



# 3D Face Recognition Using Multi-Region Summation Invariants

Wei-Yang Lin, Kin-Chung Wong, Nigel Boston, Yu Hen Hu

Dept. of Electrical & Computer Engineering  
University of Wisconsin-Madison

This research is partially supported by the NSF under Grant No. CCF-0434355

# Agenda

## ***1. Introduction***

- Focus of FR research

## 2. Invariant Theory

## 3. MSI Algorithm and Results

## 4. Conclusion

# Focus of FR Research

1. Currently focus on 3D FR algorithm
2. A novel family of geometrical invariants based on the method of moving frame
3. Extract summation invariants from local profiles on multi-regions
4. LDA-based fusion method

# Agenda

1. Introduction

***2. Invariant Theory***

- Method of Moving Frame
- Summation Invariants
- Feature Extraction on 3D surface

3. MSI Algorithm and Results

4. Conclusion

# Method of Moving Frame

- A power tool for finding invariants under group actions (É. Cartan, 1935).
- Definition: a **moving frame** is a  $G$ -equivariant mapping  $\rho : M \rightarrow G$ , i.e.

$$\rho(g \circ x) = g\rho(x)$$

# Summation Invariants

- Procedures
  1. Given a transformation over points of a curve, surface, etc.
  2. Define *jet space*.
  3. Solve the moving frame from the *normalization equations*.
  4. Invariants can be derived by applying moving frame on jet space.
- A systematical way to derive geometrical invariants for pattern recognition

# Example

STEP 1 : A point  $(x, y)$  under rotation

STEP 2 : Define jet space as  $(\bar{x}, \bar{y})$

STEP 3 : Normalization equation

$$\bar{y} = x \sin \theta + y \cos \theta = 0 \rightarrow \theta = \arctan\left(\frac{-y}{x}\right)$$

STEP 4 : Apply moving frame to jet space

$$\bar{x} = x \cos \theta - y \sin \theta = \sqrt{x^2 + y^2}$$

# Example : Euclidean Summation Invariants for Curves

STEP 1 : Given a curve  $(x[n], y[n])$  under Euclidean transformation

$$\begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} x[n] \\ y[n] \end{bmatrix} + \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} \bar{x}[n] \\ \bar{y}[n] \end{bmatrix}$$

STEP 2 : Jet space is defined as

$$\underbrace{(\bar{x}[1], \bar{y}[1], \bar{y}[N], P_{1,0}, P_{0,1}, \dots)} \quad P_{i,j} = \sum_{n=1}^N x^i[n] \cdot y^j[n]$$



# Example : Euclidean Summation Invariants of Curves

STEP 3 : We can find a **moving frame** by solving the normalization equations

$$(\bar{x}[1], \bar{y}[1], \bar{y}[N]) = (0,0,0)$$

STEP 4 : Invariants can be obtained by applying moving frame

$$\eta_{i,j} = \sum_{n=1}^N \bar{x}^i[n] \cdot \bar{y}^j[n]$$

where  $\bar{x} = \rho^{-1} \circ x$  and  $\bar{y} = \rho^{-1} \circ y$

# Example : Euclidean Summation Invariants of Curves

- The first-order summation invariants are explicitly shown below

$$\eta_{1,0} = P_{1,0}(x_1 - x_0) + P_{0,1}(y_1 - y_0) + Nx_0(x_0 - x_1) + Ny_0(y_0 - y_1)$$

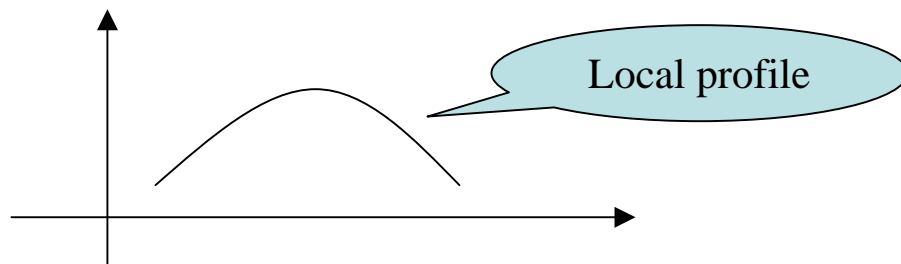
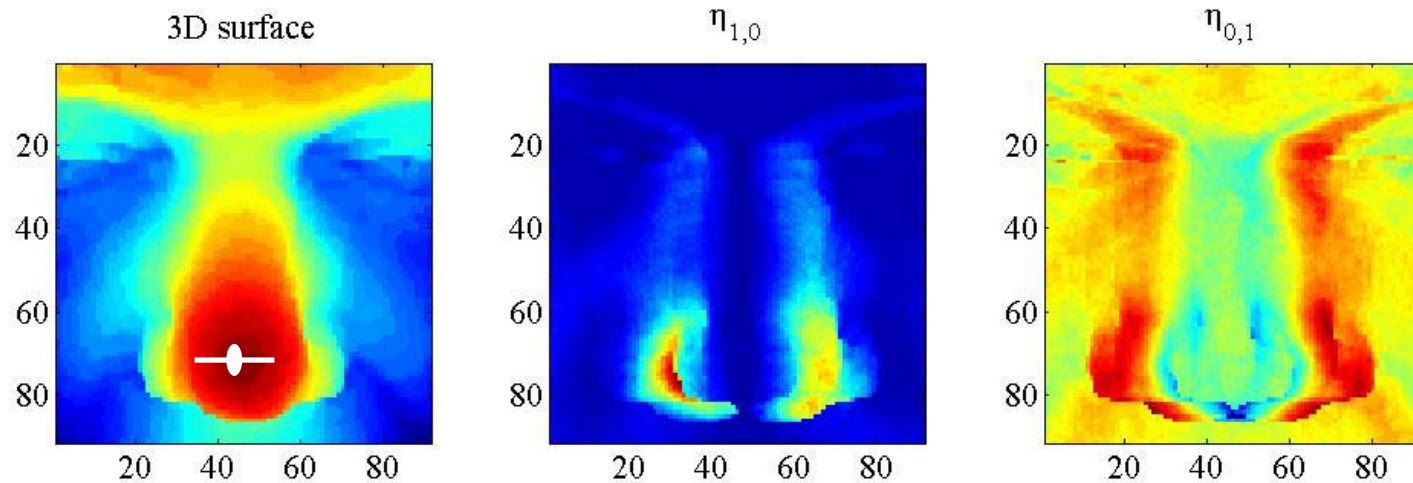
$$\eta_{0,1} = P_{1,0}(y_1 - y_0) + P_{0,1}(x_0 - x_1) + N(x_1y_0 - x_0y_1)$$

