

Embedded functions in combinatorial testing: Progress in automating test design

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Combinatorial testing

- A way to design tests to verify many inputs and configurations, with a small number of test cases
 - E.g. 30 factors: 6 with 3 values, 24 with 2 values
 - Possible test cases: $3^6 2^{24} = 12,230,590,464$
 - Pairwise test cases: 15 (2103 interactions)
 - Most failures are caused by only 1 or 2 factors
- Automated process based on some analysis
- Black-box perspective (requirements, interfaces)






Topics

- Origins of combinatorial testing and early tools
- Constraint needs and approaches
- Embedded functions concepts and uses
 - Combination functions
 - Substitution functions
- Higher strength needs and approaches
- An example. Designs to verify:
 - Equivalence classes (ECs) of expected results
 - Boundaries of expected results ECs
 - Interactions between EC boundaries



Origins of combinatorial testing

| | | | |
|------|---|---|-----------------------|
| 1920 | | | |
| |  | Design of Experiments | |
| 1940 | |  | Orthogonal Arrays |
| | |  | OAs for Manufacturing |
| 1960 | | | |
| | | | Covering Arrays |
| 1980 | | | OAs for Software |
| | | OATS, CATS, AETG | Pairwise testing |
| 2000 | | ... | ... |
| | | Many more tools | Much more testing |
| 2020 | | | |



Orthogonal arrays

| | | |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |

\mathcal{N} size, number of rows or test cases

k number of columns or test factors

u number of symbols or values for each column

t strength

In $OA(\mathcal{N}; u^k)$ every $\mathcal{N} \times t$ subarray contains every t -tuple from u symbols an equal number of times

In $OA(\mathcal{N}; u_1^{k_1} u_2^{k_2} \dots u_s^{k_s})$ every $\mathcal{N} \times t$ subarray contains every t -tuple an equal number of times



Orthogonal arrays for software testing

- Keizo Tatsumi, Robert Mandl & others applied these ideas to software testing
- Taguchi visited Bell Labs and collaborated with Madhav Phadke
- Orthogonal Array Test System (OATS) offered statistical coverage where exhaustive testing was not possible



StarLAN and StarGROUP

- Client installation test
 - 101 combinations of PCs and OS versions
 - 15 network cards
- Tried to use OATS to reduce number of test configurations
- OATS could not handle our complex configuration constraints correctly
- Generated impossible test configurations



Constraint violation example

The example has 3 factors:

- 2 OS values
- 3 Browser values
- 3 Application values

These test cases cover all pairs of factor values:

- 6 OS-Browser pairs
- 6 OS-Application pairs
- 9 Browser-Application pairs

But the Linux-IE pair is disallowed

What to do?

Skip test case 6? Then Linux-App1 pair and IE-App1 pair are not covered

Change any 1 value in test case 6? Then at least one of the pairs still is not covered

Solution: Restrict search to allowed combinations only

| Constraint violation yields impossible test case | | | |
|---|-----------|----------------|--------------------|
| <i>Test Case</i> | <i>OS</i> | <i>Browser</i> | <i>Application</i> |
| 1 | Windows | Chrome | App1 |
| 2 | Windows | Firefox | App2 |
| 3 | Windows | IE | App3 |
| 4 | Linux | Chrome | App2 |
| 5 | Linux | Firefox | App3 |
| 6 | Linux | IE | App1 |
| 7 | Windows | Chrome | App3 |
| 8 | Windows | Firefox | App1 |
| 9 | Windows | IE | App2 |



Constrained array test system (CATS)

- Search among allowed test cases to handle constraints
- Find next “best” test case to minimize uncovered combinations
- Higher strength: $t \leq k$
- Secondary result: fewer test cases than OATS
- CATS was searching for covering arrays, not orthogonal arrays



Orthogonal & covering arrays

\mathcal{N} size, number of rows or test cases

k number of columns or test factors

u number of symbols or values for each column

t strength

In $OA(\mathcal{N}; u_1^{k_1} u_2^{k_2} \dots u_s^{k_s})$ every $\mathcal{N} \times t$ subarray contains every t -tuple **an equal number of times**

In $CA(\mathcal{N}; u_1^{k_1} u_2^{k_2} \dots u_s^{k_s})$ every $\mathcal{N} \times t$ subarray contains every t -tuple **at least once**



Covering array templates

| Covering arrays can be smaller than orthogonal arrays | | |
|--|---------------------------------------|--|
| | <i>Covering arrays</i> | <i>Orthogonal Arrays</i> |
| <i>Fixed values</i> | CA(12; 3 ⁷) | OA(18; 3 ⁷) |
| <i>Mixed values</i> | CA(6; 3 ¹ 2 ⁴) | OA(12; 3 ¹ 2 ⁴) |

Many authors have found covering arrays using diverse methods
Charlie Colbourn, Arizona State University:

Covering array tables for $t = 2, 3, 4, 5, 6$

www.public.asu.edu/~ccolbou/src/tabby/catable.html



Constraint needs

- Invalid configurations can mask valid combinations, e.g. Linux with Internet Explorer
- Combinations of valid inputs can be invalid, as in the date Feb 30, 2017
- Combinations of factor values determine ECs of expected results

