

Determination of the Time-Dependence of Ω_{NBS} Using the Quantized Hall Resistance

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Abstract—The quantum Hall effect is being used to monitor the U.S. legal representation of the ohm, or as-maintained ohm, Ω_{NBS} . Measurements have been made on a regular basis since August 1983. Individual transfers between the quantized Hall resistance R_H and the five 1- Ω resistors which comprise Ω_{NBS} can now be made with a total one standard deviation (1σ) uncertainty of ± 0.014 ppm. This uncertainty is the root-sum-square of 32 individual components. The time-dependent expression for R_H in terms of Ω_{NBS} is: $R_H = 25\,812.8[1 + (1.842 \pm 0.012) \times 10^{-6} + (0.0529 \pm 0.0040)(t - 0.7785) \times 10^{-6}/\text{year}] \Omega_{\text{NBS}}$, where t is measured in years from January 1, 1987. The value of Ω_{NBS} is, therefore, decreasing at the rate of (0.0529 ± 0.0040) ppm/year.

I. INTRODUCTION

IN THE integral quantum Hall effect [1], the Hall resistance $R_H(i)$ of the i th integer plateau of a fully quantized two-dimensional electron gas is given by

$$R_H(i) = \frac{V_H(i)}{I} = \frac{R_H}{i} = \frac{h}{e^2 i}. \quad (1)$$

Here $V_H(i)$ is the Hall voltage of the i th plateau, I is the current through the sample, and R_H is the quantized Hall resistance, equal to $h/e^2 \cong 25\,812.8 \, \Omega$ where h is the Planck constant and e is the elementary charge. For precision measurements, the quantum integer i is usually chosen to be either 2 or 4.

The quantized Hall resistance of a plateau at a temperature of absolute zero is an invariant of nature. Therefore, to the best of our knowledge, it can be used to determine the time-dependence of an artifact-based national representation of the unit of resistance. One such representation is the U.S. Legal Ohm, Ω_{NBS} , which is defined in terms of the mean resistance of five 1- Ω Thomas-type wire-wound resistors maintained in a 25°C oil bath at the National Bureau of Standards (NBS) in Gaithersburg, MD. Measurements of $R_H(4)$ in terms of Ω_{NBS} have been carried out on a regular basis since August 1983, yielding reliable values of $d\Omega_{\text{NBS}}/dt$ and $\Omega_{\text{NBS}}(t)$. This paper discusses these measurements.

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II. QUANTUM HALL EFFECT DEVICE PARAMETERS

Three GaAs/AlGaAs heterostructure devices have been used. They were designated as GaAs(7), GaAs(8), and GaAs(9). The GaAs/Al_xGa_{1-x}As heterostructures were grown by molecular beam epitaxy with $x = 0.29$. They were optimized for $R_H(4) = 6453$ 20- Ω steps. The centers of the $i = 4$ steps occur in the magnetic flux density range 5.6–6.0 T for the three devices; this corresponds to electron densities in the range 5.4 – $5.8 \times 10^{11} \text{ cm}^{-2}$. The zero magnetic field mobilities at 4.2 K were $\sim 100\,000 \text{ cm}^2/(\text{V} \cdot \text{s})$ for GaAs(7) and GaAs(8), and $\sim 75\,000 \text{ cm}^2/(\text{V} \cdot \text{s})$ for GaAs(9).

The Hall bar geometries of GaAs(7) and GaAs(8) were made by removing the heterostructure material by sand-blasting the area of the sample not covered by a metal mask. The resulting mesas were ~ 4 6-mm long and ~ 0.4 -mm wide. There were three sets of Hall potential probes placed along the mesa. The two outer probe sets were symmetrically displaced ± 1.0 mm along the channel from the center set. The GaAs(9) mesa was defined by photolithography. It was about one-half the size of GaAs(7) and GaAs(8).

Electrical contacts were made to the two-dimensional electron gases of the GaAs(7) and GaAs(8) samples by alloying indium into the heterostructures at 425°C for 5 min. Evaporated AuGeNi contact windows were used for GaAs(9). Gold wires with 25- μm diameters were soldered to the contacts of all three devices.