where

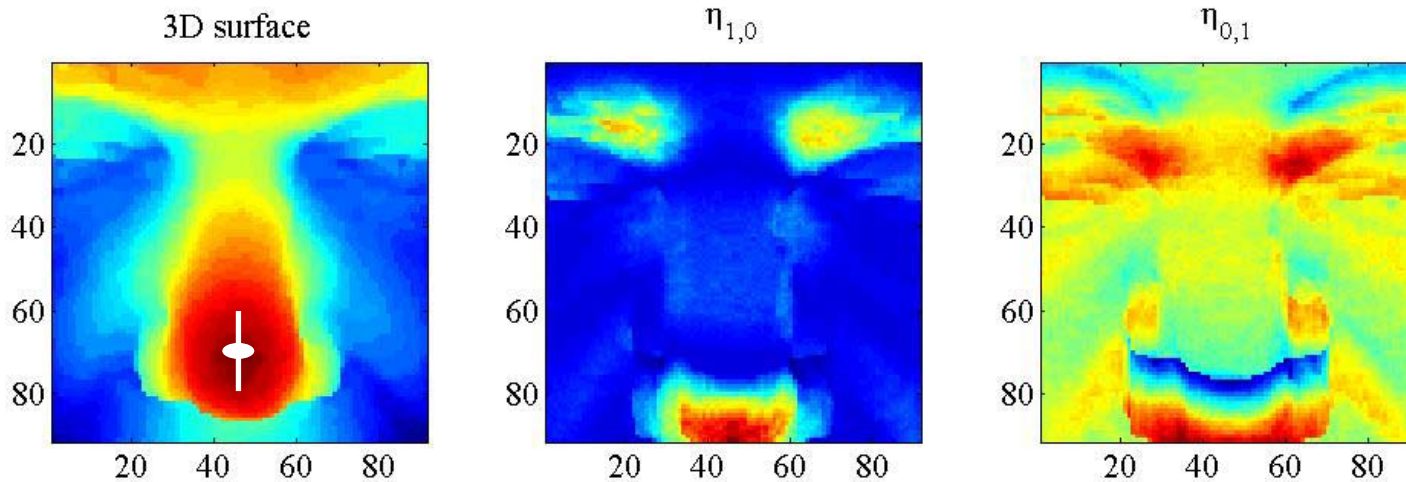
$$x_0 = x[1], x_1 = x[N], y_0 = y[1], y_1 = y[N]$$