Constraint approaches

- Logic based: (OS = "Windows") =>
(Browser = "IE" || Browser = "Firefox" || Browser = "Chrome")
- Embedded functions: fBrowser(\$OS) represents Browser value according to \$OS value



Functionally dependent test factor values

- Constraints can be described using functionally dependent test factor values
- Functional dependence:
 - 1 or more values of a dependent factor are identified by other, determinant factors
 - Determinant factors' values \rightarrow dependent factor values
- Example: The last day of any month is identified by its month and year
 - Month, Year \rightarrow Last day values
 - ℓ = number of determinant factors ($\ell = 2$ in this example)
- Use Direct Product Block (DPB) notation with or without embedded combination functions



Direct product block (DPB) notation

Fixed values form

```
Calendar Example without last_day function
Month
Day
Year
#ok All good dates
jan feb mar apr may jun jul aug sep oct nov dec
1 10
2015 2016 2017
+ long month last day
jan mar may jul aug oct dec
31
2015 2016 2017
+ short month last day
apr jun sep nov
30
2015 2016 2017
+ feb last day
feb
28
2015 2017
+ leap day
feb
29
2016
```

- Valid calendar dates example with boundary checking
- Factor values are on separate lines
- All combinations in a block are allowed
- Partition of multiple blocks includes union of their allowed combinations



Direct product block (DPB) notation

Fixed values form

Calendar Example without last_day function

Month
Day
Year
#ok All good dates
jan feb mar apr may jun jul aug sep oct nov dec
1 10
2015 2016 2017
+ long month last day
jan mar may jul aug oct dec
31
2015 2016 2017
+ short month last day
apr jun sep nov
30
2015 2016 2017
+ feb last day
feb
28
2015 2017
+ leap day
feb
29
2016

Functionally dependent form

Calendar Example with last_day function

\$month
Day
\$year
#ok All good dates
jan feb mar apr may jun jul aug sep oct nov dec
1 10 last_day(\$month,\$year)
2015 2016 2017

- Month, Year → Last day value
- Factors renamed as variables for function arguments:
\$month \$year
- Day values:
1 10 last_day(\$month,\$year)
- 5 blocks now represented by only 1 block



Combination functions

- `last_day($month,$year)` is a combination function
- Combination functions return dependent values for all allowed combinations of determinant factor values
- Generator uses these fixed values to construct test cases
- `last_day($month,$year)` needs to return the last day for any month in the years 2015 2016 2017
- PHP built-in function `cal_days_in_month` is reused:

```
function last_day($month,$year) {  
    $mo_num=array('jan'=>1,'feb'=>2,'mar'=>3,'apr'=>4,'may'=>5,'jun'=>6,  
                 'jul'=>7,'aug'=>8,'sep'=>9,'oct'=>10,'nov'=>11,'dec'=>12);  
    return(cal_days_in_month(CAL_GREGORIAN,$mo_num[$month],(int)$year));  
}
```



Substitution functions

- A substitution function returns a value for each test case after test case generation
- A substitution function value can be determined by other factor values its test case
- Substitution functions can identify equivalence classes for the **expected results** of each test case
 - To evaluate expected results classes automatically
 - To assess coverage of expected results classes
- Equivalence class functions help manage verification of results classes



Interaction rule

- Studies show a high proportion of failures are caused by faults in only 1 or 2 factors
- Higher strength designs ($t > 2$) can find more failures
 - at a diminishing rate
 - with an increase in test cases (\mathcal{N})

Higher strength needs

- Nature of failure, more determinant factors
- ECs of expected results depend on more inputs
- Difficulty conforming to constraints for valid test cases
- Inadequate specifications, e.g. interfaces, behavior



Higher strength approaches

- Increase strength until no more failures are found
 - thorough & expensive: $\mathcal{N} \sim u^t \log k$
- Apply higher strength to a subset of factors
 - less expensive (smaller \mathcal{N})
 - test factor risk estimate
- Use embedded functions to constrain factor values
 - even less expensive ($t = 2$)
 - EC functions to verify classes of expected results
 - simplifies constraints for valid test cases



BMI report requirements

- R1. The listed input data will be stored in the patient database table.
- a. Age in years (integer) $2 \leq \text{Age} < 130$
 - b. Weight in pounds (integer) $20 \leq \text{Weight} < 500$
 - c. Height in inches (integer) $30 \leq \text{Height} < 90$
 - d. Sex (female, male)
 - e. Intake in kilocalories per day (integer) $1 \leq \text{Intake} < 10000$
- R2. The BMI will be calculated (in kilograms per meter squared) as $703.06957964 \text{ Weight} / \text{Height}^2$ and stored in the patient database table.
- R3. If Age is 65 years or older, the Medicare report will be generated.
- R4. If Age is younger than 20 years, the Child report containing the BMI percentile will be generated for the corresponding listed classification.
- a. Girl, from the female BMI-age table
 - b. Boy, from the male BMI-age table
- R5. If Age is 20 years or older, the Adult report will be generated for the corresponding listed classification.
- a. Underweight BMI < 18.5
 - b. Normal $18.5 \leq \text{BMI} < 25.0$
 - c. Overweight $25.0 \leq \text{BMI} < 30.0$
 - d. Obese $30.0 \leq \text{BMI}$