The samples were mounted on gold-plated, stainless steel, twelve pin T08 headers which plug into Textool sockets. All components of the sample probes (e.g., polytetrafluoroethylene (PTFE or Teflon)-coated wires, sockets, connectors, etc.) have leakage resistances that were greater than $10^{14} \, \Omega$.

III. QUANTUM HALL EFFECT MEASUREMENT PROCEDURE

We cooled the samples slowly, in the dark, from room temperature to 4.2 K. The cooling rate was adjusted such that the source-drain resistance R_{SD} decreased linearly with time over at least a 30-min time interval. This procedure avoided freezing the electrons into higher subbands or into the conduction band.

The samples were never exposed to cold air in order to minimize stresses on the ohmic contacts. Two-terminal contact resistances were measured at room temperature.

and at 4.2 K. No sample deterioration was observed over a five-year time period.

Two different Hall probe sets were used for each of the three samples, the center set, and an off-center set. These four potential points form a loop consisting of two pairs of V_H and two pairs of V_x contacts, where V_x is the voltage drop along the channel. The voltages summed to zero around the loop to within the measurement uncertainty.

The quantum Hall plateaus of all three samples were flat to within ± 0.01 ppm over a magnetic field range that was ~ 2 percent of the central field values when the devices were cooled to ~ 1.2 K. See [2, Fig. 3] for a digital mapping of a plateau.

The quantum Hall steps and V_x curves were plotted on a chart recorder at a 1-ppm accuracy level every day that R_H was measured. This ensured that the sample was still satisfactory and that we remained on the center of the step. We also measured the minimum value of the voltage drop along the channel V_x^{\min} each day that R_H was measured. The reason for this measurement will become clear in the next section.

IV. QUANTUM HALL RESISTOR TEMPERATURE AND CURRENT DEPENDENCIES

Many laboratories [3], [4] have verified that there is a temperature dependence of both the quantized Hall resistance R_H and V_x^{\min} . The dependence can be determined by measuring the correction to R_H for various values of V_x^{\min} for each Hall probe set and for both magnetic field directions. This temperature dependence may be mainly due to the finite widths of the Hall potential probes [5]. It also includes any effects due to conduction parallel to the two-dimensional electron gas.

The temperature-dependence corrections were intensively studied for GaAs (7) and GaAs (8), and adequately studied for GaAs (9). The corrections were reproducible over many different cool-downs from room temperature. The largest temperature-dependence correction ever required was (0.026 ± 0.002) ppm for one particular Hall voltage measurement at 1.2 K. The typical correction was (0.000 ± 0.002) ppm.

The sample current I must be in a range where the values of R_H and $R_x = V_x^{\min}/I$ are current-independent, and well below the critical breakdown current I_c [6], [7]. We used $I = 25 \mu\text{A}$ for all three samples and verified current-independence at $10 \mu\text{A}$. The critical currents of these samples were within the range 300–500 μA . The current-dependence corrections were negligible, and were assigned the value (0.000 ± 0.001) ppm.

V. 6453.20- Ω WIRE-WOUND REFERENCE RESISTORS

Four different 6453.20- Ω wire-wound reference resistors were used in comparisons with the quantized Hall resistances. Each reference resistor was made up of series and parallel combinations of Evanohm resistors wound on mica cards. The Evanohm resistors were hermetically sealed in silicone fluid-filled containers and were manufactured by Electro Scientific Industries. We then trimmed

the resistors to within a few parts per million of the value of R_H and placed them in portable NBS-built temperature-regulated air bath enclosures. The enclosures were controlled to within $\pm 0.002^\circ\text{C}$ at a nominal temperature of 27.4°C under constant ambient conditions and with no power load on the resistor. British Post Office (BPO) connectors were used as terminations to the resistors.

VI. QUANTUM HALL EFFECT MEASUREMENT SYSTEMS

Three different measurement systems were used to compare the value of $R_H(4)$ with that of the 27.4°C nominal temperature, 6453.20- Ω wire-wound reference resistors. The original measurement system was a manually-operated potentiometric comparator [2], [8]. The second quantum Hall effect (QHE) measurement system was an automated, guarded, modified Wheatstone bridge [9]. The third measurement system was an automated potentiometric comparator [10]. All three QHE measurement systems have been thoroughly examined and are in excellent agreement.