$$P_{i,j} = \sum_{n=1}^N x^i[n] \cdot y^j[n]$$

# Feature extraction on 3D surface



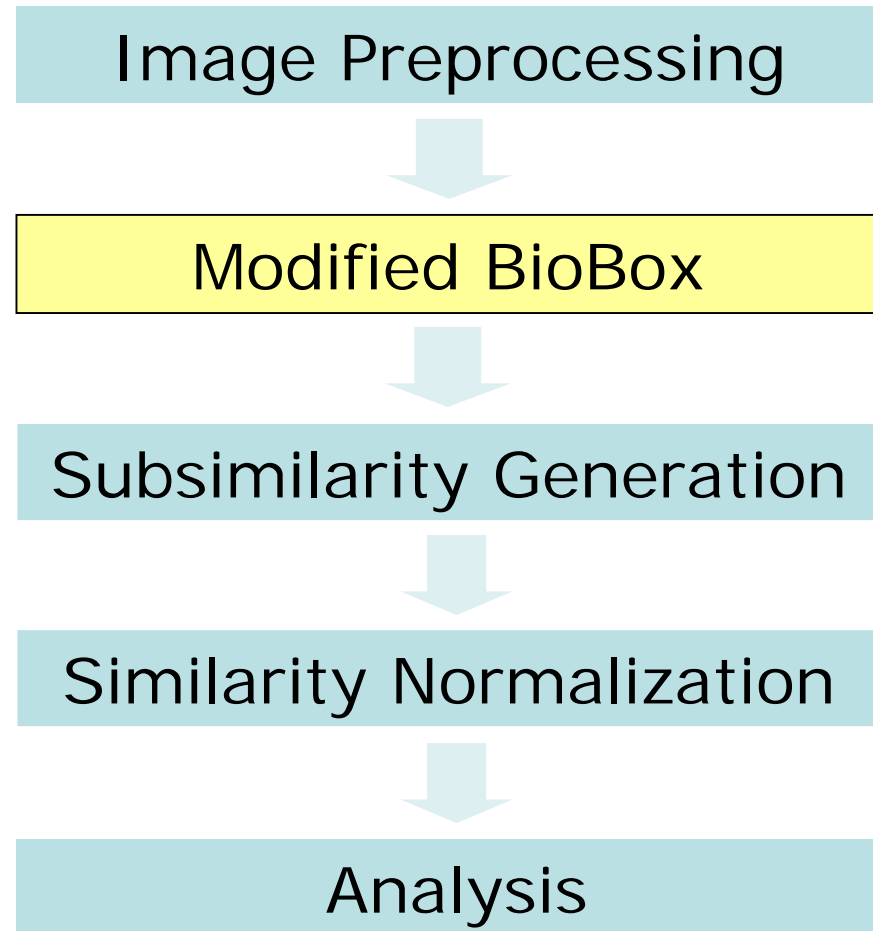
# Feature extraction on 3D surface



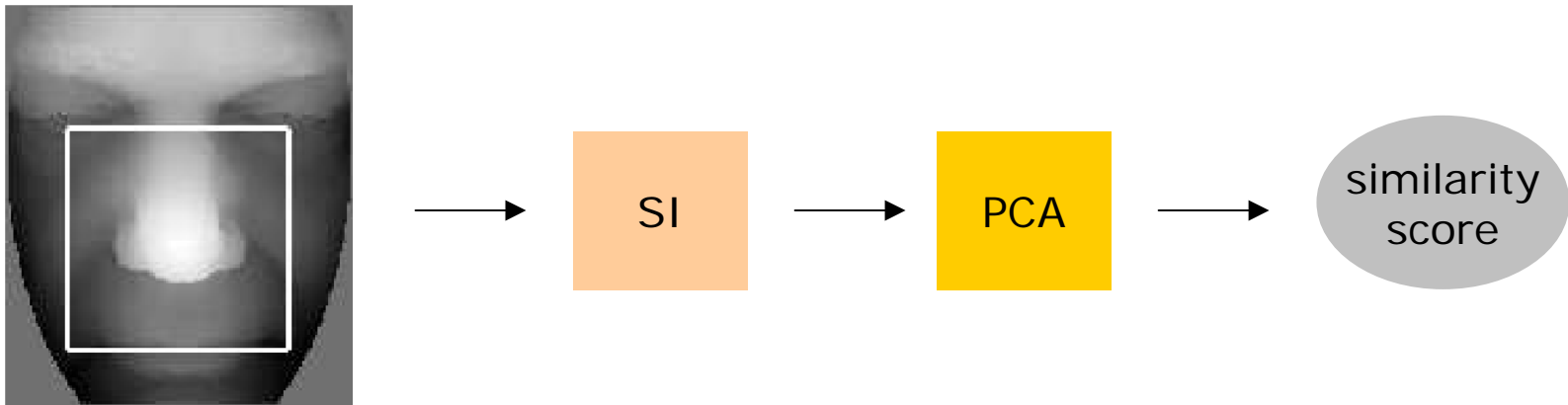
# Agenda

1. Introduction
2. Invariant Theory
- 3. *MSI Algorithm and Results***
