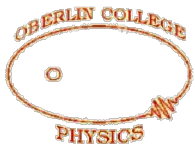


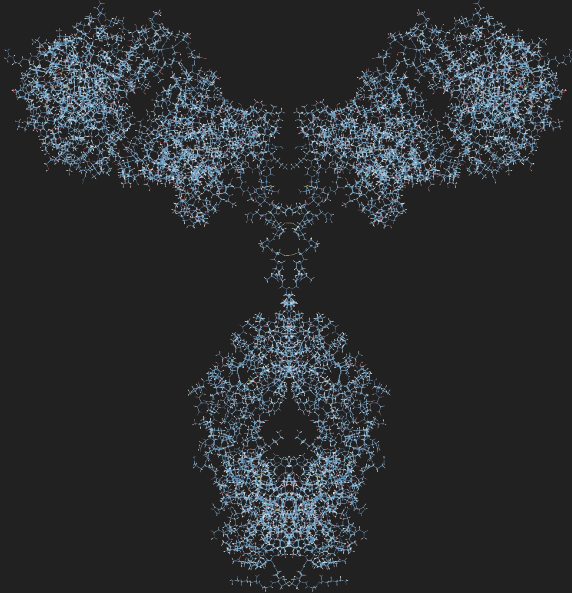
# SASCalc

## Periodic Boundary Conditions

Ian Hunt-Isaak, Joseph E. Curtis, Steven C. Howell

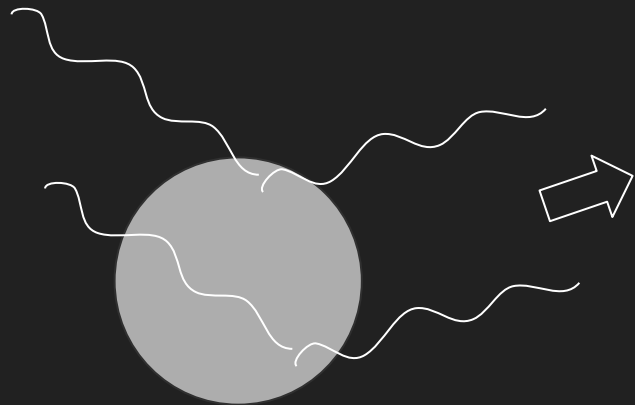


# Why are concentrated proteins so viscous?

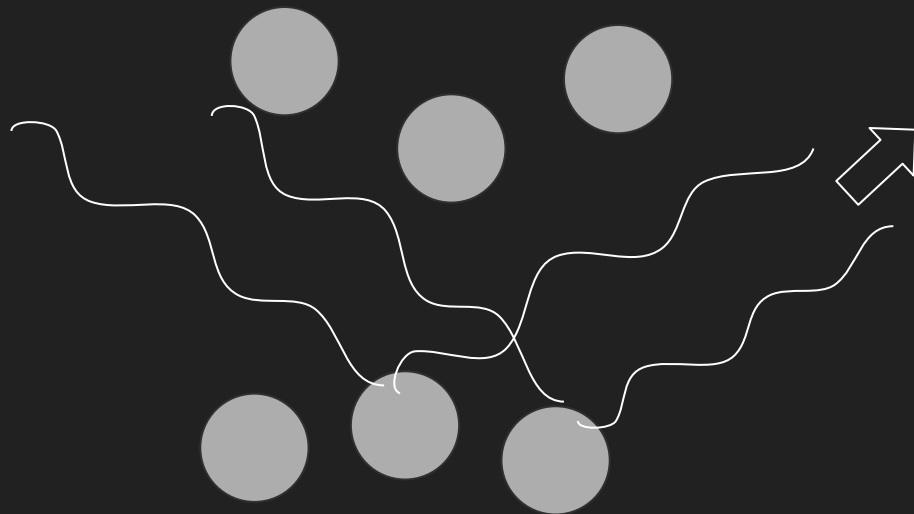


# Small-Angle Scattering (SAS) Measures

Form factor

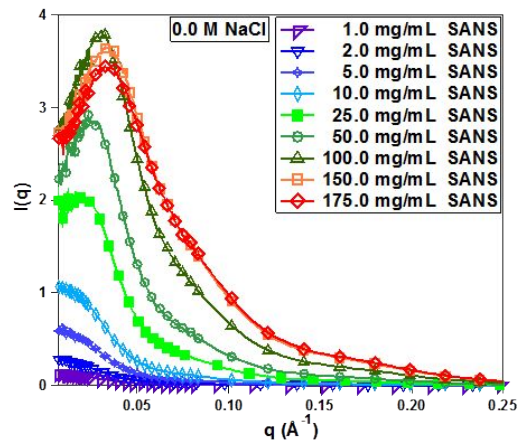
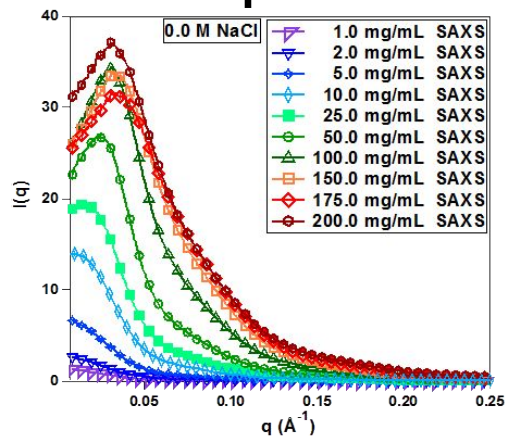