BMI report equivalence classes

- Equivalence classes group test factor combinations by similar expected results
- Classes help insure test design coverage
Example: The Medicare, Child and Adult reports each have multiple, valid equivalence classes

| Report | Valid equivalence classes | | | | |
|-----------------|---------------------------|-------------|--------|------------|-------|
| <i>Medicare</i> | no | yes | | | |
| <i>Child</i> | no | girl | boy | | |
| <i>Adult</i> | no | underweight | normal | overweight | obese |

- Equivalence classes are functionally dependent
Input, configuration values → result → equivalence class
- Report classes can be expressed as 3 functions



Equivalence class functions 1

```
function Medicare_report($Age) {  
    if($Age>=65) return('yes');  
    if($Age>0) return('no');  
}
```

/* Medicare_report equivalence class function */
/* Medicare_report is expected result */
/* Medicare_report is not expected result */

```
function Child_report($Age,$Sex) {  
    if($Age>0) {  
        switch($Sex) {  
            case 'female':  
                if($Age<20) return('girl');  
                else return('no');  
            case 'male':  
                if($Age<20) return('boy');  
                else return('no');  
        }  
    }  
}
```

/* Child_report equivalence class function */
/* Child_report for girl is expected result */
/* Child_report is not expected result */
/* Child_report for boy is expected result */
/* Child_report is not expected result */



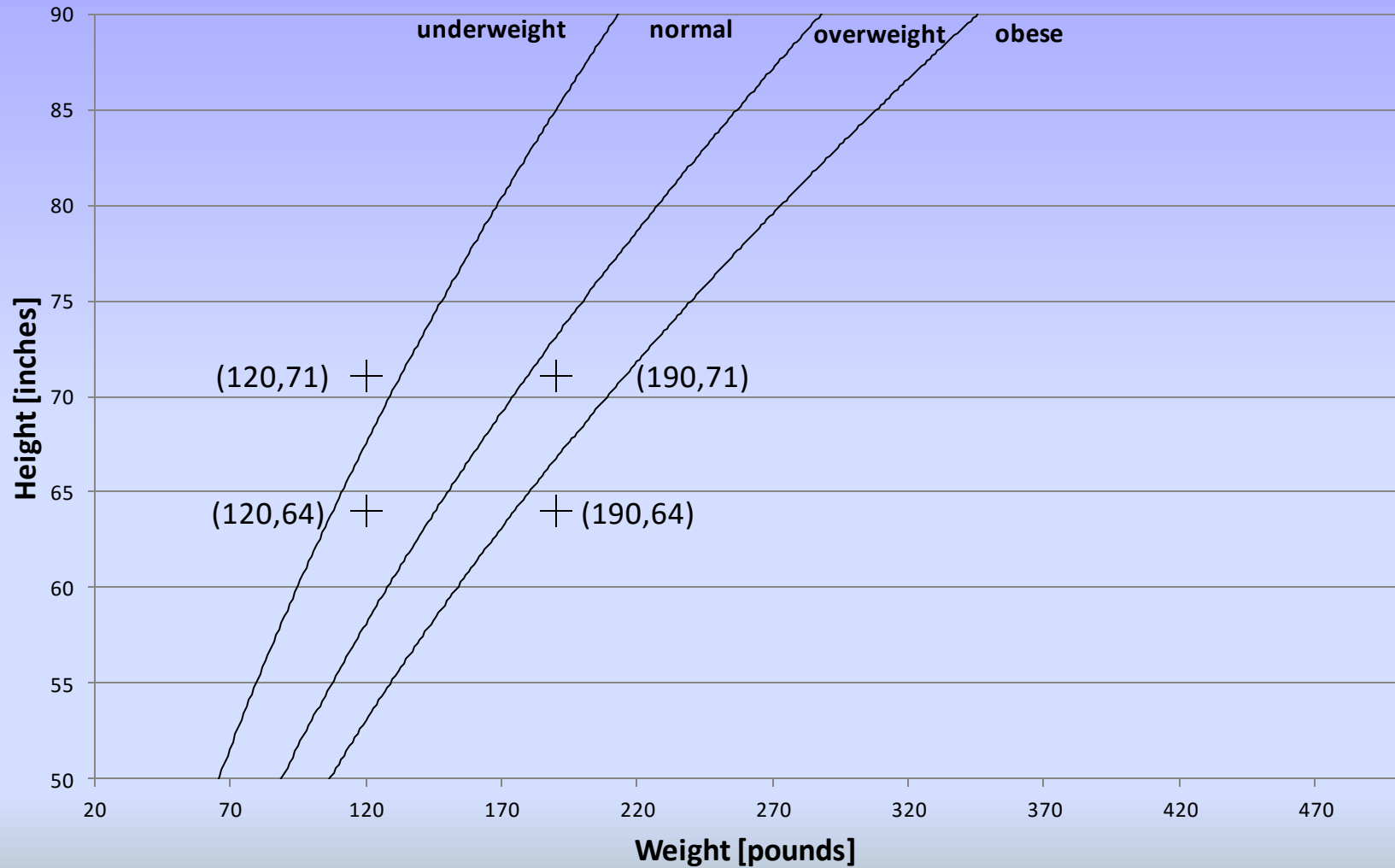
Equivalence class functions 2

```
function Adult_report($Age,$Weight,$Height) { /* Adult_report equivalence class function */
    $bmi_value=BMI($Weight,$Height); /* Use BMI function to get BMI value */
    if($Age>=20&&$bmi_value>0) {
        if($bmi_value>=30) return('obese'); /* Adult_report for obese is expected result */
        if($bmi_value>=25) return('overweight'); /* Adult_report for overweight is expected result */
        if($bmi_value>=18.5) return('normal'); /* Adult_report for normal is expected result */
        if($bmi_value>0) return('underweight'); /* Adult_report for underweight is expected result */
    }
    if($Age>0&&$bmi_value>0) return('no'); /* Adult_report is not expected result */
}
```

```
function BMI($Weight,$Height) { /* BMI function for Adult_report */
    if($Weight>0&&$Height>0) {
        $bmi_value=703.06957964*$Weight
        /$Height/$Height;
        return($bmi_value); /* return valid BMI value */
    }
}
```