4. Conclusion

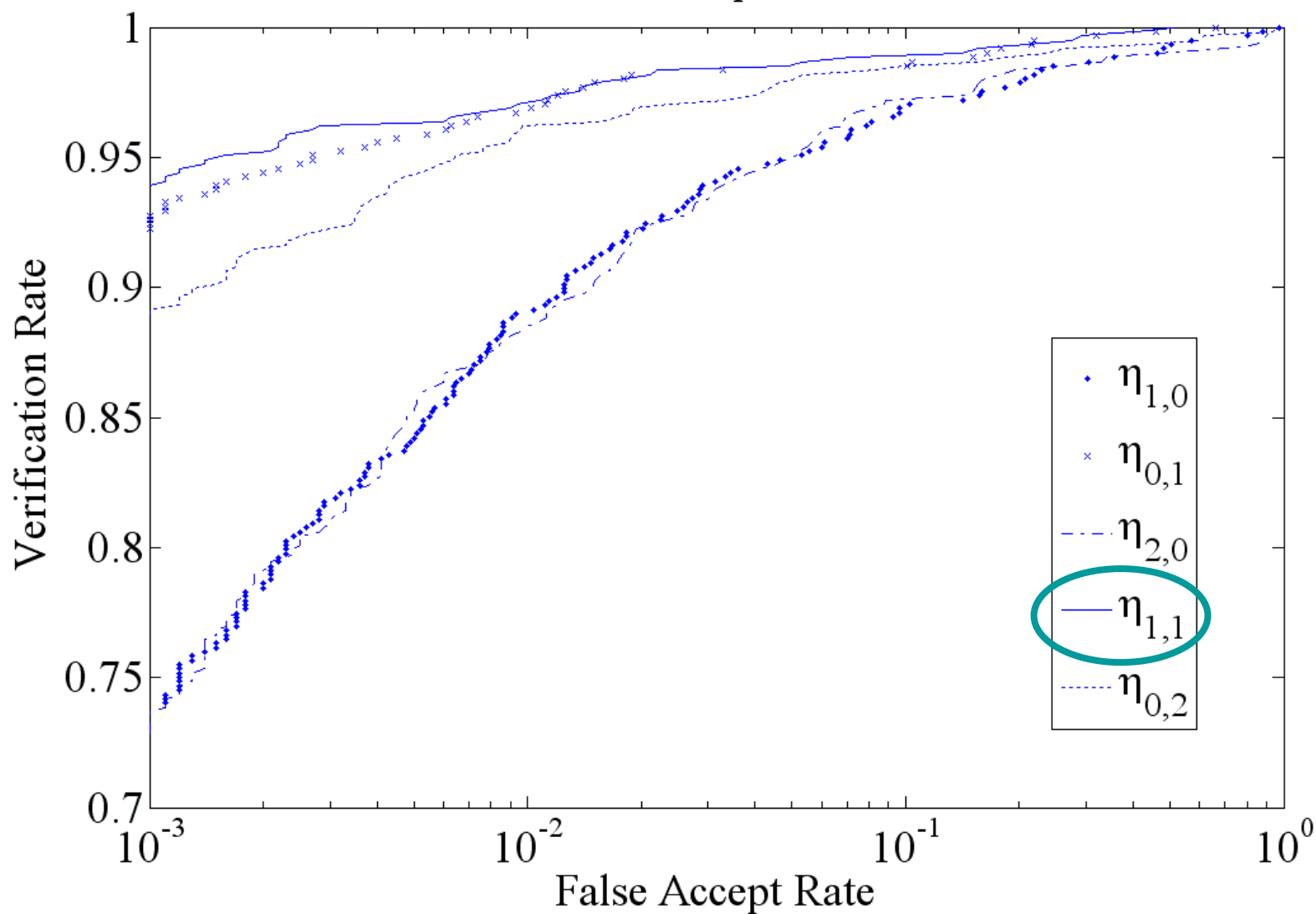
# Block diagram



# Single Region Algorithm

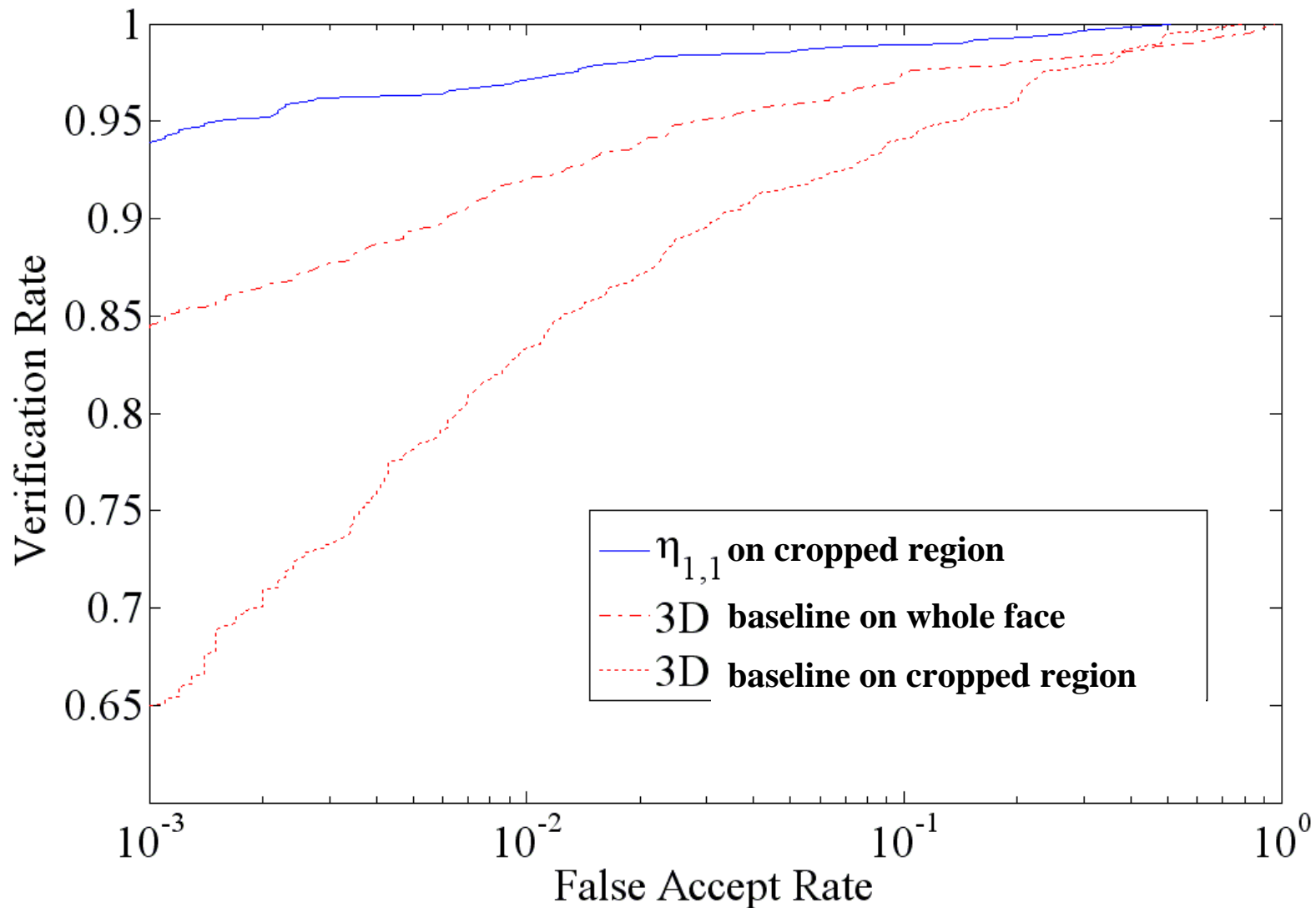


### FRGC 1.0 Experiment 3s



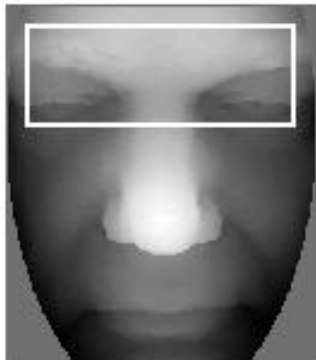