Structure factor

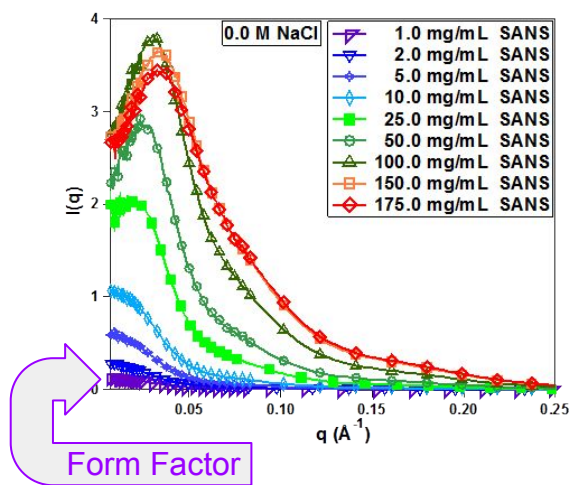
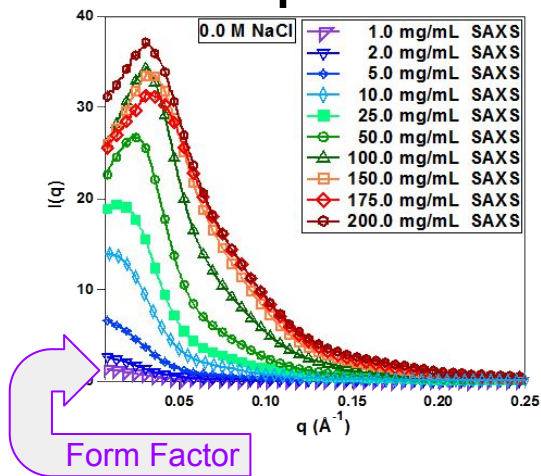


$$I(Q) = P(Q)S(Q)$$

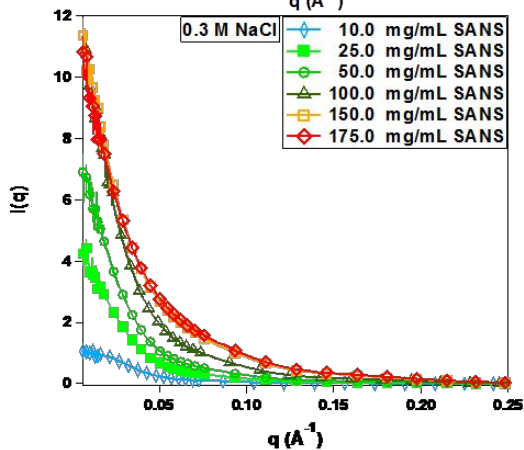
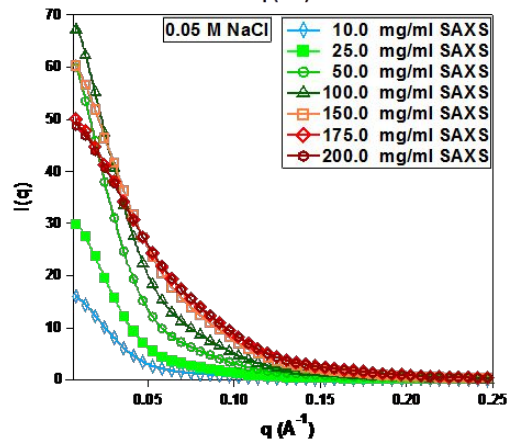
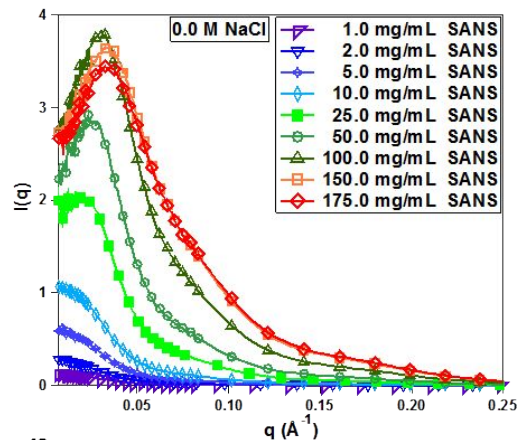
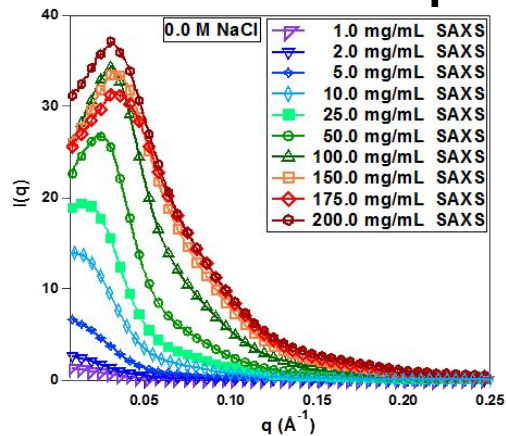
# SAS measures protein interactions