Weight-height values for BMI classes



Equivalence class substitution functions

Accidental equivalence class coverage

\$Age
\$Weight
\$Height
\$Sex
Intake
Medicare equivalence class
Child equivalence class
Adult equivalence class

15 42 67
120 190
64 71
female male
2000 3000
Medicare_report(\$Age)
Child_report(\$Age,\$Sex)
Adult_report(\$Age,\$Weight,\$Height)

Incomplete equivalence class coverage

\$Age
\$Weight
\$Height
\$Sex
Intake
Medicare equivalence class
Child equivalence class
Adult equivalence class

42 67 15
120 190
64 71
female male
2000 3000
Medicare_report(\$Age)
Child_report(\$Age,\$Sex)
Adult_report(\$Age,\$Weight,\$Height)

- Equivalence class functions return expected ECs from determinant factors in generated test cases
- Same pairwise requests, but with Age values rotated



Equivalence class substitution functions

| Accidental equivalence class coverage | | | | | | | | |
|---------------------------------------|---------------|--------|--------|--------|--------|---------------------------|--------------------|--------------------|
| | Input factors | | | | | Equivalence class factors | | |
| Test Case | Age | Weight | Height | Sex | Intake | Medicare ^s | Child ^s | Adult ^s |
| 1 | 67 | 120 | 71 | male | 3000 | yes | no | underweight |
| 2 | 67 | 190 | 64 | female | 2000 | yes | no | obese |
| 3 | 42 | 190 | 71 | male | 2000 | no | no | overweight |
| 4 | 15 | 120 | 64 | male | 2000 | no | boy | no |
| 5 | 42 | 120 | 64 | female | 3000 | no | no | normal |
| 6 | 15 | 190 | 71 | female | 3000 | no | girl | no |
| Equivalence classes covered: | | | | | | 2 of 2 | 3 of 3 | 5 of 5 |

| Incomplete equivalence class coverage | | | | | | | | |
|---------------------------------------|---------------|--------|--------|--------|--------|---------------------------|--------------------|--------------------|
| | Input factors | | | | | Equivalence class factors | | |
| Test Case | Age | Weight | Height | Sex | Intake | Medicare ^s | Child ^s | Adult ^s |
| 1 | 15 | 120 | 71 | male | 3000 | no | boy | no |
| 2 | 15 | 190 | 64 | female | 2000 | no | girl | no |
| 3 | 67 | 190 | 71 | male | 2000 | yes | no | overweight |
| 4 | 42 | 120 | 64 | male | 2000 | no | no | normal |
| 5 | 67 | 120 | 64 | female | 3000 | yes | no | normal |
| 6 | 42 | 190 | 71 | female | 3000 | no | no | overweight |
| Equivalence classes covered: | | | | | | 2 of 2 | 3 of 3 | 3 of 5 |

- Same input factors in test cases, but with Age values rotated
- Different expected ECs: underweight & obese classes missing
- Strength $t = 2 < 3 = \ell$, number of Adult determinant factors



Equivalence class combination functions

Equivalence class coverage using EC factors

\$Age

\$Weight

\$Height

\$Sex

Intake

Medicare equivalence class

Child equivalence class

Adult equivalence class

#

15 42 67

120 190

64 71

female male

2000 3000

Medicare_report(\$Age)

Child_report(\$Age,\$Sex)

Adult_report(\$Age,\$Weight,\$Height)

- Same pairwise request, but with EC combination functions
- EC functions return results classes from determinant factors for test cases
- 1 functionally dependent block → 10 fixed values blocks
- Every allowed EC value appears in at least one test case with a combination of its determinant factor values



