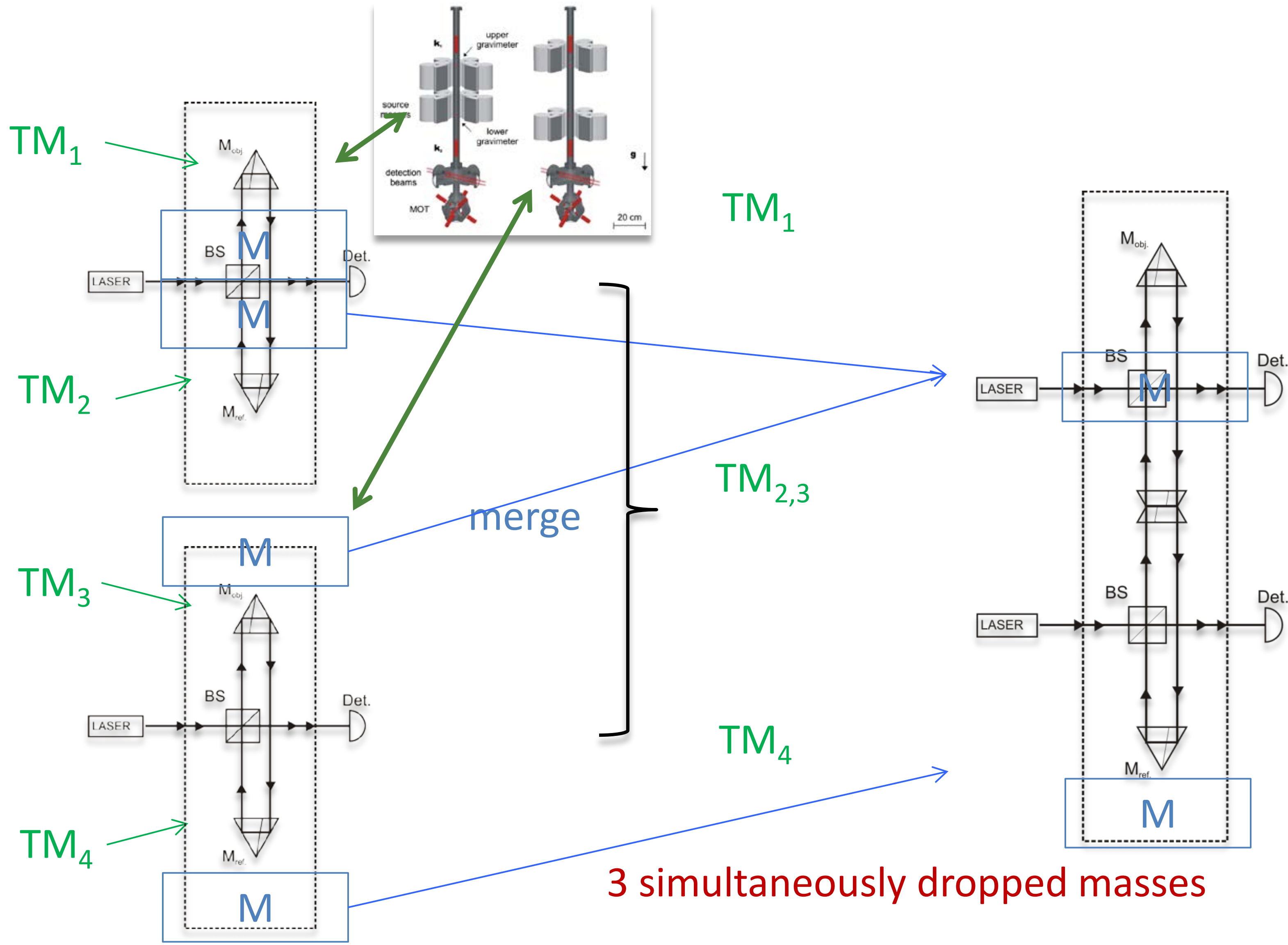
Source mass:

~516 kg tungsten

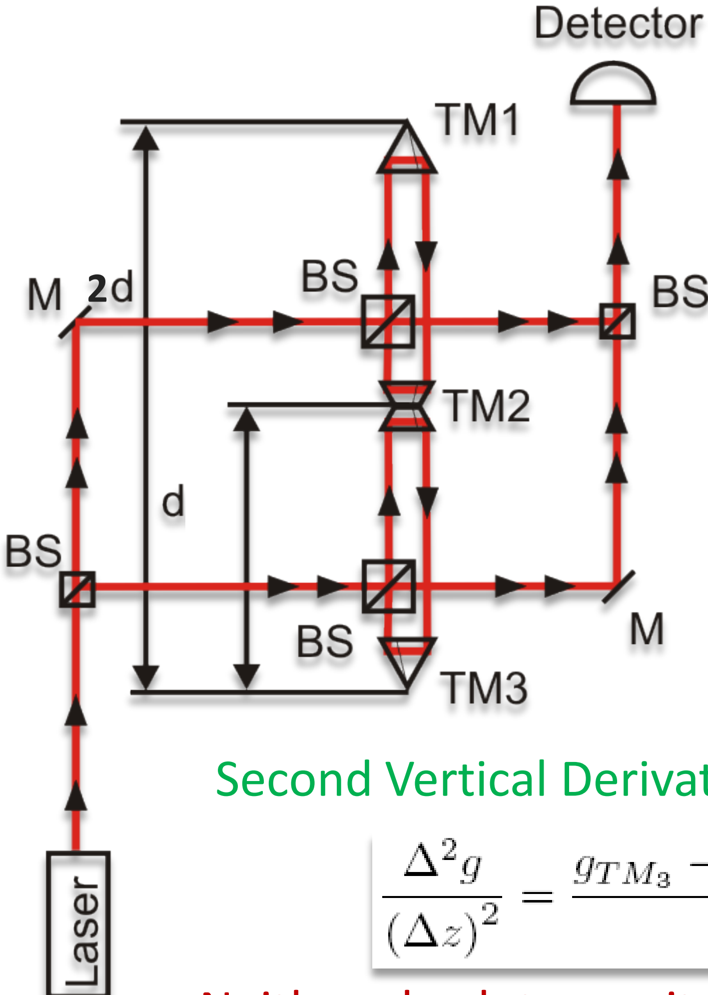
Result: $\Delta G/G = 1.5 \cdot 10^{-4}$

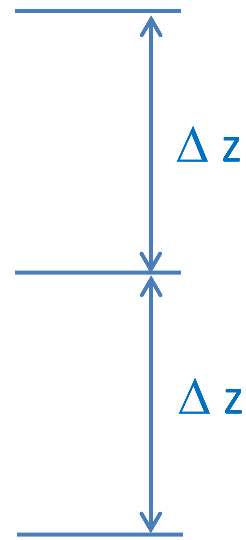
G Rosi et al. (2014). Precision measurement of the Newtonian gravitational constant using cold atoms. *Nature*, 510, 518–521.

Differential gradiometer



Differential gradiometer





height 1: g_T

$$\frac{\Delta g_T}{\Delta z} = \frac{g_{TM_2} - g_{TM_1}}{\Delta z}$$

height 2: g_M

$$\frac{\Delta g_B}{\Delta z} = \frac{g_{TM_3} - g_{TM_2}}{\Delta z}$$

height 3: g_B

Second Vertical Derivative of g (SVD)

$$\frac{\Delta^2 g}{(\Delta z)^2} = \frac{g_{TM_3} - 2g_{TM_2} + g_{TM_1}}{(\Delta z)^2}$$

Neither absolute gravity value nor gradient detectable

Null instrument

Rothleitner Ch & Francis O. (2014). Measuring the Newtonian constant of gravitation with a differential free-fall gradiometer: A feasibility study. *Rev. Sci. Instrum,* **85**, 044501.