# SAS measures protein interactions

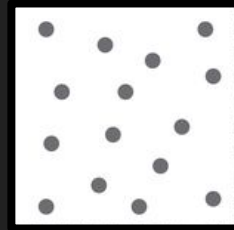


# Co-solutes modulate protein interactions



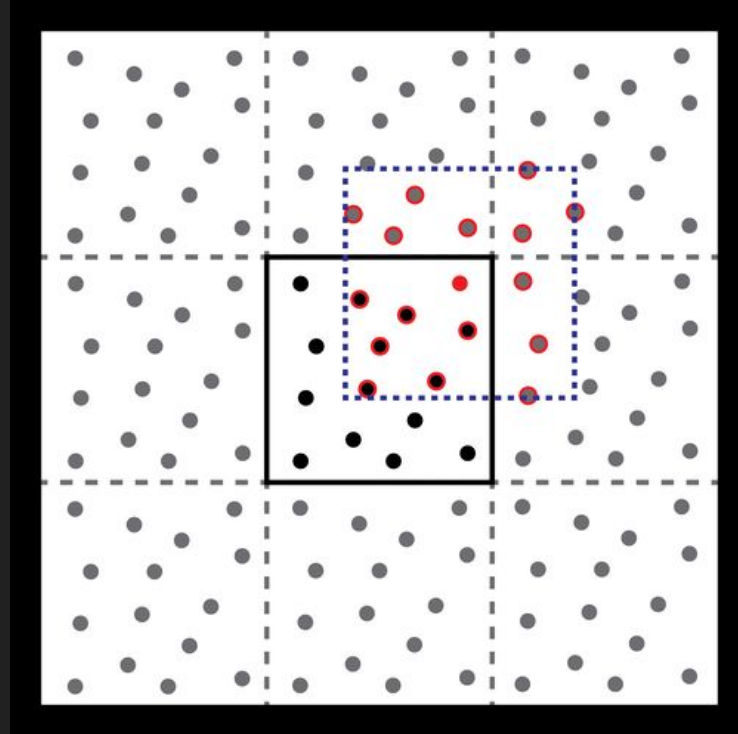
Change in scattering is not dominated by  $P(q)$  or  $S(q)$  alone.