Equivalence class combination functions

| Equivalence class coverage using equivalence class factors | | | | | | | | |
|--|---------------|--------|--------|--------|--------|---------------------------|--------------------|--------------------|
| Test Case | Input factors | | | | | Equivalence class factors | | |
| | Age | Weight | Height | Sex | Intake | Medicare ^c | Child ^c | Adult ^c |
| 1 | 42 | 190 | 71 | female | 2000 | no | no | overweight |
| 2 | 67 | 120 | 64 | male | 3000 | yes | no | normal |
| 3 | 15 | 120 | 64 | male | 2000 | no | boy | no |
| 4 | 15 | 190 | 64 | female | 3000 | no | girl | no |
| 5 | 67 | 120 | 71 | female | 2000 | yes | no | underweight |
| 6 | 42 | 190 | 64 | male | 3000 | no | no | obese |
| 7 | 67 | 190 | 71 | male | 3000 | yes | no | overweight |
| 8 | 42 | 120 | 71 | male | 3000 | no | no | underweight |
| 9 | 42 | 120 | 64 | female | 2000 | no | no | normal |
| 10 | 15 | 190 | 71 | male | 2000 | no | boy | no |
| 11 | 67 | 190 | 64 | female | 2000 | yes | no | obese |
| 12 | 15 | 120 | 64 | female | 2000 | no | girl | no |
| 13 | 15 | 190 | 64 | male | 3000 | no | boy | no |
| 14 | 15 | 190 | 71 | female | 2000 | no | girl | no |
| Equivalence classes covered: | | | | | | 2 of 2 | 3 of 3 | 5 of 5 |

- All 10 expected ECs covered by constraints in pairwise design
- Equivalence classes are paired with nondeterminant factor values



Age boundary values

| Age boundary and edge values | | | | | |
|------------------------------|-------|---------------------------|-------|-------|--|
| Boundaries and edges | | Equivalence class factors | | | |
| Age | Limit | Medicare | Child | Adult | |
| 129 | max | yes | | | |
| 65 | min | | | | |
| 64 | max | no | | | |
| 20 | min | | | | |
| 19 | max | | | | |
| 2 | min | | girl | | |

- Boundaries separate ECs
- Edges are extreme values
- Same pairwise request, now with Age boundary values
- EC combination functions cover expected results as before

Coverage of univariate Age boundaries using EC factors

```

$Age
$Weight
$Height
$Sex
Intake
Medicare equivalence class
Child equivalence class
Adult equivalence class
#
19 20 64 65
120 190
64 71
female male
2000 3000
Medicare_report($Age)
Child_report($Age,$Sex)
Adult_report($Age,$Weight,$Height)
    
```



Age boundaries with EC factors

| Coverage of univariate equivalence class boundaries using equivalence class factors | | | | | | | | |
|---|---------------|--------|--------|--------|--------|---------------------------|--------------------|--------------------|
| Test Case | Input factors | | | | | Equivalence class factors | | |
| | Age | Weight | Height | Sex | Intake | Medicare ^c | Child ^c | Adult ^c |
| 2 | 65 | 120 | 64 | male | 3000 | yes | no | normal |
| 6 | 65 | 120 | 71 | female | 2000 | yes | no | underweight |
| 8 | 65 | 190 | 71 | male | 3000 | yes | no | overweight |
| 12 | 65 | 190 | 64 | female | 2000 | yes | no | obese |
| 4 | 64 | 190 | 64 | female | 3000 | no | no | obese |
| 7 | 64 | 120 | 71 | male | 2000 | no | no | underweight |
| 15 | 64 | 190 | 71 | male | 2000 | no | no | overweight |
| 16 | 64 | 120 | 64 | male | 2000 | no | no | normal |
| 1 | 20 | 190 | 71 | female | 2000 | no | no | overweight |
| 9 | 20 | 120 | 64 | female | 2000 | no | no | normal |
| 10 | 20 | 190 | 64 | male | 3000 | no | no | obese |
| 13 | 20 | 120 | 71 | male | 3000 | no | no | underweight |
| 3 | 19 | 120 | 64 | male | 2000 | no | boy | no |
| 5 | 19 | 120 | 71 | female | 3000 | no | girl | no |
| 11 | 19 | 190 | 71 | male | 2000 | no | boy | no |
| 14 | 19 | 120 | 64 | female | 2000 | no | girl | no |
| 17 | 19 | 190 | 64 | male | 3000 | no | boy | no |
| 18 | 19 | 190 | 71 | female | 2000 | no | girl | no |
| Equivalence classes covered: | | | | | | 2 of 2 | 3 of 3 | 5 of 5 |

- Test cases are sorted to show age boundaries
- All 10 expected ECs covered by constraints in pairwise design



Weight function for BMI boundary value

$$\text{BMI} = 703.06957964 \cdot \text{Weight} / \text{Height}^2$$
$$\text{Weight} = \text{Height}^2 \cdot \text{BMI} / 703.06957964$$