## FRGC 1.0 Experiment 3s : Comparison with baseline

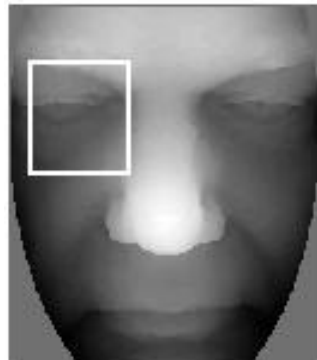


# Multi-Region Summation Invariants (MSI) Algorithm

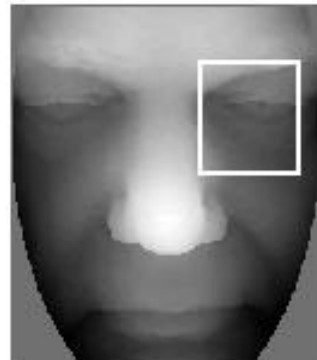
Region 1



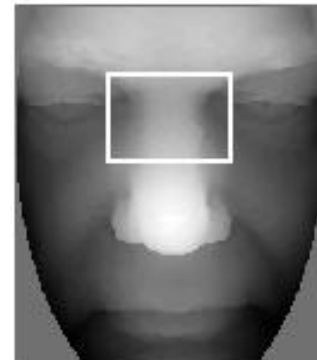
Region 2



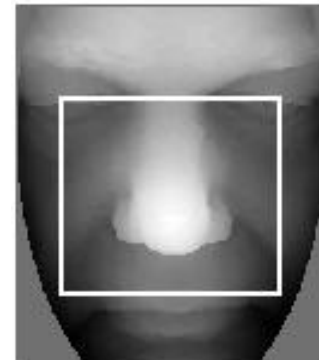
Region 3