# Molecular Simulation

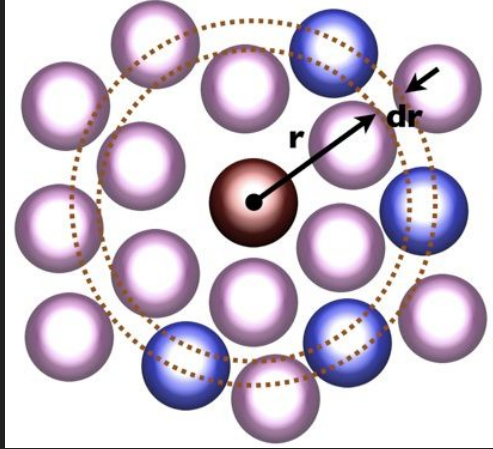




# Molecular Simulation



# Fourier Transform: $g(r) \rightarrow S(Q)$

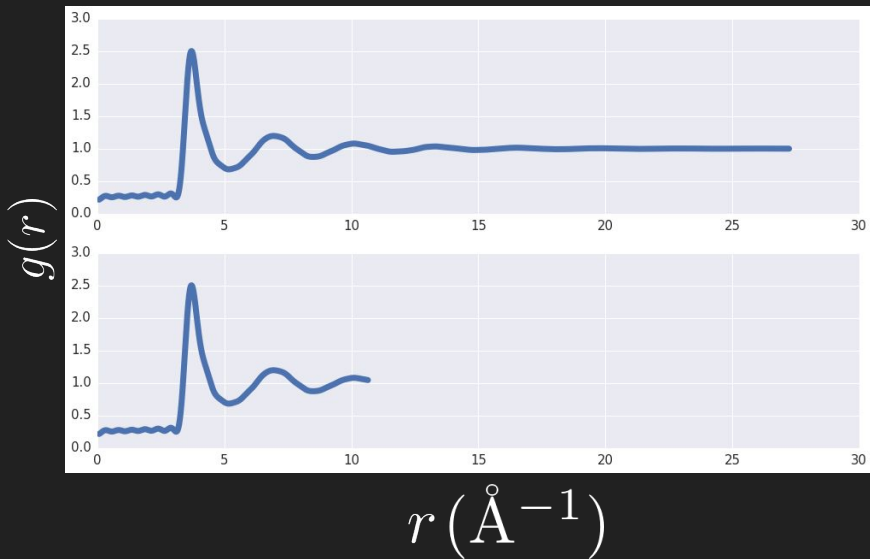


$$S(Q) = 1 + 4\pi\rho\frac{1}{q} \int (g(r) - 1) \cdot r \sin(qr) dr$$

- Limited to spherical systems

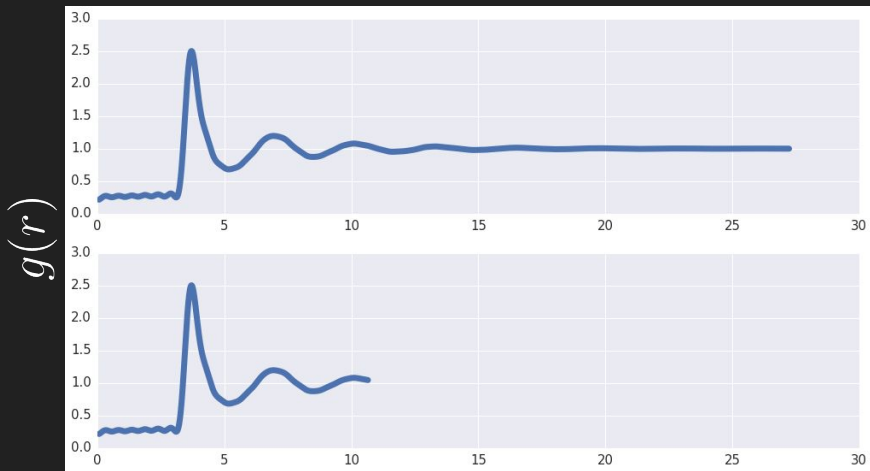
# Finite size effects

$$S(Q) = 1 + 4\pi\rho\frac{1}{q} \int (g(r) - 1) \cdot r \sin(qr) dr$$

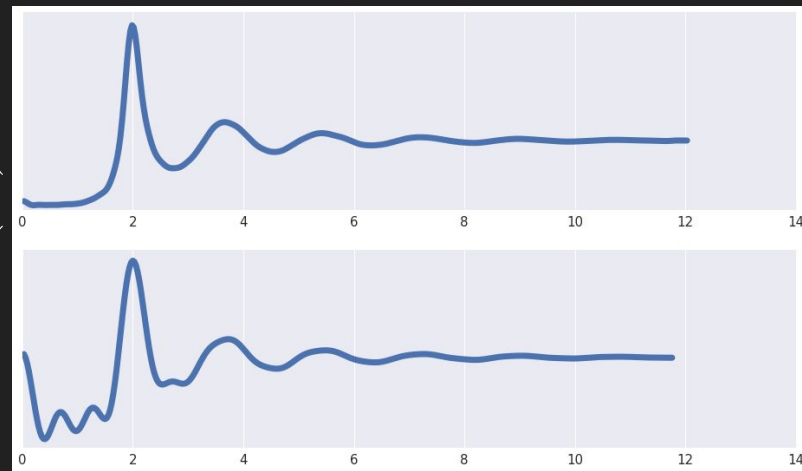


# Finite size effects

$$S(Q) = 1 + 4\pi\rho\frac{1}{q} \int (g(r) - 1) \cdot r \sin(qr) dr$$



$S(Q)$



$r (\text{\AA}^{-1})$

$Q (\text{\AA}^{-1})$

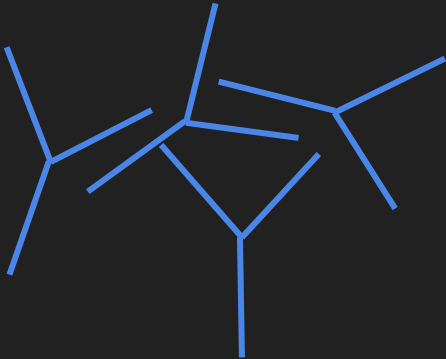
# Desired Features of Calculator

- Simultaneously calculate both  $P(Q)$  and  $S(Q)$
- Avoid finite size effects
- Fast

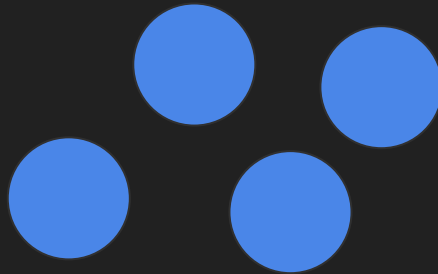
# Debye Formula

$$I(q) = \sum_{j=1}^N \sum_{k=1}^N f_i(q) f_j(q) \frac{\sin(qr_{ij})}{qr_{ij}}$$

Not universally applicable:



vs

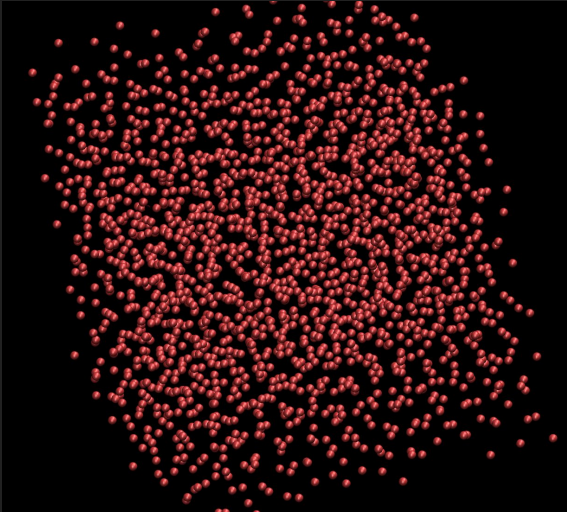


# Explicit Fourier Transform

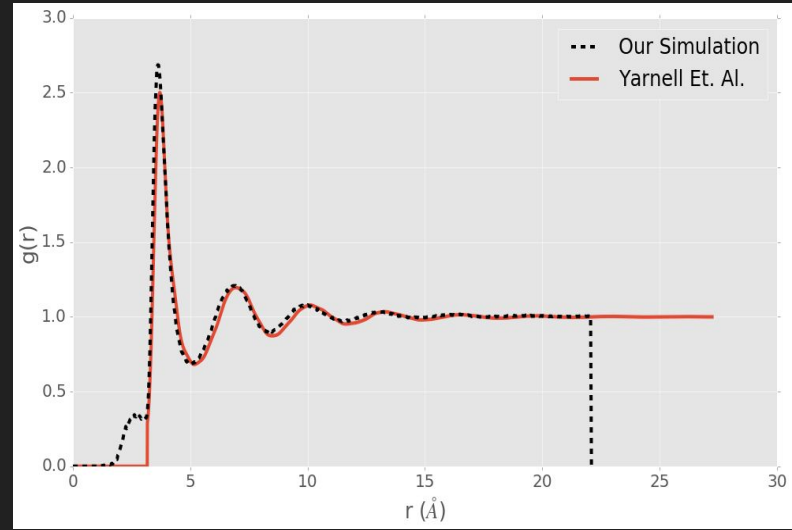
- Most general
  - Contains P(Q) and S(Q)

$$I(q) = I_1(q) \left\langle \sum_{j=1}^N \sum_{k=1}^N e^{-i\mathbf{q}\mathbf{r}_{jk}} \right\rangle = I_1(q) \left\langle \sum_{j=1}^N \sum_{k=1}^N \sin \mathbf{q}\mathbf{r}_{jk} \right\rangle$$