A. Random Measurement Uncertainties

The manually operated potentiometric comparator has a ± 0.011 -ppm random, or type *A*, uncertainty for a 1-h measurement period with a device current of 25 μA . The random uncertainties of the automated resistance bridge and the automated potentiometric comparator were both ± 0.006 ppm for the same measurement period. All three of these uncertainties were known with great confidence because they were each based on a set of about one thousand measurements.

A May 1988 comparison of R_H with the 6453.20- Ω wire-wound reference resistors is representative of recent measurements. The random QHE measurement uncertainty for a 4.3-h measurement period was ± 0.0029 ppm.

B. Systematic Measurement Uncertainties

There were three principal systematic, or type *B*, uncertainties associated with each of the three QHE measurement systems. They were due to leakage currents, detector gain instabilities, and interchange errors.

The leakage resistances of all three measurement systems were greater than $10^{12} \Omega$, and were humidity-dependent. The one standard deviation (1σ) uncertainty due to leakage currents was typically ± 0.006 ppm for the first two measurement systems, and ± 0.004 ppm for the automated potentiometric comparator.

There was an uncertainty in calibrating the gains of the electronic detector-digital voltmeter pair. The day-to-day gains of the Leeds and Northrup 9829 Linear Amplifiers varied by ~ 0.1 percent when the room temperature was controlled to $\pm 1^\circ\text{C}$. The standard-deviation of a set of detector gain measurements taken over a two-year period for the new potentiometric measurement system was ± 0.047 percent. Therefore, the uncertainty of its gain was about ± 0.001 ppm when the reference resistor values were 2–3 ppm different from R_H . The gain uncertainty was about ± 0.003 ppm for the old manual potentiometric sys-

TABLE I
ESTIMATED ONE STANDARD DEVIATION UNCERTAINTIES FOR THE MAY 10, 1988 STEP-DOWN TO Ω_{NBS} . THE TOTAL ROOT-SUM-SQUARE UNCERTAINTY IS ± 0.014 ppm FOR THIS STEP-DOWN

QHE Sample	
Temperature-Dependence (ppm)	Current-Dependence (ppm)
0.0020	0.0010

R_H to 6.453.2- Ω Measurements				
Measurement System	Type A Random (ppm)	Type B Uncertainties		
		Leakage (ppm)	Gain (ppm)	Interchange (ppm)
Potentiometric	0.0029	0.0050	0.0012	0.0041

6.453.2- Ω to 1- Ω Resistance Scaling Measurements								
Measurement System	Type A Random (ppm)	Temperature (ppm)	Pressure (ppm)	Type B Uncertainties				53 2- Ω (ppm)
				Power (ppm)	Leakage (ppm)	Leads (ppm)	Linearity (ppm)	
6.453 2- Ω	0.0046	0.0015	0.0010	0.0020	0.0030	0.0020	0.0020	0.0022
100- Ω	0.0043	0.0015	0.0010	0.0010	0.0010	0.0020	0.0020	
1- Ω	0.0026	0.0015	0.0020	0.0010	0.0010	0.0030	0.0020	

6.453.2- Ω Resistors			
Self-heating (ppm)	Type B Uncertainties		
	Drift (ppm)	Transport (ppm)	Temperature (ppm)
0.0050	0.0000	0.000	0.0000

tem. There were three detectors in the automated bridge system, and their gains were correlated. We estimated that the resistance bridge gain uncertainty was typically ± 0.011 ppm.

There was a correction due to a measurement system offset, or interchange error in which the value of R_H depended on whether it was measured in the quantized Hall resistance position of the measurement circuit or in the reference resistor position. The error was often larger than that expected from dc leakage current effects. It seemed to be independent of the detector input current. The correction could be determined by interchanging the positions of the quantum Hall device and the reference resistor. The error varied from day-to-day, so this interchange was done every day that R_H was measured. The position-dependent error was sometimes as large as (0.025 ± 0.016) ppm for the old potentiometric system and (0.019 ± 0.011) ppm for the resistance bridge. It had been as large as (0.013 ± 0.004) ppm for the new potentiometric system, but was typically (0.000 ± 0.005) ppm.