Height values: 64 71

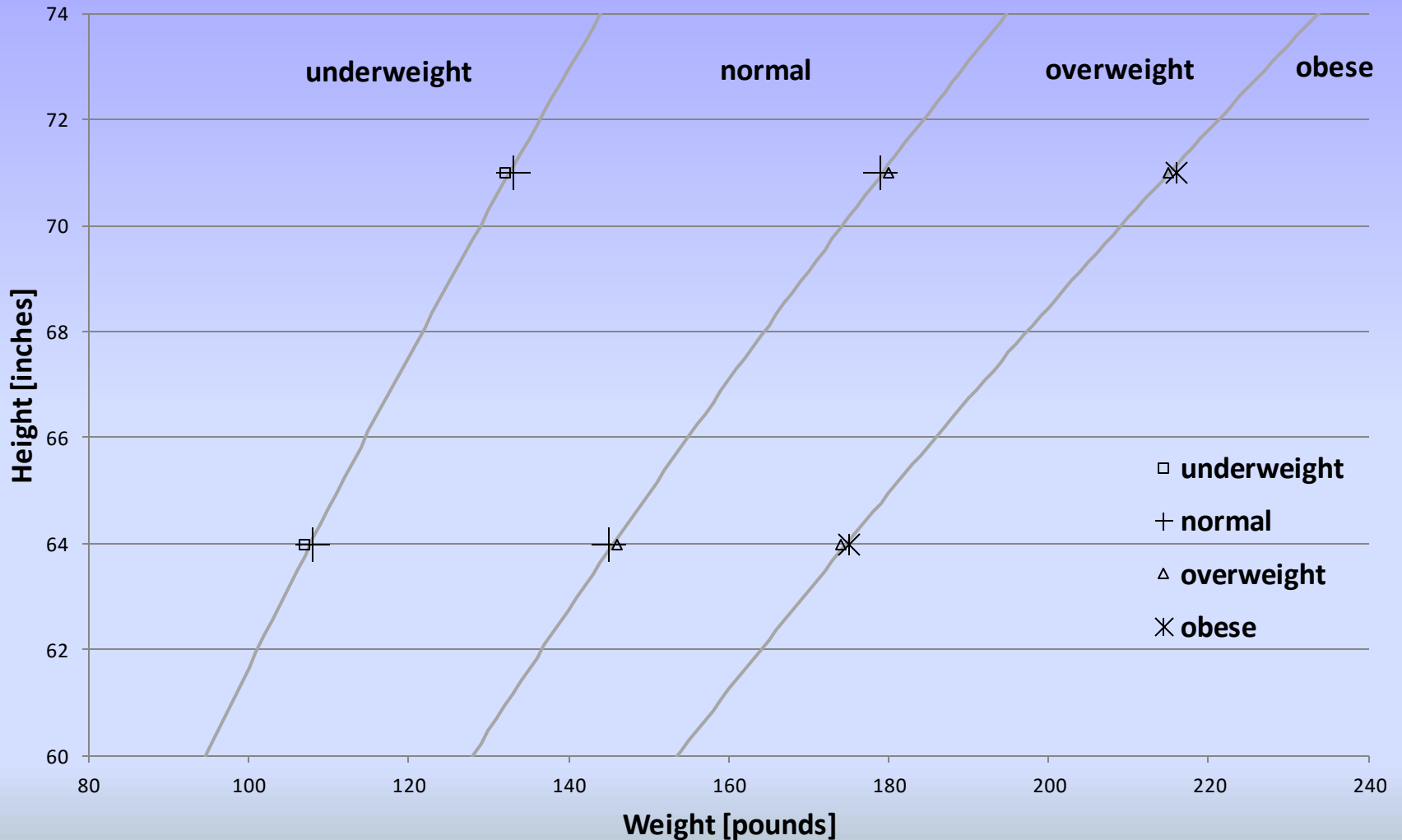
BMI_boundary values: 18.5 25 30

Input_limit values: min max

```
function Weight_boundary($Height,$BMI_boundary,$Input_limit) { /* compute weight input */
  if($Height>0&&$BMI_boundary>0) {
    $w_hi=ceil($Height*$Height*$BMI_boundary/703.06957964); /* round up $w_hi */
    switch($Input_limit) { /* so BMI >= $BMI_boundary */
      case 'min':
        return($w_hi); /* return min integer value */
      case 'max':
        $w_lo=$w_hi-1; /* for higher BMI class */
        return $w_lo; /* return max integer value */
    } /* for lower BMI class */
  }
}
```



BMI boundary values



Age & BMI boundaries – all corners

Coverage of Age & BMI boundaries using BV factors with dependent Weight values – all corners

\$Age

\$Weight

\$Height

\$Sex

Intake

Medicare equivalence class

Child equivalence class

Adult equivalence class

\$BMI_boundary

\$Input_limit

#

19 20 64 65

Weight_boundary(\$Height,\$BMI_boundary,\$Input_limit)

64 71

female male

2000 3000

Medicare_report(\$Age)

Child_report(\$Age,\$Sex)

Adult_report(\$Age,\$Weight,\$Height)