# First Test System - Lennard Jones Particles



Verify Radial Distribution of Simulation

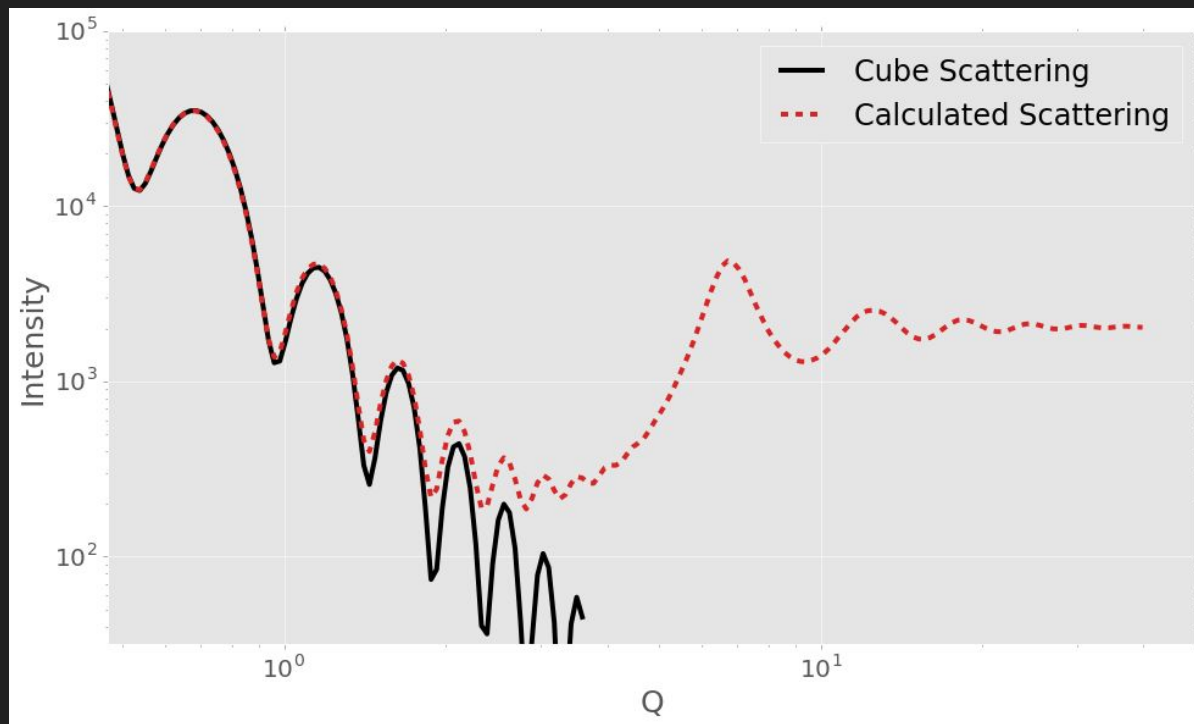




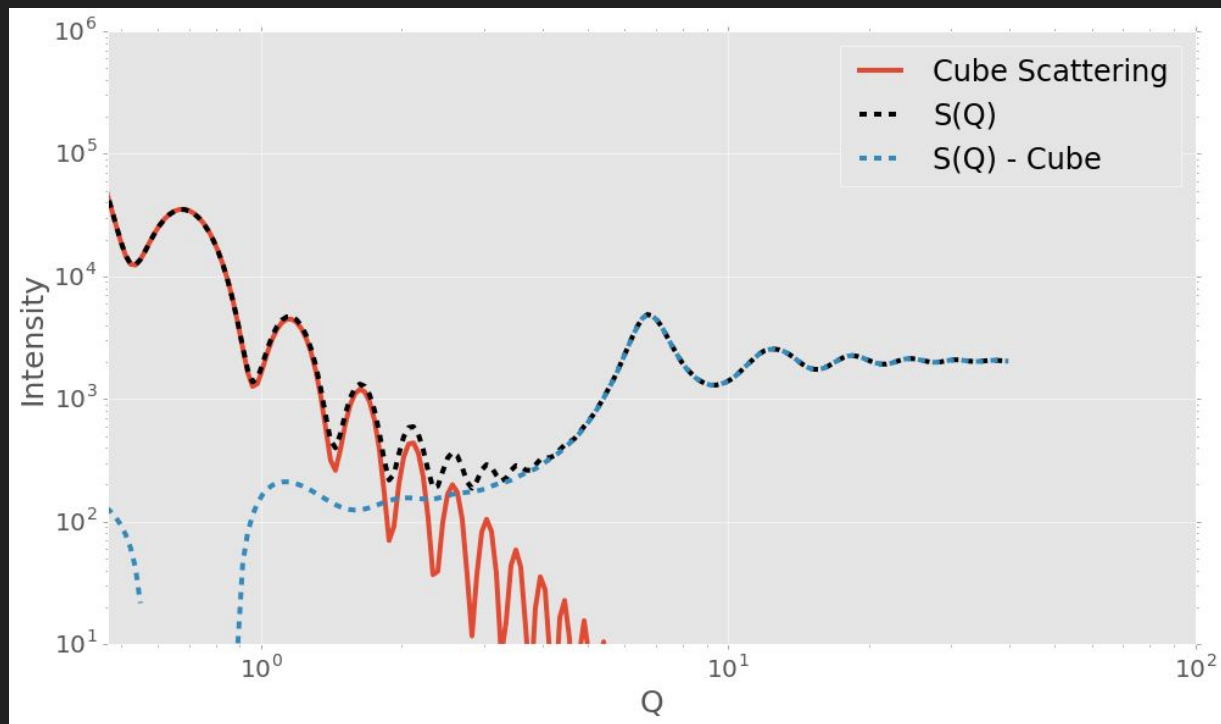
# Solution

- Fancy Stat Mech -  $g(r)$ 
  - Limited to spherical systems
- Bigger Box
  - Num Atoms  $\sim (\text{Box Length})^3$
- Calculate Box scattering and remove