The type B QHE measurement uncertainties for the May 1988 comparison of R_H with the 6453.20- Ω wire-wound reference resistors were ± 0.0050 ppm for leakage currents, ± 0.0012 ppm for gain and linearity variations,

and ± 0.0041 ppm for interchange errors. The May 1988 QHE measurement uncertainties are listed in Table I.

VII. QUANTUM HALL EFFECT MEASUREMENTS

Fig. 1 shows comparisons of $i = 4$ quantized Hall resistances of the three GaAs devices against one of the nominally-valued 6453.20- Ω reference resistors during a 60 month time interval starting in May 1983. These data are independent of the Hall device, the Hall probe set, the magnetic field direction, and the quantum Hall effect measurement system.

This figure demonstrates the improvements in measurement accuracy. The manually-operated potentiometric comparator, which was used from May 1983 until May 1985, had an uncertainty of about ± 0.02 ppm. The automated bridge system was used from May 1985 until August 1986. It had an uncertainty of about ± 0.015 ppm. The automated potentiometric comparator had an uncertainty of about ± 0.007 ppm, and had been used since August 1986. The total root-sum-square uncertainty of the May 1988 QHE measurement uncertainties listed in Table I was ± 0.0072 ppm.

A weighted linear least squares fit, which takes into account the root-sum-square uncertainty of each measure-

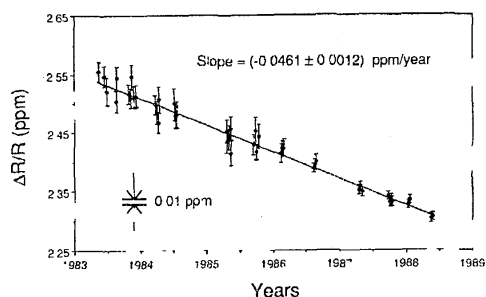


Fig. 1 Relative comparisons as a function of time of the resistance of the $i = 4$ steps of three different quantum Hall devices with that of a nominally-valued 6453.20- Ω wire-wound reference resistor R_R . $\Delta R/R = (V_H - V_R)/V_R$. The value of this particular reference resistor is increasing by (0.0461 ± 0.0012) ppm/year.

ment, shows that the resistance of this particular reference resistor is increasing at a rate of (0.0461 ± 0.0012) ppm/year. This unusually small and linear drift rate $(0.000\ 126 \pm 0.000\ 002\ 7)$ ppm/day enables us to continuously monitor the reliability of the three QHE measurement systems. The values of the other three 6453.20- Ω reference resistors are also drifting linearly with time, but with drift rates that are between two and three times larger.

VIII. HAMON TRANSFER STANDARDS FOR STEP-DOWNS FROM THE 6453.20- Ω REFERENCE RESISTORS TO Ω_{NBS}

In order to monitor the NBS ohm, the nominally-valued 6453.20- Ω reference resistors are calibrated in terms of the set of five 1- Ω resistors which define Ω_{NBS} . This is done in two stages: the first stage uses a 6453.20- Ω to 100- Ω series/parallel Hamon network [11] configuration consisting of eight 800- Ω resistors plus a series connected 53.2- Ω resistor; the second stage uses a 100- to 1- Ω Hamon network consisting of ten 10- Ω resistors.

Four NBS-built Hamon transfer standards are used in the resistance scaling process. Hamon HQHA is the primary transfer standard for scaling from 6453.20 to 100 Ω . It contains nine series connected card-type resistors sealed in an aluminum box filled with silicone fluid. The first eight resistors have a nominal value of 800 Ω and the ninth has a value of 53.20 Ω to make the total 6453.20 Ω . The eight 800- Ω resistors are connected in parallel for the 100- Ω measurements. BPO connectors are used for the terminations. The shields of these connectors are isolated from the metal case so that they can be driven by a guard potential in order to reduce measurement errors resulting from leakage currents. HQHA has a drift rate of ~ 1.0 ppm/year.

Hamon HQHB is the check standard for scaling from 6453.20 to 100- Ω . It is similar in construction to HQHA. The resistors in HQHB were overheated because of a defective oven during its construction phase, and, as a result, the corrections of all the resistors shifted ~ 200 ppm. It is not as stable as HQHA, with a drift rate of ~ 2.0 ppm/year.