18.5 25 30

min max



Age & BMI – all corners – 1 of 2

Coverage of Age & BMI boundaries using boundary value factors with dependent Weight values – all corners

| Test Case | Input factors | | | | | Equivalence class factors | | | BMI BV factors | |
|-----------|---------------|---------------------|--------|--------|--------|---------------------------|--------------------|--------------------|----------------|-------|
| | Age | Weight ^c | Height | Sex | Intake | Medicare ^s | Child ^s | Adult ^s | BMI | Limit |
| 1 | 19 | 180 | 71 | female | 2000 | no | girl | no | 25 | min |
| 2 | 65 | 174 | 64 | male | 3000 | yes | no | overweight | 30 | max |
| 3 | 20 | 132 | 71 | male | 2000 | no | no | underweight | 18.5 | max |
| 4 | 64 | 108 | 64 | female | 3000 | no | no | normal | 18.5 | min |
| 5 | 20 | 216 | 71 | female | 3000 | no | no | obese | 30 | min |
| 6 | 19 | 145 | 64 | male | 3000 | no | boy | no | 25 | max |
| 7 | 64 | 215 | 71 | male | 2000 | no | no | overweight | 30 | max |
| 8 | 65 | 179 | 71 | female | 2000 | yes | no | normal | 25 | max |
| 9 | 20 | 146 | 64 | male | 2000 | no | no | overweight | 25 | min |
| 10 | 65 | 133 | 71 | female | 2000 | yes | no | normal | 18.5 | min |
| 11 | 19 | 107 | 64 | female | 2000 | no | girl | no | 18.5 | max |
| 12 | 19 | 175 | 64 | female | 2000 | no | girl | no | 30 | min |
| 13 | 64 | 146 | 64 | female | 3000 | no | no | overweight | 25 | min |
| 14 | 65 | 180 | 71 | male | 3000 | yes | no | overweight | 25 | min |
| 15 | 64 | 132 | 71 | female | 3000 | no | no | underweight | 18.5 | max |
| 16 | 19 | 179 | 71 | male | 3000 | no | boy | no | 25 | max |
| 17 | 64 | 216 | 71 | male | 2000 | no | no | obese | 30 | min |
| 18 | 20 | 215 | 71 | female | 3000 | no | no | overweight | 30 | max |
| 19 | 19 | 133 | 71 | male | 3000 | no | boy | no | 18.5 | min |
| 20 | 19 | 174 | 64 | female | 2000 | no | girl | no | 30 | max |
| 21 | 65 | 107 | 64 | male | 3000 | yes | no | underweight | 18.5 | max |
| 22 | 65 | 145 | 64 | female | 2000 | yes | no | normal | 25 | max |
| 23 | 65 | 175 | 64 | male | 3000 | yes | no | obese | 30 | min |
| 24 | 20 | 108 | 64 | male | 2000 | no | no | normal | 18.5 | min |



Age & BMI – all corners – 2 of 2

Coverage of Age & BMI boundaries using boundary value factors with dependent Weight values – all corners

| Test Case | Input factors | | | | | Equivalence class factors | | | BMI BV factors | |
|-----------|---------------|---------------------|--------|--------|--------|---------------------------|--------------------|--------------------|----------------|-------|
| | Age | Weight ^c | Height | Sex | Intake | Medicare ^s | Child ^s | Adult ^s | BMI | Limit |
| 25 | 20 | 180 | 71 | male | 2000 | no | no | overweight | 25 | min |
| 26 | 64 | 180 | 71 | female | 3000 | no | no | overweight | 25 | min |
| 27 | 19 | 132 | 71 | female | 2000 | no | girl | no | 18.5 | max |
| 28 | 65 | 132 | 71 | female | 2000 | yes | no | underweight | 18.5 | max |
| 29 | 20 | 179 | 71 | male | 2000 | no | no | normal | 25 | max |
| 30 | 64 | 179 | 71 | female | 3000 | no | no | normal | 25 | max |
| 31 | 19 | 216 | 71 | female | 2000 | no | girl | no | 30 | min |
| 32 | 65 | 216 | 71 | female | 2000 | yes | no | obese | 30 | min |
| 33 | 19 | 215 | 71 | female | 2000 | no | girl | no | 30 | max |
| 34 | 65 | 215 | 71 | female | 2000 | yes | no | overweight | 30 | max |
| 35 | 20 | 133 | 71 | male | 2000 | no | no | normal | 18.5 | min |
| 36 | 64 | 133 | 71 | female | 3000 | no | no | normal | 18.5 | min |
| 37 | 20 | 174 | 64 | male | 2000 | no | no | overweight | 30 | max |
| 38 | 64 | 174 | 64 | female | 3000 | no | no | overweight | 30 | max |
| 39 | 20 | 107 | 64 | male | 2000 | no | no | underweight | 18.5 | max |
| 40 | 64 | 107 | 64 | female | 3000 | no | no | underweight | 18.5 | max |
| 41 | 19 | 146 | 64 | female | 2000 | no | girl | no | 25 | min |
| 42 | 65 | 146 | 64 | female | 2000 | yes | no | overweight | 25 | min |
| 43 | 20 | 145 | 64 | male | 2000 | no | no | normal | 25 | max |
| 44 | 64 | 145 | 64 | female | 3000 | no | no | normal | 25 | max |
| 45 | 20 | 175 | 64 | male | 2000 | no | no | obese | 30 | min |
| 46 | 64 | 175 | 64 | female | 3000 | no | no | obese | 30 | min |
| 47 | 19 | 107 | 64 | female | 2000 | no | girl | no | 18.5 | min |
| 48 | 65 | 108 | 64 | female | 2000 | yes | no | normal | 18.5 | min |