Separation of inertial from gravitational forces

Measured force in a non-inertial frame:

$$\ddot{x}_k = f_k - 2a_{ik}\dot{a}_{ij}\dot{x}_j - a_{ik}\ddot{a}_{ij}x_j - \ddot{b}_k$$

Gravitat. force — Coriolis force — Euler and centrifugal force — Linear acceleration

$f_i =$ gravitational force, derived from potential $f_i = -\frac{\partial V}{\partial x_i}$

$a_{ik} =$ rotation matrix; rel. rotation between inertial and non-inertial frame

$\ddot{b} =$ rel. linear acceleration between both frames

$$\text{SVD} \quad \frac{\partial^2 \ddot{x}_i}{\partial x_i^2} = \frac{\partial^2 f_i}{\partial x_i^2} = -\frac{\partial^3 V}{\partial x_i^3}$$

⟶ inertial forces disappear; pure gravitational signal is measured

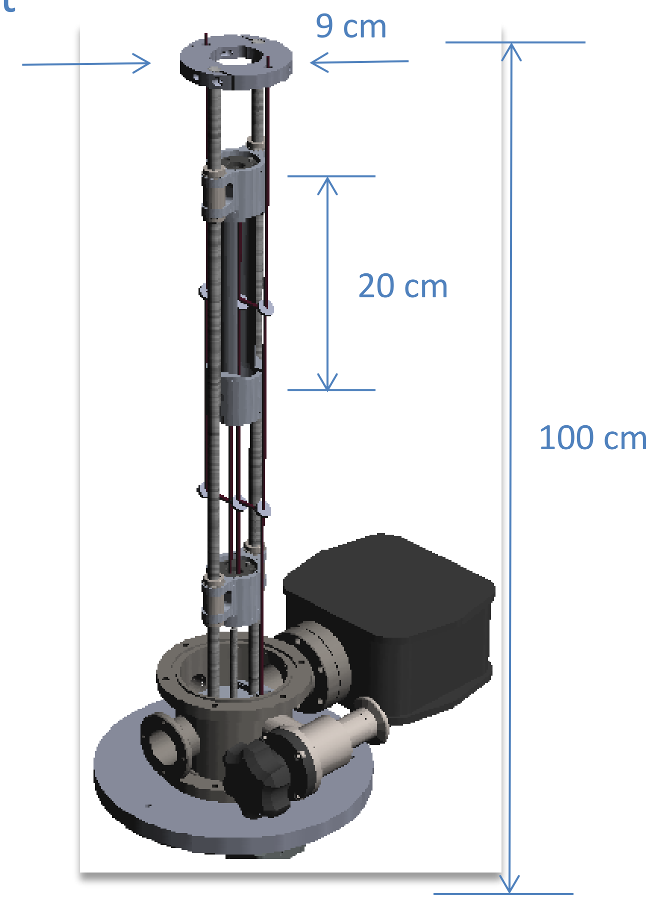
(possible applications in fundamental physics, airborne/shipborne gravimetry, navigation (gravity map matching), etc.)

(in principle no inertial stabilization necessary)

see e.g. Hofmann-Wellenhof, B & Moritz, H (2005). *Physical Geodesy.* Springer Wien New York.

Preliminary data

- First tests show statistical uncertainties of about 0.2 µGal (24 h) (standard deviation ~ 8 µGal)
- Max. drop frequency is 1/36 s



© by MicroG LaCoste

Uncertainty budget of gradiometer

TABLE III. Table of all considered sources of uncertainty given as relative standard uncertainty; signal of 100 μGal is assumed.

Error source	Relative uncertainty
Beam verticality	1×10^{-9}
Beam diffraction	1.2×10^{-9}
Laser stability	2×10^{-9}
Clock stability	1×10^{-10}
Speed of light	1×10^{-9}
Drag effect	1×10^{-6}
Outgassing	2.6×10^{-4}
Buoyancy	negligible
Temperature gradient	1.8×10^{-6}
Magnetic fields	1×10^{-6}
Electrostatic forces	1×10^{-7}
Radiation pressure	5.1×10^{-5}
Self attraction	1×10^{-5}
Environmental effects	negligible
Initial velocity of test mass	3×10^{-6}
Corner cube rotation	8.1×10^{-5}
Source mass (density/positioning)	1×10^{-5}
Beam shear	negligible
Coriolis effect	1×10^{-4}
Two-sample zerocrossing	4.4×10^{-4}
Combined standard uncertainty	5.3×10^{-4}

Could be improved with current technology

-> aimed uncertainty of 1.2×10^{-4} looks feasible

Rothleitner Ch & Francis O. (2014). Measuring the Newtonian constant of gravitation with a differential free-fall gradiometer: A feasibility study. *Rev. Sci. Instrum,* **85**, 044501.