Hamon H10A is the primary transfer standard for scaling from 100 to 1 Ω . It is constructed using ten 10- Ω resistance elements of the Rosa design. The resistance elements are individually sealed in brass cans filled with a silicone heat sink compound. The terminations of H10A are mercury-wetted contacts. The connectors for the parallel configuration consist of: 1) low-resistance amalgamated copper shorting bars for the current terminations, and 2) one of two fixtures having fan resistances of either 1 or 10 Ω for the potential terminations. Hamon H10A exhibits good stability, with a drift rate of ~ -0.18 ppm/year.

Hamon H10B is the check standard for scaling from 100 to 1 Ω . It is constructed using ten card-type 10- Ω resistors. The resistors are sealed in a thick-walled aluminum box filled with silicone fluid. The terminations for H10B are BPO connectors. Special BPO fixtures were constructed for connecting the resistors in the parallel mode.

IX. RESISTANCE MEASUREMENT SYSTEMS FOR STEP-DOWNS

Four measurement systems were used in the resistance scaling process: the 1:1 measurements at the 1- Ω level were done using the automated NBS 1- Ω direct-current comparator system [12]. H10A and H10B were compared to the NBS reference group which comprises the U.S. Legal Ohm Ω_{NBS} and to two control resistors that were stored in Oil Bath I. The temperature of the oil bath was maintained at $25.000 \pm 0.003^\circ\text{C}$. The oil bath temperature, as well as the ambient barometric pressure, was monitored during a measurement run. The Ω_{NBS} consists of five Thomas-type resistors that were constructed in 1933. All 1- Ω resistors were measured at a current level of 100 mA (10 mW/resistor). Both H10A and H10B were situated in Oil Bath II. Either one could be connected to the measurement system in Oil Bath I via a four-connector, shielded, PTFE-insulated cable.

The measurements at 100 Ω were done using an automated direct-current comparator system similar in design, construction, and operation to the 1- Ω measurement system. The 100- Ω system was located in Bath III. A four conductor cable from Bath III to Bath II connected either H10A or H10B to the 100- Ω measurement system. Another cable from Bath III to Bath IV provided for the 100- Ω measurements of either HQHA or HQHB, which were located in Bath IV. Two Hamons, e.g., H10A and HQHA, could be compared during a measurement run. As an additional check, two standard resistors and one control resistor were also measured during a test run. All 100- Ω resistors were measured at a current level of 10 mA (10 mW/resistor).

Measurements at the 6453.20- Ω level were made using a guarded, resistance-ratio bridge. The adjustable part of the bridge was a modified, commercial, direct-reading, double-ratio set situated at Bath IV. The Hamon standards and a 6453.20- Ω secondary resistor were located in the oil

bath. A battery power supply and an electronic null detector completed the bridge circuit. The 6453.20- Ω resistors were measured at a current level of 1.25 mA (~ 10.1 mW/resistor).

The 53.20- Ω measurement was not very critical since it only represented ~ 0.82 percent of the total resistance of 6453.20 Ω . Usually this measurement was made using an automatic NBS 0–100- Ω resistance thermometer bridge [13]. This bridge was calibrated at the 100- Ω level before and after the 53.20- Ω measurements. In the past, the results from this measurement system were compared to the results obtained using a dc current comparator resistance bridge. The results were in agreement to within ± 0.004 ppm of 6453 20- Ω .

A. Measurement Uncertainties for Resistance Step-Downs

There were seven uncertainties for each of the measurements at the 1-, 100-, and 6453.20- Ω levels of a step-down: one random or type-*A* uncertainty and six systematic or type-*B* uncertainties, for a total of 21 uncertainties. The type-*B* uncertainties were due to: temperature variations, pressure fluctuations, power coefficients, leakage currents, connection resistances, and linearity problems. In addition, there was one other type-*B* uncertainty, that for calibrating the 53.20- Ω resistor. The 22 resistance measurement uncertainties for the May 1988 step-down to Ω_{NBS} are listed in Table I. The root-sum-square resistance measurement uncertainty is ± 0.0105 ppm for that step-down. It has typically been ± 0.011 ppm since September 1985, and was ± 0.018 ppm before that.

These step-down uncertainties are much smaller than the ± 0.044 ppm originally assigned in the past [2], [8]. That uncertainty was a very preliminary, highly conservative number which was always expected to be reduced significantly upon the completion of a thorough but realistic evaluation of the step-down procedure as has now been done in connection with the preparation of this paper.