Age-BMI 48-corner map

| Test cases covering all Age-BMI corners | | | | | | | | | |
|---|-----|-----|------|-----|-----|-----|-----|-----|--|
| | | BMI | 18.5 | | 25 | | 30 | | |
| Height | Age | | max | min | max | min | max | min | |
| 71 | 65 | min | 28 | 10 | 8 | 14 | 34 | 32 | |
| | 64 | max | 15 | 36 | 30 | 26 | 7 | 17 | |
| | | | | | | | | | |
| | 20 | min | 3 | 35 | 29 | 25 | 18 | 5 | |
| | 19 | max | 27 | 19 | 16 | 1 | 33 | 31 | |
| | | | | | | | | | |
| 64 | 65 | min | 21 | 48 | 22 | 42 | 2 | 23 | |
| | 64 | max | 40 | 4 | 44 | 13 | 38 | 46 | |
| | | | | | | | | | |
| | 20 | min | 39 | 24 | 43 | 9 | 37 | 45 | |
| | 19 | max | 11 | 47 | 6 | 41 | 20 | 12 | |



Age & BMI boundaries – some corners

Coverage of Age & BMI boundaries using BV factors with dependent Weight values – **some corners**

\$Age

\$Weight

\$Height

\$Sex

Intake

Medicare equivalence class

Child equivalence class

Adult equivalence class

\$BMI_boundary

\$Input_limit

#

19 20 64 65

Weight_boundary(\$Height,\$BMI_boundary,\$Input_limit)

64 71

female male

2000 3000

Medicare_report(\$Age)

Child_report(\$Age,\$Sex)

Adult_report(\$Age,\$Weight,\$Height)

18.5 25 30

min max



Age & BMI – some corners

Coverage of Age & BMI boundaries using boundary value factors with dependent Weight values – some corners

| <i>Test Case</i> | <i>Input factors</i> | | | | | <i>Equivalence class factors</i> | | | <i>BMI BV factors</i> | |
|------------------|----------------------|---------------------------|---------------|------------|---------------|----------------------------------|--------------------------|--------------------------|-----------------------|--------------|
| | <i>Age</i> | <i>Weight^s</i> | <i>Height</i> | <i>Sex</i> | <i>Intake</i> | <i>Medicare^s</i> | <i>Child^s</i> | <i>Adult^s</i> | <i>BMI</i> | <i>Limit</i> |
| 1 | 65 | 179 | 71 | male | 2000 | yes | no | normal | 25 | max |
| 2 | 19 | 216 | 71 | female | 3000 | no | girl | no | 30 | min |
| 3 | 20 | 107 | 64 | male | 2000 | no | no | underweight | 18.5 | max |
| 4 | 64 | 145 | 64 | female | 3000 | no | no | normal | 25 | max |
| 5 | 64 | 216 | 71 | male | 2000 | no | no | obese | 30 | min |
| 6 | 65 | 108 | 64 | female | 3000 | yes | no | normal | 18.5 | min |
| 7 | 20 | 216 | 71 | female | 3000 | no | no | obese | 30 | min |
| 8 | 19 | 108 | 64 | female | 2000 | no | girl | no | 18.5 | min |
| 9 | 20 | 146 | 64 | male | 3000 | no | no | overweight | 25 | min |
| 10 | 19 | 179 | 71 | male | 2000 | no | boy | no | 25 | max |
| 11 | 65 | 174 | 64 | male | 2000 | yes | no | overweight | 30 | max |
| 12 | 64 | 132 | 71 | male | 3000 | no | no | underweight | 18.5 | max |



Age-BMI 12-corner map

| Test cases covering some Age-BMI corners | | | | | | | | |
|--|-----|-----|------|-----|-----|-----|-----|-----|
| | | BMI | 18.5 | | 25 | | 30 | |
| Height | Age | | max | min | max | min | max | min |
| 71 | 65 | min | | | 1 | | | |
| | 64 | max | 12 | | | | | 5 |
| | | | | | | | | |
| | 20 | min | | | | | | 7 |
| | 19 | max | | | 10 | | | 2 |
| 64 | 65 | min | | 6 | | | 11 | |
| | 64 | max | | | 4 | | | |
| | | | | | | | | |
| | 20 | min | 3 | | | 9 | | |
| | 19 | max | | 8 | | | | |



Conclusions

- Decades of progress: orthogonal arrays, covering arrays, constraints, greedy searches, higher strength
- Embedded functions:
 - constraints: simple functions in a familiar language
 - control: higher strength for determinant factors
 - efficiency: fewer test cases
 - flexibility: variety of test objectives
 - automation: less manual analysis
- 21st century testing: increasing dependency on software, networks and distributed applications