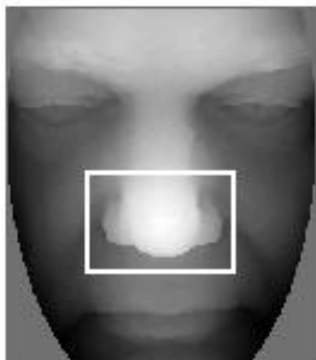
Region 4



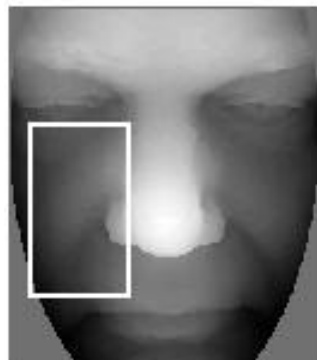
Region 5



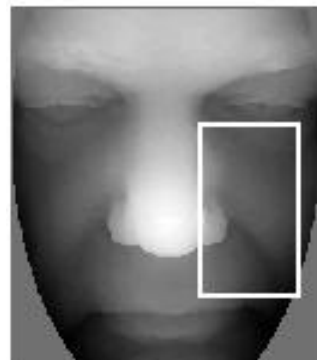
Region 6



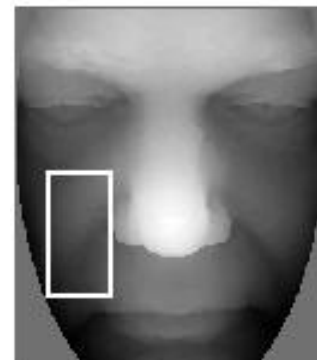
Region 7



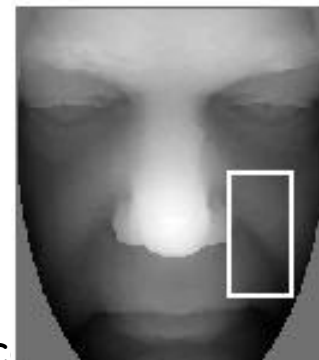
Region 8