# Removing Box Effects

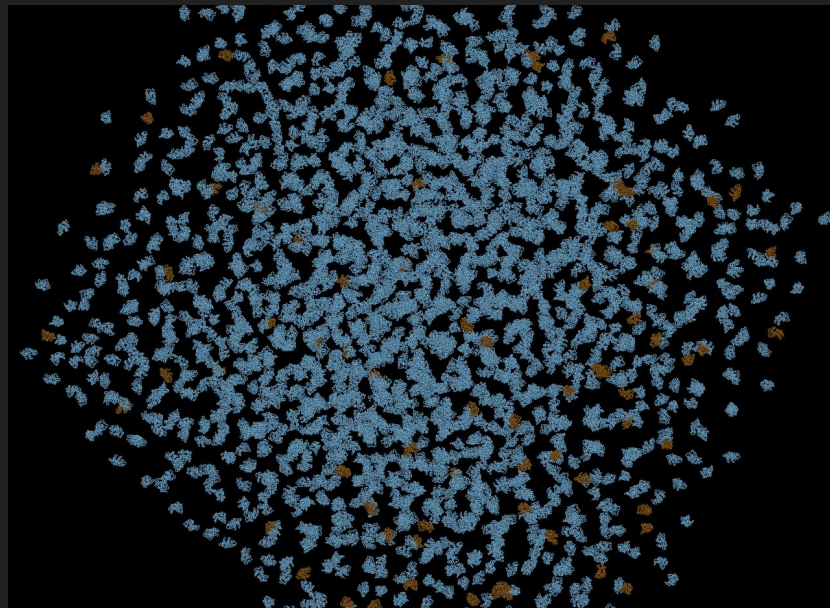
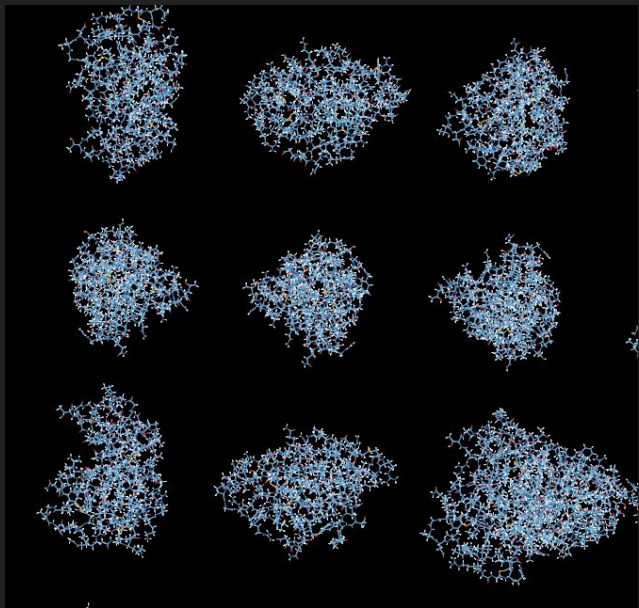


# Removing Box Effects

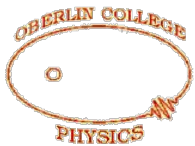


# Next Steps

- Apply this to a periodic box of proteins (lysozyme, mAb)
- Automate algorithm to subtract box effects
- Parallelize using GPUs