One check of the accuracy of the step-down procedure was to replace the two primary Hamon transfer standards with the two check standards. They were in complete agreement, well within the ± 0.011 -ppm uncertainty assigned to the NBS process. Another check tested the complete scaling procedure by replacing the two primary Hamon transfer standards with the 6453.20- to 1- Ω Hamon transfer standard that was designed, constructed, and used at the National Measurement Laboratory (NML) in Australia [14]. This Hamon network is comprised of eighty-three 80.333- Ω resistors. The series configuration consists of eighty resistors in series with the remaining three in parallel. The parallel configuration uses the same eighty resistors in parallel and the same three resistors in series. The difference [15] between the NBS and NML methods of scaling from the 6453.20- Ω level to the 1- Ω level was again within the ± 0.011 ppm uncertainty assigned to the NBS process.

X. 6453.20- Ω REFERENCE RESISTOR POWER COEFFICIENTS AND TRANSPORT UNCERTAINTIES

The current used in the step-downs was 1.25 mA (10 mW) for the 6453.20- Ω reference resistors, whereas it was 25 μ A in the QHE resistance comparisons. This additional current caused a self-heating effect in the reference resistors. The reference resistors were maintained in constant-temperature air baths, which enhanced the self-heating effect. At higher currents, the self-heating increased the temperature of the silicone fluid surrounding the reference resistors, and therefore, changed the values of the resistors.

In the past this self-heating effect was measured indirectly, and a ($+0.02 \pm 0.02$) ppm correction was applied to the values of the 6453.20- Ω reference resistors in the step-down procedure. We have recently directly measured the self-heating effects of the reference resistors, and can now apply a correction to within a ± 0.005 -ppm uncertainty for all previous measurements. The typical correction is now ($+0.009 \pm 0.005$) ppm.

The values of the reference resistors change with time in a very predictable way, e.g., see Fig. 1. We, therefore, corrected for the small changes in resistance due to the few days difference between the mean dates for the QHE measurements and the resistance scaling measurements. The correction was ± 0.0020 ppm for the May 1988 step-down, with a negligible uncertainty.

The values of the reference resistors could shift during transport between the QHE and the resistance scaling laboratories. Therefore, the reference resistors were inter-compared (using the QHE measurement system) both before and after transport. Transport shifts could be detected if they were larger than the random uncertainty of the QHE measurement system. If, as happened on one occasion, a transport shift was detected, that step-down measurement was rejected. We assigned no uncertainties (other than the QHE random uncertainty) to successful transports.

The air bath temperature of the reference resistors could be different in the QHE and in the resistance scaling laboratories. We, therefore, monitored this temperature, and applied a correction if necessary. No correction was necessary, but there was an uncertainty due to reading the temperature. This uncertainty was ± 0.010 ppm from 1983–1984, ± 0.003 ppm from 1985–1986, and has been negligible since. The 6453.20- Ω reference resistor uncertainties for the May 1988 step-down are also listed in Table I.

XI. STEP-DOWNS TO Ω_{NBS}

Measurements involving the entire sequence (quantized Hall resistance comparisons with nominally-valued 6453.20- Ω reference resistors and then step-downs to Ω_{NBS}) were made on eleven occasions over a 57 month interval commencing in August 1983. The data are listed in Table II. The total 1- σ root-sum-square uncertainty has

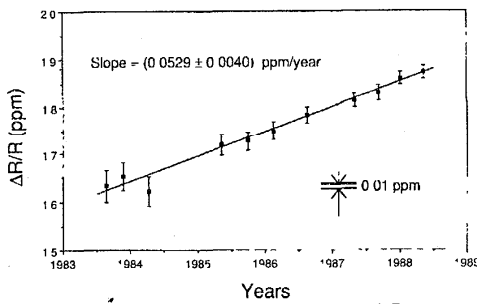


Fig. 2 Monitoring as a function of time the value of R_H expressed as a difference in ppm from a reference value of $25\,812.8\,\Omega_{\text{NBS}}$. $\Delta R/R = (R_H - 25\,812.8\,\Omega_{\text{NBS}})/25\,812.8\,\Omega_{\text{NBS}}$. These data indicate that the U.S. Legal Ohm, Ω_{NBS} , is decreasing by (0.0529 ± 0.0040) ppm/year.