Similarity to atom gravimeters

- Dropper chamber can have the same dimension -> same source mass
- Common (but also different) uncertainty sources -> comparison; detection of systematic errors

Picture from Lamporesi, 2006, PhD thesis

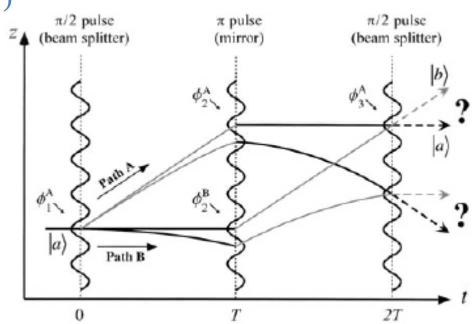
Analogy in the measurement functions of corner cube and atom gravimeters

atom interferometer

$$P = \frac{1}{2}\left(1 - \cos(\phi_{tot})\right)$$

probability

*)



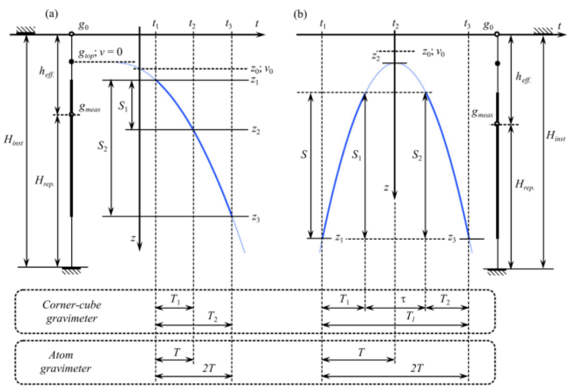
$$g_{meas} = \frac{1}{k_{eff} T^2}(\phi_1 - 2\phi_2 + \phi_3)$$

*) figure from Peters et al. (2001). High-precision gravity measurements using atom interferometry. Metrologia. 38, 25-61.

classical laser interferometer

$$I(t) = 2I_0\left(1 + \cos(\phi(t))\right)$$

intensity

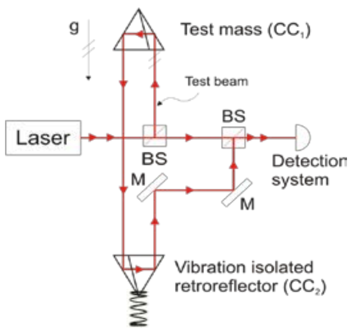


$$g_{meas} = \frac{2}{T_2 - T_1}\left(\frac{S_2}{T_2} - \frac{S_1}{T_1}\right)$$

describes three level system

k_{eff} : effective Raman wavenumber

λ: wavelength of laser



$$\phi(t) = 2\pi \frac{z(t)}{\lambda/2} = \frac{4\pi}{\lambda} z(t)$$

$$g_{meas} = \frac{1}{(4\pi/\lambda)T^2}(\phi_1 - 2\phi_2 + \phi_3)$$

$T_1 = t_2 - t_1,\ T_2 = t_3 - t_1$

$S_1 = z_2 - z_1 \quad S_2 = z_3 - z_1$

$T_1 = T \quad T_2 = 2T$

Rothleitner, Ch., Svitlov, S. (2012). On the evaluation of systematic effects in atom and corner-cube absolute gravimeters. *Phys. Lett. A., 376,* 1090-1095.

Proposal

Perform a Big G measurement by

- Using an atom and a classical gradiometer
- Using the same source mass
- Compare uncertainty budgets
- Identify and eliminate systematic errors if present

Advantages of free-fall experiments

- The classical and the atom gravimeter can have the same physical sizes;

  -> same source masses can be used

- The experiment can be realized with two different technologies / physical laws

  -> proof of physical theories possible

- Both technologies are under intense research

  -> good know-how; immediate start possible

- Benefit for industry

  -> use in other areas of science and technology possible

- Reflects the popular story about Newton´s apple

  -> Attractive for popular readership

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