# Thank you!



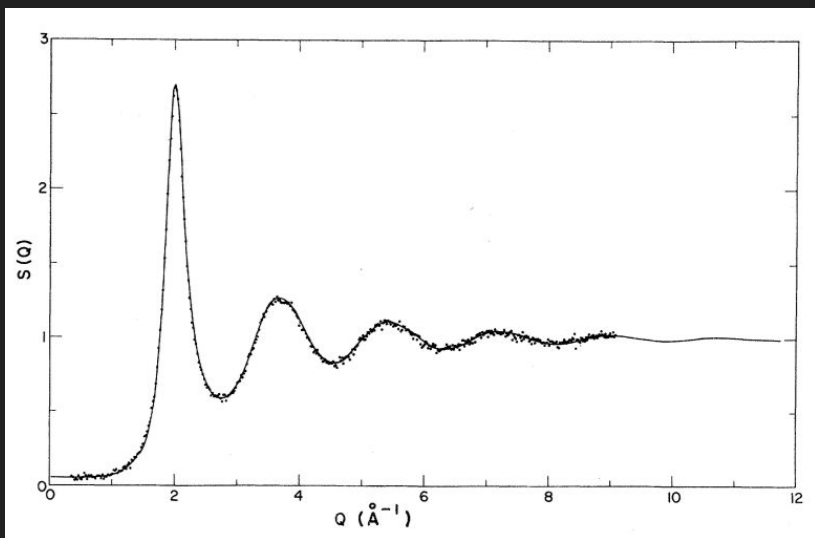
**NIST**



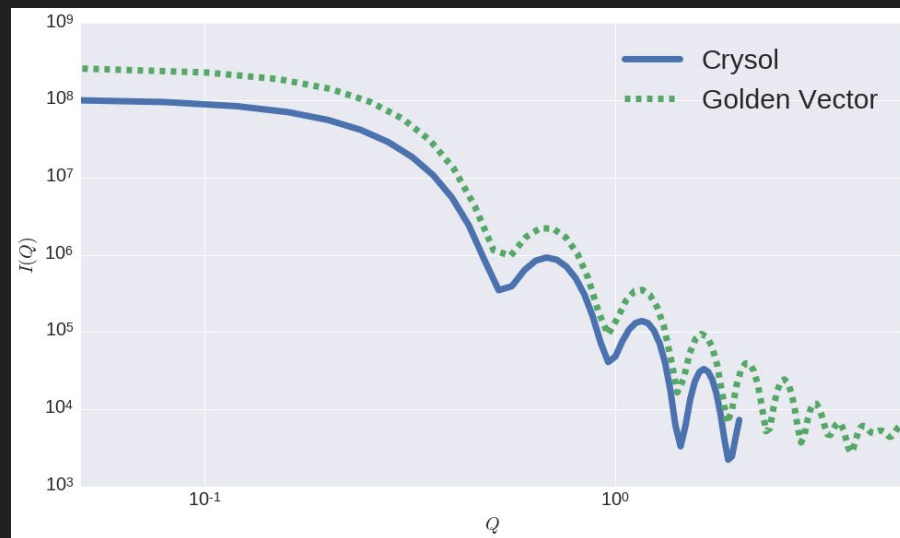
Backup Slides

# Existing Calculators

Experiment



Simulation



Watch out  $S(Q) \neq I(Q)$ .

## S(Q) via Fourier Transform

- $g(r)$  simple to calculate
- Can extend  $g(r)$  to large  $r$

$$h(r) = g(r) - 1$$

$$S(Q) = \frac{1}{1 - \tilde{h}(q)}$$



# Sascalc - Golden Vector

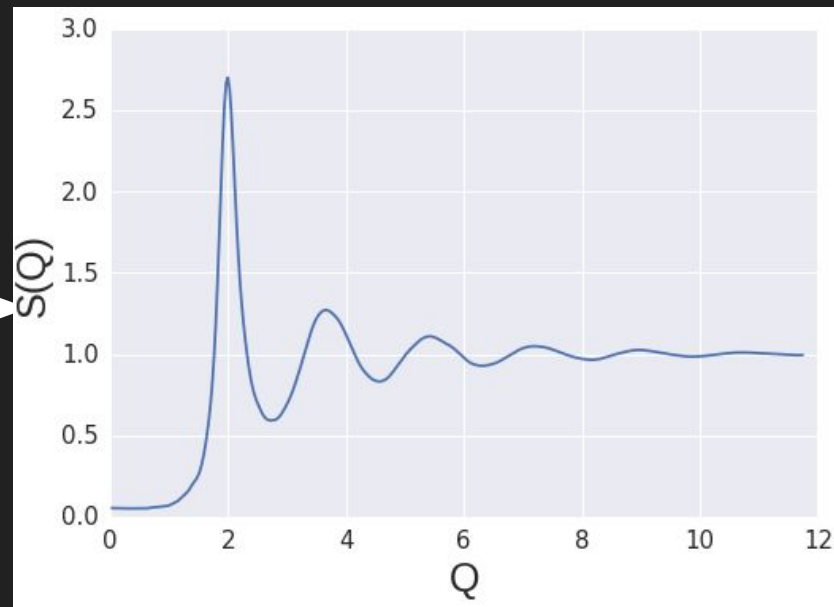
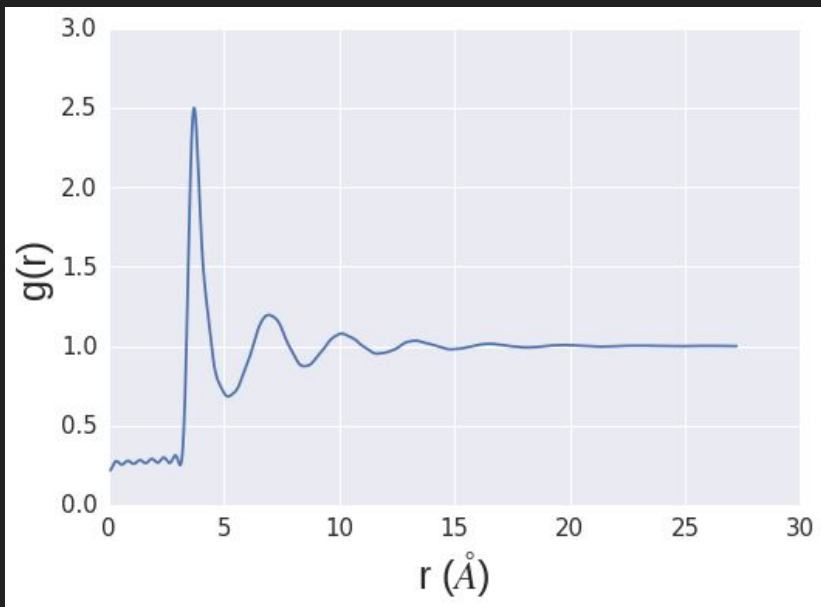
$$I(\mathbf{q}) = \left[ \sum_j^N b_j \cos(\mathbf{q} \cdot \mathbf{r}_j) \right]^2 + \left[ \sum_j^N b_j \sin(\mathbf{q} \cdot \mathbf{r}_j) \right]^2$$

- Multiple Molecules?
- Separation of S and P?