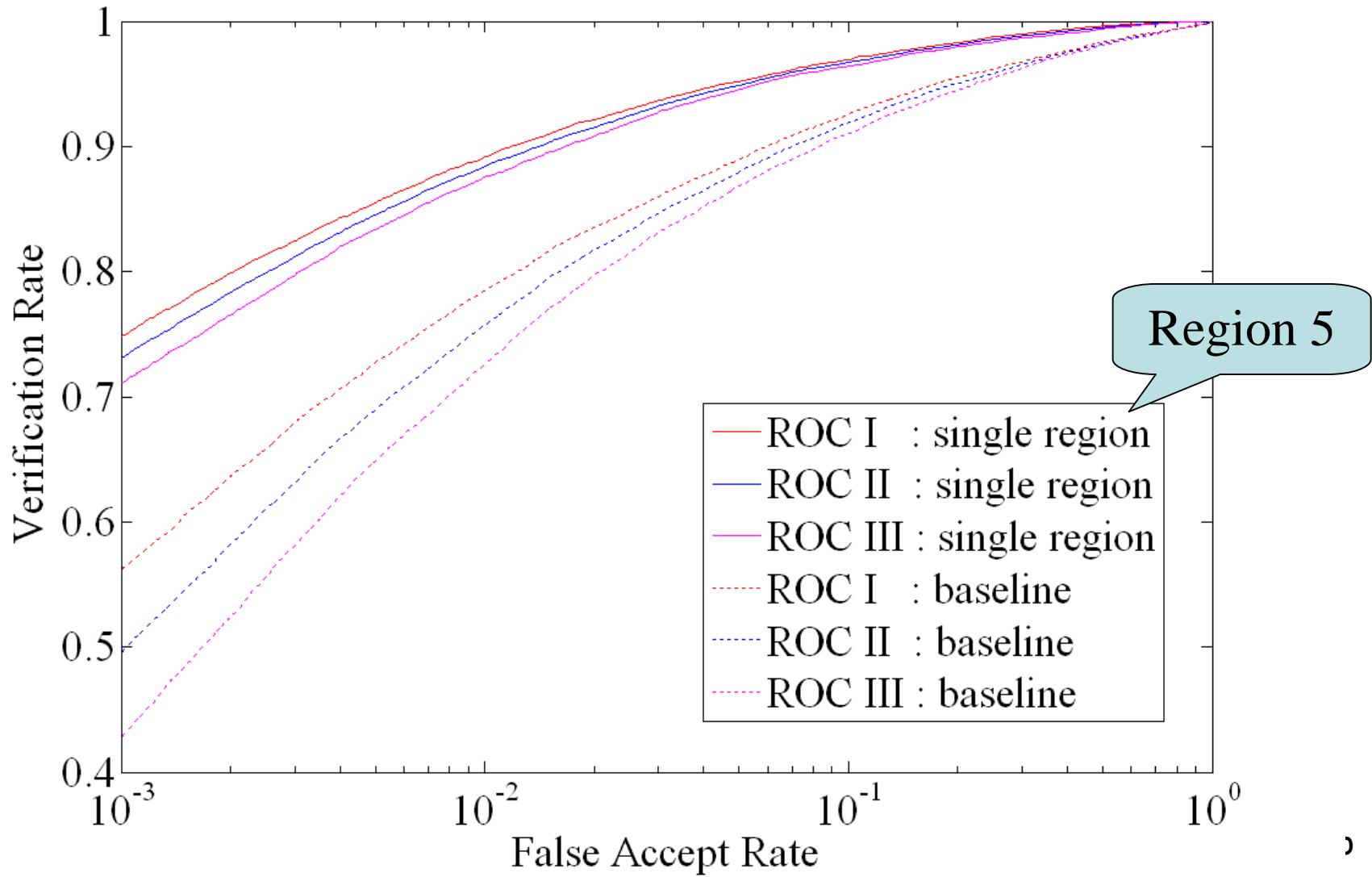
Region 9



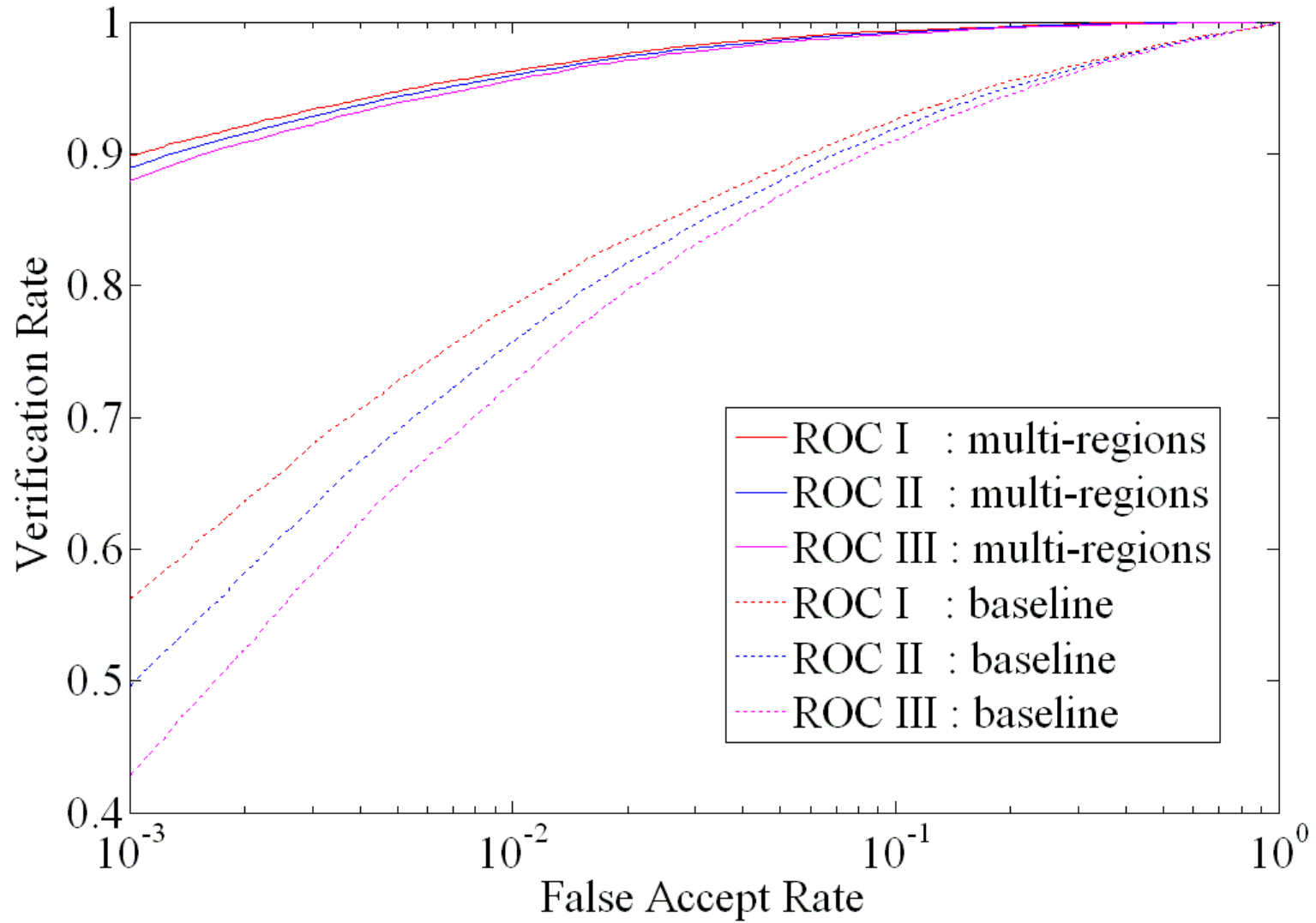
Region 10



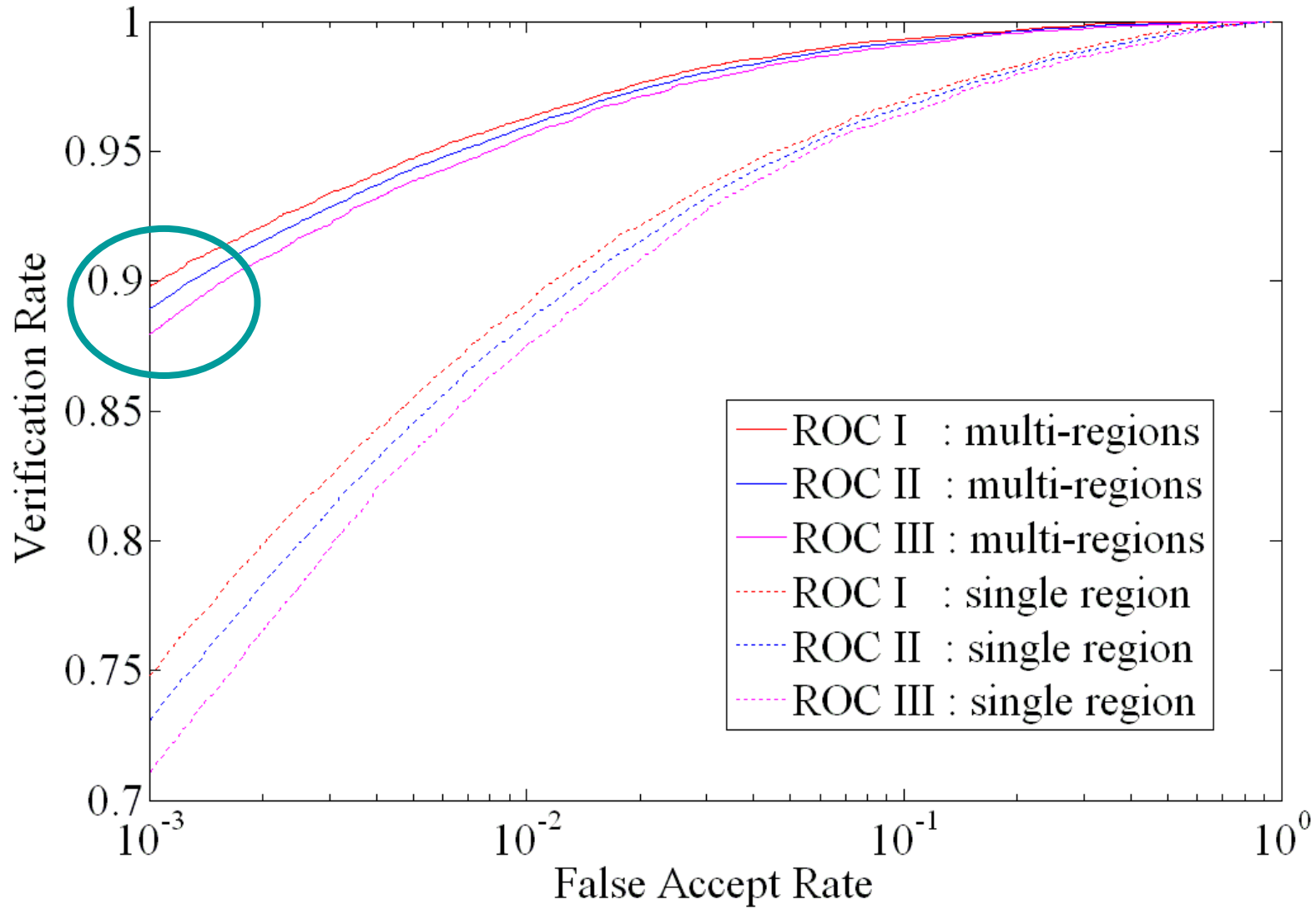
### FRGC 2.0 Experiment 3s : Single Region



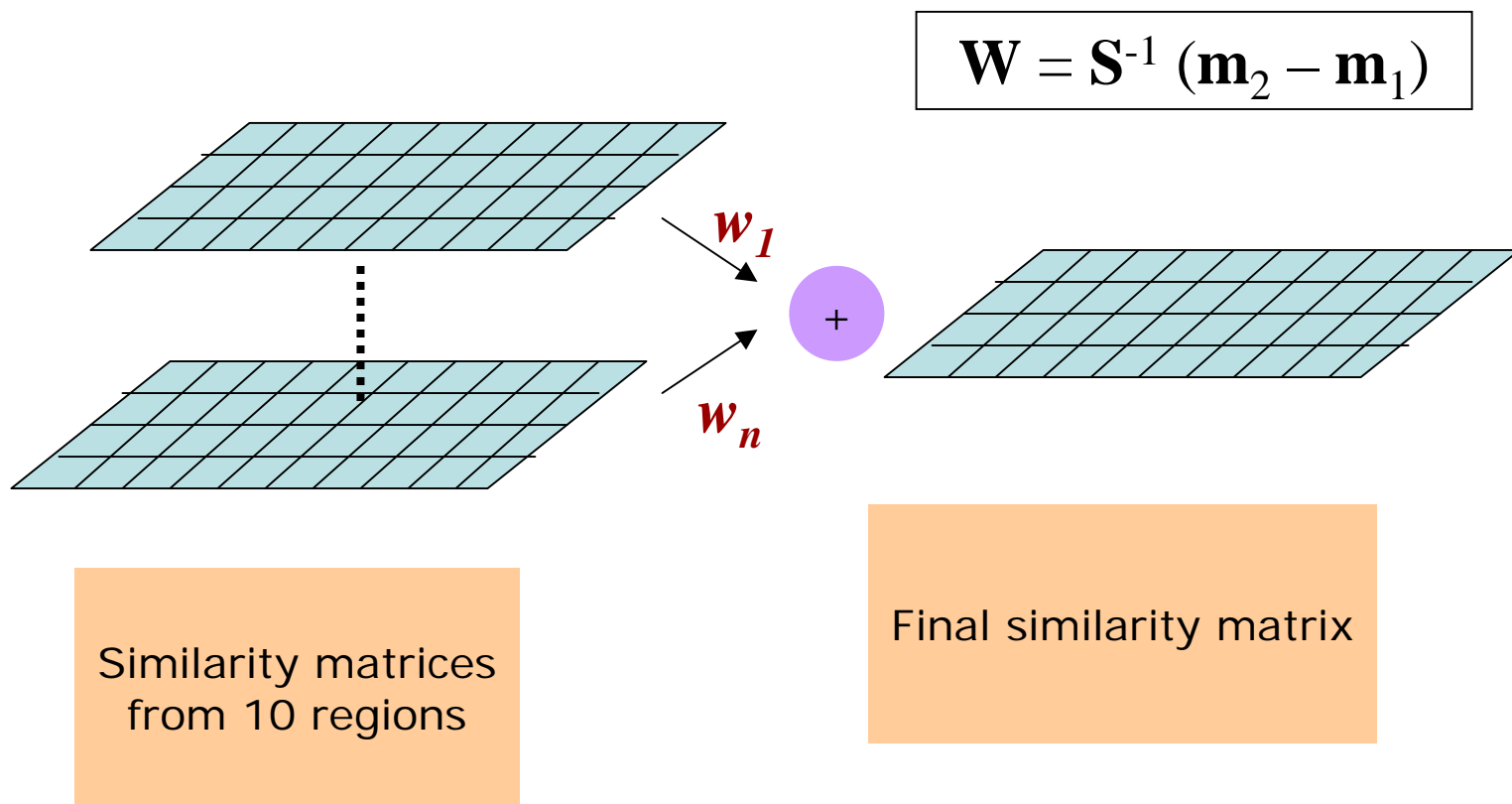
### FRGC 2.0 Experiment 3s : Multi-Regions