TABLE II

RESULTS OF THE STEP-DOWN TO THE U.S. LEGAL OHM, $R_H \rightarrow \Omega_{\text{NBS}}$. THE VALUES ARE EXPRESSED AS THE DIFFERENCE IN ppm FROM $25\,812.8\,\Omega_{\text{NBS}}$. THE ROOT-SUM-SQUARE TOTAL UNCERTAINTIES ARE AT THE ONE STANDARD DEVIATION (68 PERCENT) CONFIDENCE LEVEL.

Mean Date	Value (ppm)	Total Uncertainty (ppm)
22 August 1983	1.633	0.033
22 November 1983	1.652	0.029
6 April 1984	1.622	0.030
8 May 1985	1.719	0.023
25 September 1985	1.727	0.020
13 February 1986	1.749	0.019
13 August 1986	1.782	0.017
28 April 1987	1.814	0.015
9 September 1987	1.830	0.016
5 January 1988	1.860	0.013
10 May 1988	1.873	0.014

improved by a factor of 2.4 over this period. It was ± 0.014 ppm for the May 1988 measurement. This total uncertainty includes the QHE random measurement uncertainty, the three resistance scaling random uncertainties, and the 28 type-B systematic uncertainties described in the sections above, for a total of 32 type-A and type-B uncertainty components.

Fig. 2 shows the results of these measurements. The Ω_{NBS} is decreasing with time. A weighted linear least squares fit which takes into account the correlations between the uncertainties [16] yields a drift rate of $d\Omega_{\text{NBS}}/dt = -(0.0529 \pm 0.0040)$ ppm/year. The value $d\Omega_{\text{NBS}}/dt = -(0.0632 \pm 0.0043)$ $\mu\Omega$ /year, which we obtain from the results of international comparisons of 1- Ω resistance standards with the National Measurement Laboratory of Australia (NML) through the International Bureau of Weights and Measures (BIPM) over the period 1973–1987, is not inconsistent with this result. (The NML ohm representation is based on periodic calculable capacitor realizations of the ohm and thus is time-independent.)

The time-dependent expression for R_H in terms of Ω_{NBS} is:

$$R_H = 25\,812.8[1 + (1.842 \pm 0.012) \times 10^{-6} + (0.0529 \pm 0.0040)(t - 0.07785) \times 10^{-6}/\text{year}] \Omega_{\text{NBS}} \quad (2)$$

where t is measured in years from January 1, 1987. Note that: 1) because a correlated-error least squares fit was used, the intercept and slope uncertainties represent their respective total uncertainties, and 2) equation (2) has been written in such a way that these two uncertainties are uncorrelated (i.e., the resulting 2×2 error matrix of the least squares fit has been diagonalized). This is the reason for the factor $(t - 0.07785)$ in the equation. For January 1, 1987 (i.e., $t = 0$) we find from (1) that $R_H = 25\,812.8[1 + (1.801 \pm 0.012) \times 10^{-6}] = 25\,812.84\,649(31)\,\Omega_{\text{NBS}}$. The least squares fit to the eleven data points of Fig. 2 is quite satisfactory. The χ^2 is 6.6, well below the expected value of 9.0. The uncertainties given in (1) have not been reduced by the implied multiplicative scale factor $[\chi^2/9.0]^{1/2} = 0.86$.

We shall continue to carry out these measurements to monitor the time-dependence of Ω_{NBS} in anticipation of the world-wide adoption beginning January 1, 1990 of a recommended value of R_H for the purpose of basing a reproducible representation of the ohm on the quantized Hall effect.

XII. SI REPRESENTATION OF THE OHM

These measurements express R_H in terms of Ω_{NBS} . The NBS calculable capacitor experiment [17], [18] can be used to realize the SI representation of the ohm, and thus the SI value of R_H , and a value of the fine-structure constant, α . The SI value of R_H and a value of α can also be derived from the determination, in NBS electrical units, of the proton gyromagnetic ratio by the low-field method $\gamma'_p(\text{low})$ [19] and the Josephson frequency-to-voltage quotient, $2e/h$ [20]. These results are described in [18]–[21].

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