Two questions:

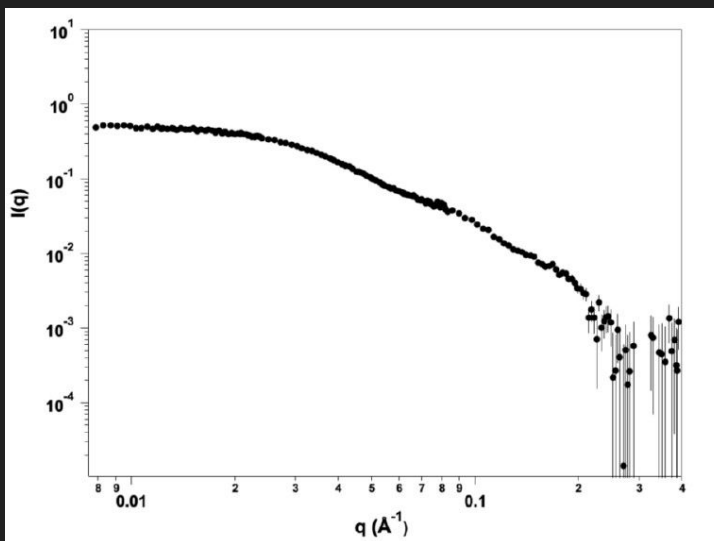
- Can Extract S?
- Periodic Boundary Conditions effects?

$$\int_0^{\infty} (g(r) - 1) \cdot r \sin(qr) dr$$

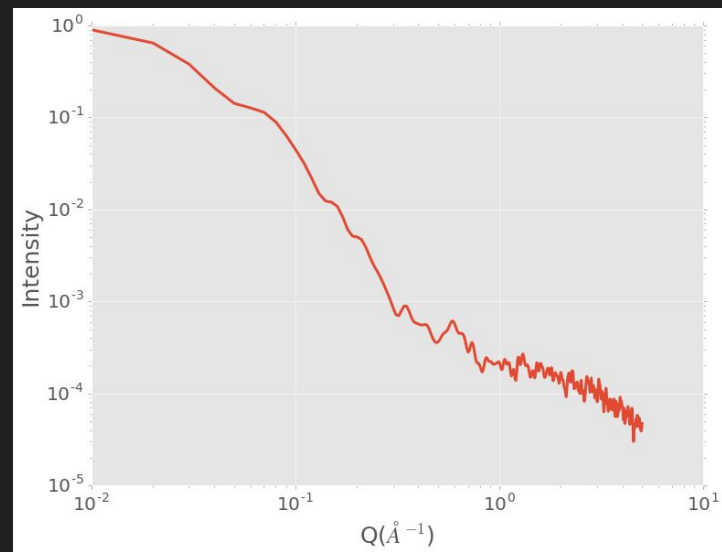


# How do we study?

Experiment



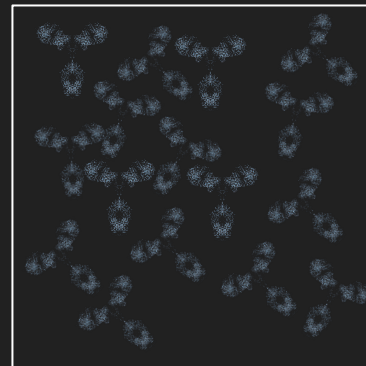
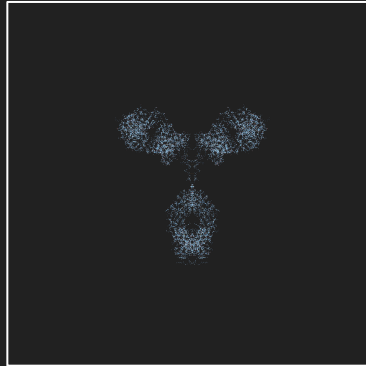
Simulation



# Current Limitations

	10 mg/ml	100 mg/ml
Of Medical interest?	YES	YES
Can we calculate Scattering?	YES	NO (somewhat)

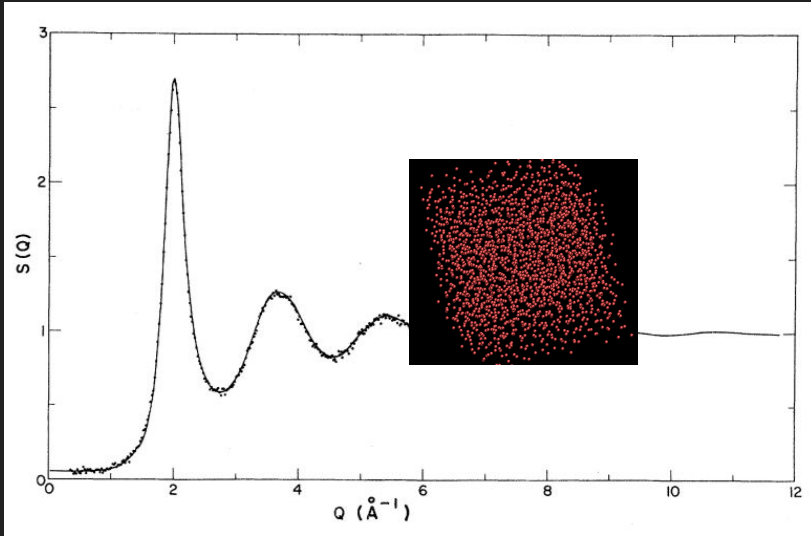
Simulation Boxes:



# Methods to Calculate $S(Q)$

- Pair Distance Distribution
- Debye Formula
- Explicit Fourier Transform

# First Test System - Lennard Jones Particles



Taken from Structure Factor and radial Distribution Function for Liquid Argon at 85 K - Yarnell, Et. al.

## Verify Pair Distance Distribution of Simulation