### FRGC 2.0 Experiment 3s : Multi-Regions vs. Single Region



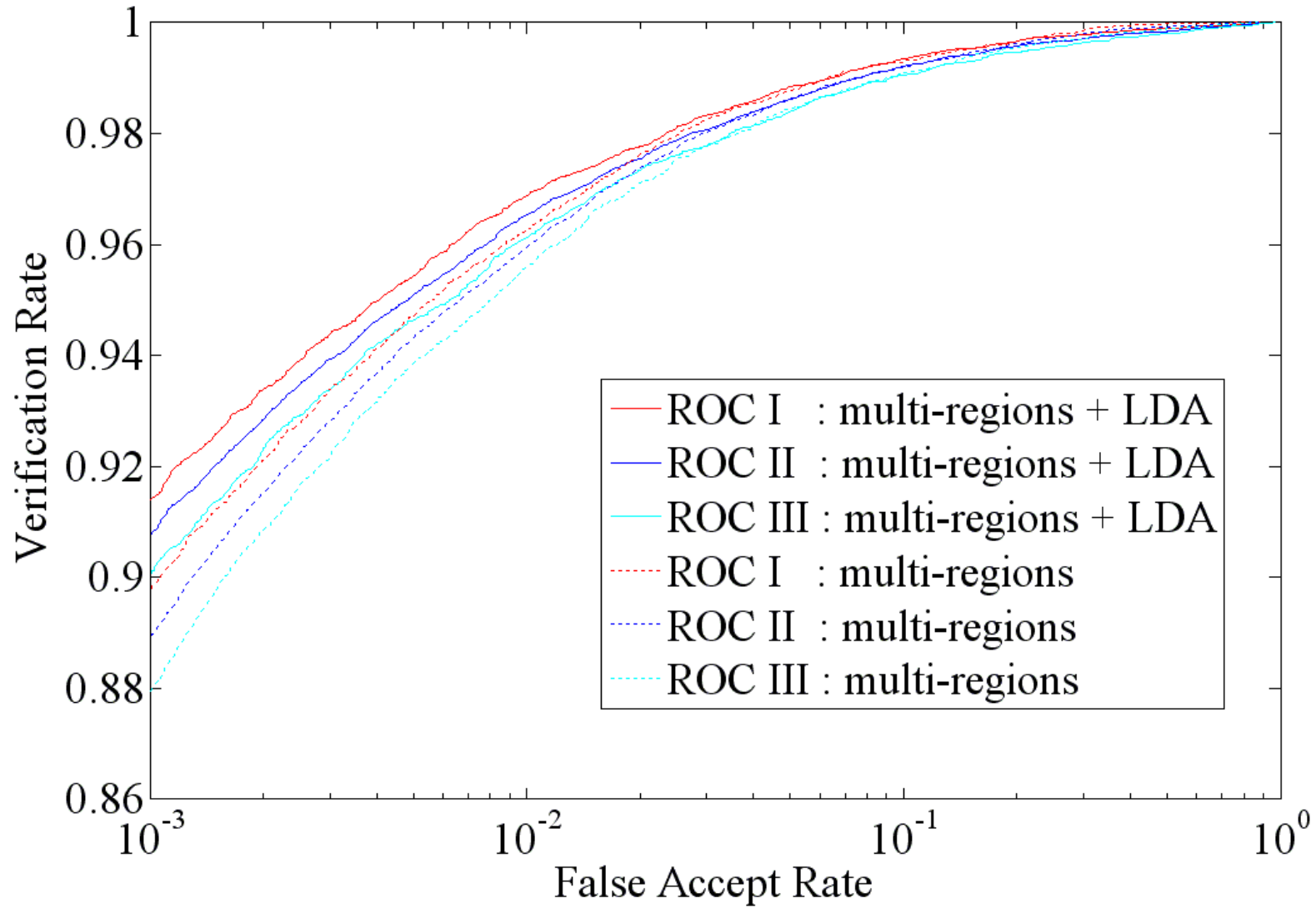
# Optimal Fusion with LDA



# LDA Fusion Results and Feature Selection

# of regions	Weight on each region										V.R. @ FAR = .001		
	#1	#2	#3	#4	#5	#6	#7	#8	#9	#10	ROC I	ROC II	ROC III
9 regions	.3586	.3222	.2665	.3661	.4849	.4008	.2396	.2749	.1831		<b>.9185</b>	<b>.9120</b>	<b>.9045</b>
8 regions	.3630	.3162	.2675	.3784	.4958	.4067	.2564	.2763			.9065	.8992	.8910
7 regions	.3708	.3125	.2709	.3759	.6225		.2881	.2819			.8867	.8751	.8614
6 regions	.3922	.2400		.3906	.6289		.2994	.3044			.9046	.8984	.8918
5 regions	.3980	.3736		.3923	.6679			.3192			.9086	.9034	.8983
4 regions	.4052	.3754		.3950	.7339						.8863	.8771	.8663
3 regions	.4692			.4374	.7671						.8450	.8302	.8133
2 regions	.5367				.8437						.8630	.8536	.8414
1 regions	1.0										.7591	.7429	.7242

FRGC 2.0 Experiment 3s : Multi-Regions + LDA vs. Multi-Regions





# Conclusion and Future Works

- Summation invariants
  - A systematical way to derive geometric invariants for pattern recognition
  - extract useful shape information
- Fusion of multiple regions
  - LDA can improve performance
- Future work
  - Apply SI on non-normalized shapes

Lin *et. al.*, “Fusion of Summation Invariants in 3D Human Face Recognition”, accepted to appear in CVPR’06